ANSWERS CHAPTER 6

THINK IT OVER

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TIO 6.1: Random implies the number is not specifically chosen. Variable implies the number can have any value. Therefore, random variable implies the number is not chosen and can have any value.

TIO 6.2: In theory there is no smallest number. A smaller number can always be found.

TIO 6.3: The expected value of the variable x is equal to adding together all of the values found when x is multiplied by the probability of x occurring.

TIO 6.4: Bi means two and nomial means number, i.e. two numbers. The binomial distribution only deals with two outcomes.

TIO 6.5: Ideally assumptions should be based on past experiences. Sometimes this is not possible so a 'best guess' is made which should be made after some investigation into situations which are similar but not identical.

TIO 6.6: Using $\pi^x(1-\pi)^{(n-x)}$ where $x = 2$, $\pi = 0.15$, $n = 5$, 0.15^2 (1–015)$^{5-2} = 0.0138$

TIO 6.7: 0.2% chance of selling three boards. Where have you gone wrong in your calculation?

You forgot to include $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ in your calculation. It should have been:

$10 \times 0.003375 \times 0.7225 = 0.0244$ which equates to a 2.4% probability of making a sale.

TIO 6.8: $E(x)$ = mean = median = $0.5(a + b)$.

TIO 6.9: Good places for coffee shops are where a lot of people congregate: shops, theatres, attractions, etc., so you can expect more coffee shops in these prime locations. Also, there are only so many customers in a particular location, so too many coffee shops reduces the number of customers per shop and hence reduces the profitability of the business. You have gone wrong in assuming that the locations of the other coffee shops are an independent process. The location of one coffee shop will be affected by the proximity of the others.

TIO 6.10: They are the same.

TIO 6.11: If the value of the standard deviation is relatively high compared with the value of the mean, the normal distribution will be 'flatter' (to use the technical word, the curve is platykurtic). If the value of the standard deviation compared with the value of the mean is low, the distribution will be 'peaky' (leptokurtic if you want the technical term). Software such as SPSS can give you values for kurtosis and skewness, but they require careful consideration.

TIO 6.12: If we standardise our data we can compare different samples from possibly different populations. By standardising we effectively remove the possibility of misinterpretation when comparing different distributions.

EXERCISES

1. $E(X) = (0)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (0)\left(\frac{1}{6}\right) + (40)\left(\frac{1}{6}\right) + (0)\left(\frac{1}{6}\right) + (-30)\left(\frac{1}{6}\right) = 5.$

2. $P(1\text{boy}) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{1}{2}$ $\qquad$ $P(2\text{boys}) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$P(1\text{boy}) = \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{1}{4}$ $\qquad$ $P(1\text{boy}) = \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$

$P(\text{at least 1 boy}) = P(1\text{boy}) + P(2\text{boys}) + P(3\text{boys}) + P(4\text{boys})$

$= \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16} = \frac{15}{16}.$

(b) $P(\text{at least 1 boy and at least 1 girl}) = 1 - P(\text{no boys}) - P(\text{no girls}) = 1 - \frac{1}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}.$

3. Let X denote the number of people who suffer a bad reaction. Assume X is Poisson distributed.

$$P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$$

where $\mu = np = (2000)\,(0.001) = 2$

(a) $P(X = 3) = \frac{2^3 e^{-2}}{3!} = 0.18.$

(b) $P(X > 2) = 1 - \left[P(X = 0) + P(X = 1) + P(X = 2)\right]$

$$= 1 - \left[\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right]$$

$$= 1 - 5e^{-2} = 0.323.$$

4. In this example a success is deemed to be if the inspector finds a defective saucepan ($x = 1$).

The inspector is randomly selecting 3 of the 12 saucepans, in stats speak he performs three trials ($n = 3$).

The number of defective saucepans in the population of 12 ($N = 12$) is 5 and since a success is finding defective saucepans, $r = 5$.

$$r = 5,\ x = 1,\ N = 12,\ n = 3$$

$$p(x) = \frac{\binom{5}{1}\binom{7}{2}}{\binom{12}{3}} = \frac{\left(\frac{5!}{1!4!}\right)\left(\frac{7!}{2!5!}\right)}{\left(\frac{12!}{3!9!}\right)} = \frac{(5)(21)}{220} = 0.4773$$

5. (a) 0.496 in standard units $= \dfrac{0.496 - 0.502}{0.005} = -1.2$

0.508 in standard units $= \dfrac{0.508 - 0.502}{0.005} = 1.2$

Proportion of acceptable bottles = area under normal curve between $z = -1.2$ and $z = 1.2$

$$= 0.3849 + 0.3849 = 0.7698$$

Therefore, the percentage of defective bottles is 1–0.7698 = 0.2302.
Approximately 23%.

