Appendix A

Review of Basic Mathematical Operations

> *I never did very well in math—I could never seem to persuade the teacher that I hadn't meant my answers literally.*
>
> —Calvin Trillin

2 Introduction

Many of you undoubtedly have avoided taking a statistics class because you believed that the mathematics involved would be too difficult for your meager skills. After many years of teaching undergraduate statistics courses, we have probably heard all the stories.

Some students protest, "I'm not very good at math, so how can I ever hope to pass a statistics course? Statistics is nothing but math!" Others are more pessimistic: "I've *never* been good at math. I did lousy in junior high, high school, and college. I just have a mental block against doing math!" Others are only slightly more optimistic, claiming that they are simply rusty: "I haven't had a math course since high school. I've forgotten everything since then!"

This anxiety you brought with you to the course was probably only made worse when you thumbed through the chapters of this book, seeing all the equations, formulas, and strange symbols. Even letters in a different alphabet! "Boy," you thought, "I am sunk. Maybe I should change my major or start planning for summer school!" Put your mind at rest; you need do none of those things. The study of statistics does require some mathematical skills, but they are no more than the ability to add, subtract, multiply, and divide. Let us assure you that if you can do these simple mathematical operations, you can do statistics.

In this statistics text, we have emphasized the conceptual and logical dimension of statistical analyses of crime data. Most complex statistical analyses are now performed by computer programs. You will undoubtedly learn one of these programs in this or some other course. The study site for this text introduces you to one such statistical software program called SPSS. This stands for the Statistical Package for the Social Sciences. This is only one such statistical package that will do the calculations for you for the statistics described in this book. There are many others available, and all of them perform high-speed and accurate calculations of simple and complex statistics.

Although computer software programs can perform the calculations for us much quicker than we could by hand and with far greater accuracy, we need to know some basics about statistics so that we know which statistical analyses to perform in which situations. We also need to know how to interpret and diagnose the mass of statistical information most computer programs spit out for us. In other words, no matter how fast, accurate, or sophisticated the statistical computer package you use, *you still need to know what you are doing.* Therefore, in this statistics course, you need to learn how to hand calculate the various statistical procedures.

The hand calculation of statistics is not that daunting a task. Again, all you need to know how to do mathematically is add, subtract, multiply, and divide. The task will be made simpler by two things we have provided in each chapter of the text:

1. Clear and simplified examples.
2. A step-by-step approach in which even the most difficult statistical procedures are broken down into simple steps.

In addition, you will probably find it necessary to use a hand calculator to do the numerical operations for you. There are a great many kinds of calculators on the market now. Some of these calculators seem as complex as personal computers with graphic screens and everything! Others, in addition to basic mathematical operations, actually calculate some of the statistics in this book for you such as standard deviations and correlation coefficients.

We would recommend that you use a calculator for your calculations. You do not, however, need a very fancy or expensive one. All you really need is a calculator that, in addition to mathematical operations such as adding and subtracting, has a square root key ($\sqrt{}$) and a square key (x^2). The square key will enable you to square (multiply by itself) any number. A simple calculator that does these things is all you really need to work the problems described in this text.

Before we describe some simple mathematical operations, we would like to show you some common symbols used in statistics. Mathematical operations involve many symbols in their own right; as if this were not difficult enough, many statistics are symbolized by Greek letters. To help you through the symbolism, the following are some common math symbols and Greek letters you will find in this text:

Common Mathematical Symbols

$+$	Addition	$>$	Is greater than
$-$	Subtraction	$\geq$	Is greater than or equal to
$\times$	Multiplication	$\approx$	Is approximately equal to
/ or ÷	Division	x^2	The number x squared
$=$	Equals	$\sqrt{x}$	The square root of the number x
$\neq$	Is not equal to	$\ln x$	The natural log of the number x
$\pm$	Plus or minus	$\log x$	The common log of the number x
$<$	Is less than	$\lvert x \rvert$	The absolute value of the number x
$\leq$	Is less than or equal to		

Common Greek Letters Used In Statistics

Uppercase	*Lowercase*	
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Λ	λ	Lambda
Μ	μ	Mu
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Φ	ϕ	Phi
Χ	χ	Chi

Mathematical Operations

Most of you are familiar with the four basic mathematical operations: addition, subtraction, multiplication, and division. In this text, the operations of addition and subtraction are shown with their common symbols, + and –. In the text, the operations of multiplication and division are shown with several different symbols. For example, the operation of multiplying x by y may be shown as xy, $x \times y$, or $(x)(y)$. The operation of dividing x by y may be shown as $x \div y$ or x / y.

