ANSWERS CHAPTER 5

THINK IT OVER

think it over

TIO 5.1 Pizza $\in\{F\}$

$A \cap F$ means items that are both frozen and alcoholic. Normally alcoholic items are not frozen unless you can invent alcoholic ice cream or lollies – there's a thought!

$A \cup F$ means every item in the shopping.

TIO 5.2: This situation is known as a mathematics paradox. The barber cannot belong to the set 'men who shave themselves and not by the barber' because he is the barber. Similarly, he cannot belong to the set 'men who don't shave themselves' because he does shave himself! This scenario destroyed the life's work of a mathematician called Gottlob Frege. Frege wanted to make all mathematics based on set theory and Bertrand Russell proposed this paradox (in mathematical terms) with the result that Frege's work had to be modified.

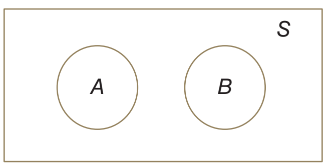
TIO 5.3: It will not make any difference to your chances of winning whichever method you choose. The winning numbers are drawn at random.

It does not make any difference how many people play and you cannot predict how many winners there will be.

TIO 5.4: It cannot predict precisely. You could use it as a guide, but when you are dealing with people, conditions can never be precisely the same. We are all different and a good job too!

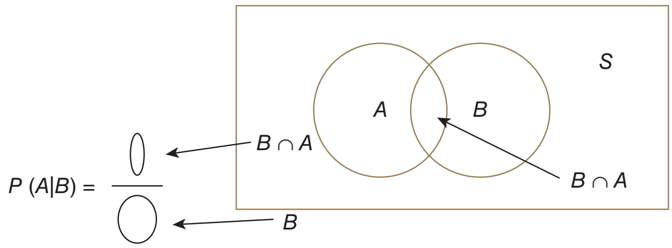
TIO 5.5: This is known as the 'empty set' and the symbol $\emptyset$ is used to denote it. Even though it does not have any members, it is still treated as a set.

$P(A\cup B) = P(A) + P(B)$. What we can say is $P(A\cap B) = P(A)\,P(B)$



Independent. The probability of *A* occurring is not affected by the probability of *B* occurring or not occurring. Independence is a very important concept in statistics.

TIO 5.6: By definition $P(A|B) \equiv \dfrac{P(B\cap A)}{P(B)}$. $P(A \mid B)$ means the probability of *A* occurring given that *B* has occurred and since *B* has occurred, it becomes the new sample space replacing the original one, *S*.



EXERCISES

1. (a) 1/16.
   (b) 1/32.

2. (a) False. Equally likely outcomes. See representativeness heuristic for psychological explanation.
   (b) Equally probable head or tail.
   (c) 1/64.

3. (a) Yes.
   (b) One method could be Bayes' Theorem.

   You might think that, with two doors left unopened, you have a 50:50 chance with either door, and so there is no point in changing doors. However, this is not the case. Let us call the situation that the prize is behind a given door A_r, A_g, and A_b. To start with, $P(A_r)=P(A_g)=P(A_b)=\frac{1}{3}$, and to make things simpler we shall assume that we have already picked the red door. Let us call B 'the presenter opens the blue door'. Without any prior knowledge, we would assign this a probability of 50%.

   - In the situation where the prize is behind the red door, the presenter is free to pick between the green or the blue door at random. Thus, $P(B|A_r) = 1/2$.
   - In the situation where the prize is behind the green door, the presenter must pick the blue door. Thus, $P(B|A_g) = 1$.
   - In the situation where the prize is behind the blue door, the presenter must pick the green door. Thus, $P(B|A_b) = 0$.

$$P(A_r \mid B)=\frac{P(B \mid A_r)P(A_r)}{P(B)}=\frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}}=\frac{1}{3}$$

$$P(A_g \mid B)=\frac{P(B \mid A_g)P(A_g)}{P(B)}=\frac{1.\frac{1}{3}}{\frac{1}{2}}=\frac{2}{3}$$

$$P(A_b \mid B)=\frac{P(B \mid A_b)P(A_b)}{P(B)}=\frac{0.\frac{1}{3}}{\frac{1}{2}}=0.$$

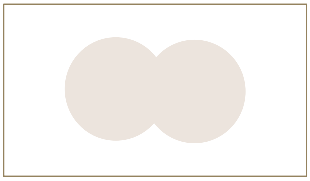
4. (a) 2/5.
   (b) 4/15.
   (c) 1/3.
   (d) 3/5.
   (e) 2/3.

5. They are all equal order and repetition does not change a set.

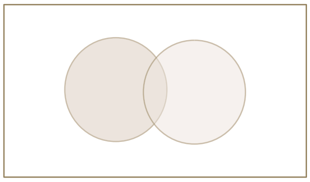
6. (a) {1,2,3,4,5,6,7}.
   (b) {4,5}.
   (c) $\{1,2,3,4,5,6,7,8,9\} = U$.
   (d) {5}.

7. (a) {5,6,7}.
   (b) {4,5,6,7,8,9}.
   (c) {4,5,6,7,8,9}.

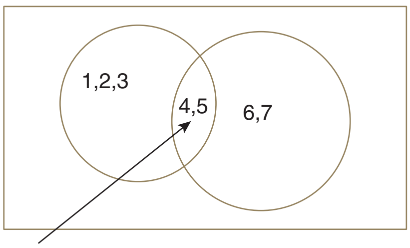
8. (a)



   (b)



9. (a)



   (b)

10. (a) 0.2.
    (b) No, since $P(A) \neq P(A|B)$.
    (c) 0.8.