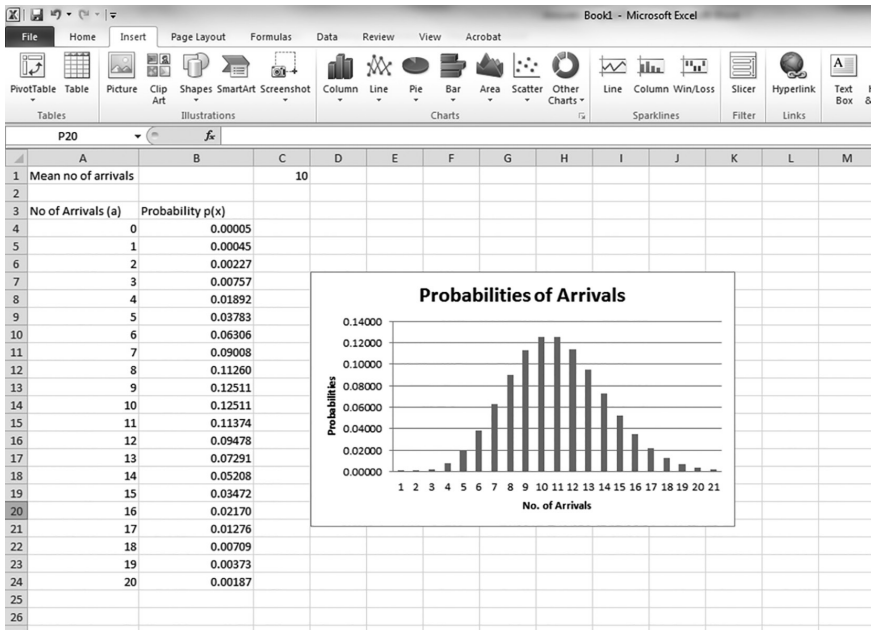
(b) This means out of every 100 bottles produced there is a possibility, on average, that 23 will be outside of the acceptable tolerance level. Which means the company will be losing money.

(c) The production lines need to be investigated to find out where the process is going wrong. This could be due to worn out machinery or poorly maintained equipment.

6. (a) The probably of a passenger is the same for any time interval. The arrival of a passenger is independent between time intervals.

(b) Poisson distribution.

(c) and (d)

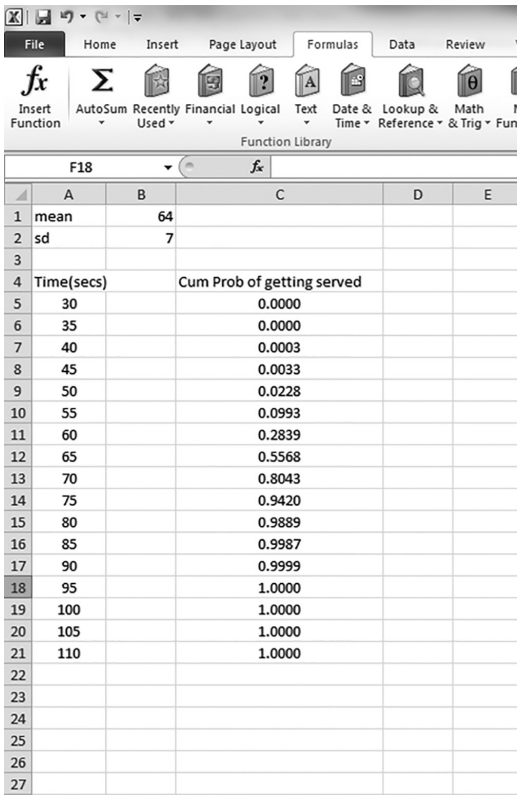


	A	B	C
1	Mean no of arrivals		10
2			
3	No of Arrivals (a)	Probability p(x)	
4	0	0.00005	
5	1	0.00045	
6	2	0.00227	
7	3	0.00757	
8	4	0.01892	
9	5	0.03783	
10	6	0.06306	
11	7	0.09008	
12	8	0.11260	
13	9	0.12511	
14	10	0.12511	
15	11	0.11374	
16	12	0.09478	
17	13	0.07291	
18	14	0.05208	
19	15	0.03472	
20	16	0.02170	
21	17	0.01276	
22	18	0.00709	
23	19	0.00373	
24	20	0.00187	
25			
26			

7.

(a) Yes, according to the analysis, every customer is fed within 95 secs.

(b) No, since it is a statistical analysis and relies upon mean values, etc.



	A	B	C	D	E
1	mean	64			
2	sd	7			
3					
4	Time(secs)		Cum Prob of getting served		
5	30		0.0000		
6	35		0.0000		
7	40		0.0003		
8	45		0.0033		
9	50		0.0228		
10	55		0.0993		
11	60		0.2839		
12	65		0.5568		
13	70		0.8043		
14	75		0.9420		
15	80		0.9889		
16	85		0.9987		
17	90		0.9999		
18	95		1.0000		
19	100		1.0000		
20	105		1.0000		
21	110		1.0000		
22					
23					
24					
25					
26					
27					

8. No answer required.
9. No answer required.