In addition to the standard operations of addition, subtraction, multiplication, and division, there are three other very frequent mathematical operations in statistics. One of these is the squaring of a number. A number squared is symbolized by the number being squared shown with a superscript of 2. For example, 4 squared is shown as 4^2, and 7 squared is shown as 7^2. When you square a number, you multiply that number by itself, so 4 squared is equal to $4 \times 4 = 16$, and 7 squared is equal to $7 \times 7 = 49$. These expressions tell us that 4 squared is equal to 16 and that 7 squared is equal to 49. One squared is equal to 1 because $1^2 = 1 \times 1 = 1$. When calculating the square of fractions, it is probably easier first to convert the fraction to a decimal and then square. For example, the square of one half $(^1/_2)^2$ would be equal to $.50^2$ or $(.50)(.50) = .25$. The square of one third $(^1/_3)^2$ would be equal to $.33^2$ or $(.33)(.33) = .1089$.

A second frequent mathematical operation in statistics is taking the square root of a number. This is symbolized by placing the number we want the square root of within something called a radical sign (√). For example, the square root of 2 is shown as $\sqrt{2}$, and the square root of 9 is shown as $\sqrt{9}$. The square root of a number is the value that, when squared, results in the original number. For example, the square root of 9 is 3 ($\sqrt{9} = 3$) because when 3 is squared, we obtain 9 ($3^2 = 3 \times 3 = 9$). The square root of 25 is 5 ($\sqrt{25} = 5$) because when 5 is squared, we obtain 25 ($5^2 = (5)(5) = 25$). As with the squaring of fractions, it will probably be easier to convert a fraction into a decimal before taking the square root. For example, the square root of one half ($\sqrt{1/2}$) is equal to $\sqrt{.5}$, which is equal to .707 because $.707^2 = .5$. The square root of a negative number, $\sqrt{-x}$, is not defined because there is no number x that, when squared (multiplied by itself), results in a negative number. This is because the multiplication of two negative numbers always results in a positive product.

The third other operation that you will frequently see in this text is the summation operation. This is actually an addition operation, but because it appears with its own symbol, we need to call special attention to it. The operation of summation is symbolized by the uppercase Greek letter sigma (Σ). The summation sign stands for "the sum of," and the operation requires you to add a series of scores for a given variable. For example, presuming that there are five scores for the variable "Age" (itself symbolized as x), the ages of five persons might be as follows:

$$x_1 = 13 \qquad x_4 = 20$$

$$x_2 = 18 \qquad x_5 = 17$$

$$x_3 = 25$$

The operation Σx instructs you to sum or add each of these x scores or ages. That is, instead of stating that you should take the first person's age and add it to the second person's age, and then add this sum to the third person's age, and so on, a formula will simply state the sum of all the x scores or Σx. In this example, then, $\Sigma x = 13 + 18 + 25 + 20 + 17 = 93$. Think of the symbol Σ, then, as a mathematical operation that says "add up all of the x scores and determine the sum."

2 Order of Operations

Many statistical formulas require you to perform several mathematical operations at once. At times, these formulas may seem very complex, requiring addition, division, squaring, square roots, and summation. Your task of comprehending statistical formulas would not be so difficult if it did not matter how all the calculations were performed as long as they were all completed. Unfortunately, however, statistical formulas require not only that all mathematical operations be conducted but also that they be conducted in the right order because you will get different results depending on the order in which the operations are performed!

For example, take the following very simple equation that requires you to add and divide a few numbers:

$$15 + 10 \div 5$$

Note that you will get completely different results depending on whether you complete the addition before dividing or do the dividing first:

$$(15 + 10) \div 5 \qquad 15 + (10 \div 5)$$

$$25 \div 5 = 5 \qquad 15 + 2 = 17$$

As you can see, the order in which you perform your mathematical operations does make a substantial difference and must, therefore, be correctly followed. Fortunately, there are some standard rules that tell you the order in which operations should be performed. Furthermore, we would like to emphasize that even the most complex formula or mathematical expression can be simplified by solving it in sequential steps. We now illustrate these rules of operation and our recommended step-by-step approach for solving mathematical expressions.

The first rule is that any operation that is included in parentheses should be performed before operations not included in parentheses. For example, for the expression

$$15 + (10 \div 5) \times (7 \times 2)$$

the order of operations would be first to divide 10 by 5 and multiply 7 by 2. We now have simplified the expression to

$$15 + 2 \times 14$$

How do we solve the remainder of this? Do we first add 15 + 2 and then multiply by 14 to get 238? Or do we first multiply 2 by 14 and then add 15 to get 43?

The second rule of the order of operations is that you should first obtain all squares and square roots, then perform multiplication and division, and last complete any addition and subtraction. Because in the expression just listed we have no squares or square roots to calculate, we know that we should first multiply the 2 and 14 to get 28:

$$15 + 28$$

After this, we should add 28 to 15 to get the final sum of 43.

To summarize, the rules of operation for solving mathematical expressions are, in order:

- Solve all expressions in parentheses.
- Determine the value of all squares and square roots.
- Perform all division and multiplication operations.
- Perform all addition and subtraction operations.

We will practice these rules with some exercises momentarily, but first we need to illustrate the parentheses rule in combination with the rule of squares.

The rules are to perform all operations within parentheses first, then squares and square roots, next multiplication and division, and then addition and subtraction. As an example, assume that we have the following six scores: 46, 29, 61, 14, 33, and 25. With these scores, examine the two expressions, Σx^2 and $(\Sigma x)^2$. These two expressions look virtually identical because they both require a summation of scores and that a number be squared. Note, however, that in the first expression there are no parentheses. We know that the summation sign tells us that we have to add the six scores. Before we do this, however, following the correct order of operations, we must first square each x score and then sum all of them:

$$\begin{aligned}\Sigma x^2 &= 46^2 + 29^2 + 61^2 + 14^2 + 33^2 + 25^2 \\ &= 2{,}116 + 841 + 3{,}721 + 196 + 1{,}089 + 625 \\ &= 8{,}588\end{aligned}$$

In this first expression, then, we have followed the order of operations by first squaring each x score and then taking the sum (squaring before addition).

Note that in the second expression, we have parentheses $(\Sigma x)^2$. As the order of operations is to conduct all calculations within parentheses first, this expression tells us first to sum the six scores and then square the sum:

$$\begin{aligned}(\Sigma x^2) &= (46 + 29 + 61 + 14 + 33 + 25)^2 \\ &= 208^2 \\ &= 43{,}264\end{aligned}$$

To reiterate the point made earlier, Σx^2, called the sum of the x squares, is obtained by first squaring each x score and then adding all squared numbers. This is different from the expression, $(\Sigma x)^2$, called the sum of the xs, squared, which is obtained by first adding up all the x scores and then squaring the sum.

2 Operations With Negative Numbers and Fractions in Denominators

In many statistical calculations, you have both positive and negative scores. Positive scores are shown with no sign at all, so that a positive 10 appears as 10. Negative numbers are shown with a minus sign in front of them, so that a negative 10 appears as –10. Negative numbers are less than zero, and positive numbers are greater than zero. It is important to keep track of the signs of numbers because it makes a substantial difference for the final result of a mathematical operation.

For example, when a positive number is added to a positive number, nothing special happens, and the sum of the two numbers can be obtained directly: $10 + 14 = 24$. When a negative number is added to a positive number, however, it has the same effect as subtraction. For example, adding a negative 14 to 10 is the same thing as subtracting 14 from 10: $10 + (-14) = 10 - 14 = (-4)$. When a positive number is subtracted from another positive number, nothing special happens, and the difference between the two numbers can be obtained directly: $25 - 10 = 15$. When a negative number is subtracted from either a positive or negative number, its sign changes to that of a positive number, so that $25 - (-10) = 25 + 10 = 35$; $(-10) - (-7) = (-10) + 7 = (-3)$. Remember, then, that the subtraction of a negative number changes the sign of the number from negative to positive.

When two positive numbers are multiplied, nothing special happens, and the product of the two numbers can be obtained directly: $6 \times 3 = 18$. When two numbers are multiplied and one is positive and the other negative, the resulting product is negative. For example: $25 \times (-3) = -75$; $(-14) \times 5 = -70$. When two negative numbers are multiplied, the resulting product is always positive: $(-23) \times (-14) = 322$. So the rule is that the multiplication of either two positive or two negative numbers results in a positive product, whereas the multiplication of one positive and one negative number results in a negative product.

The same pattern occurs when the operation is division rather than multiplication. When two positive numbers are divided, nothing special happens, and the result (the *quotient*) is positive: $125 \div 5 = 25$; $10 \div 20 = .5$. When two numbers are divided and one is positive and the other negative, the quotient is negative: $250 \div (-25) = (-10)$; $(-33) \div 11 = -3$. When two negative numbers are divided, the quotient is always positive: $(-16) \div (-4) = 4$. So the rule is that the division of either two positive or two negative numbers results in a positive quotient, whereas the division of one positive and one negative number results in a negative quotient.

Rounding Numbers Off

Whenever you are working with statistical formulas, you need to decide how precise you want your answer to be. For example, should your answer be correct to the tenth decimal place? The fifth? The third? It is also important to decide when to round up and when to round down. For example, having decided that we want to be accurate only to the second decimal place, should the number 28.355 be rounded up to 28.36 or rounded down to 28.35? It is important to make these decisions explicit because two people may get different answers to the same statistical problem simply because they employed different rounding rules.

Unfortunately, no rule about when to round off can always be hard and fast. When we are dealing with large numbers, we can frequently do our calculations with whole numbers (integers). In this case, we would not gain much precision by carrying out our calculations to one or two decimal places. When we are dealing with much smaller numbers, however, it may be necessary, to be as precise as possible, to take a number out to three or four decimal places in our calculations. With smaller numbers, there is a substantial gain in precision by including more decimal places in our calculations. Whenever possible, however, we have tried to limit our precision to two decimal places. This means that most of the time, numbers will include only two decimal places. We warn you, however, that this will not always be the case.

The question about how to round can be answered a little more definitively. When rounding, the following convention should be applied. When deciding how to round, look at the digit to the right of the last digit you want to keep. If you are rounding to the second decimal place, then, look at the third digit to the right of the decimal point. If this digit is larger than 5, you should round up. For example, 123.148 becomes 123.15 and 34.737 becomes 34.74. If this digit is less than 5, you should round down. For example, 8.923 becomes 8.92 and 53.904 becomes 53.90.

What do you do in the case where the third digit is a 5, as in 34.675, for example? Do you round up or round down? You cannot simply say that you should always round up or always round down because there will be systematic bias to your rounding decision. Your decision rule will be consistent to be sure, but it will be biased because numbers are always being overestimated (if rounded up) or underestimated (if rounded down). You would like your decision rule to be consistent but consistently fair—that is, never in the same direction. This way, sometimes the 5 will be rounded up and sometimes it will be rounded down, and the number of times it is rounded up and down will be approximately the same. One way to ensure this is to adopt the following rounding rule: If the third digit is a 5, then look at the digit immediately before the 5; if that digit (the second decimal place) is an even number, then round up; if it is an odd number, then round down. For example, the number 34.675 should be rounded down to 34.67 because the number immediately *before* the 5 is an odd number. The number 164.965 should be rounded up to 164.97 because the number before the 5 is an even number. Note that the number of occasions you will decide to round up (if the immediately preceding digit is an even number 0, 2, 4, 6, or 8) is the same as the number of occasions when you will decide to round down (if the immediately preceding digit is an odd number 1, 3, 5, 7, 9). Because even numbers should appear in our calculations as frequently as odd numbers, there is no bias to our rounding decision rule.

Examples

Let's go through a few examples step by step to make sure that we understand all the rules and procedures. We will begin by solving the following problem:

$$25 + 192 - (3 + 5)^2$$

Following the rules of operation, we first solve within the parentheses:

$$25 + 192 - (8)^2$$

Then we square the 8:

$$25 + 192 - 64$$

Now we can solve for the final answer either by adding 25 to 192 and then subtracting 64 or by subtracting 64 from 192 and then adding 25. Either way, we get the same result:

$$217 - 64 = 153$$

$$25 + 128 = 153$$

Now let's solve a more complicated-looking problem. Please note that this problem is only more *complicated-looking*. When we solve it step by step, you will see that it is very manageable and that all you really need to know is addition, subtraction, multiplication, and division:

$$[32 + 17)^2 / 10] + [\sqrt{16}/(10 - 6)^2]$$

First, we solve within parentheses:

$$[(49)^2 / 10] + [\sqrt{16}/(4)^2]$$

Then we calculate all squares and square roots:

$$(2{,}401/10) + (4/16)$$

Next we do the division:

$$240.1 + .25$$

Finally, we do the addition:

$$240.35$$

Here is one more problem, and it's probably as difficult as any you will encounter in this book:

$$\sqrt{\frac{(116-27)^2+21}{\sqrt{15+1}}} - \frac{(212-188)}{2}$$

Following the rules of operations, we first want to solve within all the parentheses:

$$\sqrt{\frac{(89)^2+21}{\sqrt{15+1}}} - \frac{24}{2}$$

Then we calculate all squares and square roots. Note, however, that in the denominator of the first term, we first have to use addition (15 + 1) before taking the square root of the sum. Note also that we cannot take the square root of the entire first term until we solve for all that is under the square root sign:

$$\sqrt{\frac{7{,}921+21}{4}} - 12$$

Now we continue to solve that part of the problem within the square root by first completing the numerator (by addition) and then dividing:

$$\sqrt{\frac{7{,}942}{4}} - 12$$

$$\sqrt{1{,}985.5} - 12$$

Finally, now that we have completed all the operations within the square root sign, we can complete that:

$$44.56 - 12$$

Note that the result for the first expression was 44.558. Because the third decimal place is greater than 5, we round the second digit up, so that 44.558 becomes 44.56. Then we complete the problem by subtracting 12:

$$32.56$$

We hope that you now feel greater confidence in solving math equations. As long as things are performed in a step-by-step manner, in accordance with the rules of operations, everything in any equation can be solved relatively easily. To make sure that you comprehend these rules, as well as to brush up on your math skills, complete the following exercises.

We have provided answers for you at the end of the section. If you can do these problems, you are ready to tackle any of the statistics problems in this text. If some of the problems in the exercises give you difficulty, simply review that section of this appendix or consult a mathematics book for some help.

2 Practice Problems

1. Calculate each of the following:
   a. $5^2 + 3$
   b. $(35/7) - 4$
   c. $\sqrt{16} + 7 - (4/2)$
   d. $[(35)(.3)] / 10 + 15$

2. Calculate each of the following:

   a. $45 + \sqrt{\dfrac{125}{15 - (3)^2}}$

   b. $18 + (12 \times 10)\sqrt{150} - 50$

   c. $(18 + 12) \times 10\sqrt{150} - 50$

   d. $[(23 + 17) - (5 \times 4)] / (8 + 2)^2$

   e. $(-5) \times 13$

   f. $(-5) \times (-13)$

   g. $[18 + (-7)] \times [(-4) - (-10)]$

   h. $125 / -5$

   i. $450 - [(-125 / -10) / 2]$

3. With the 10 scores 7, 18, 42, 11, 34, 65, 30, 27, 6, and 29, perform the following operations:

   a. Σx

   b. $(\Sigma x)^2$

   c. Σx^2

4. Round the following numbers off to two places to the right of the decimal point:

   a. 118.954

   b. 65.186

   c. 156.145

   d. 87.915

   e. 3.212

   f. 48.565

   g. 48.535

Solutions to Problems

1. a. 28

   b. 1 (Remember to do the division before the subtraction.)

   c. 13

   d. 16.05

2. a. 49.56

   b. 128

   c. 290

   d. .20 (Remember to do all operations within parentheses first, starting with the innermost parentheses.)

   e. –65

   f. 65

   g. 66

   h. –25

   i. 443.75 (Following the rules of operation, you should have divided the two negative numbers (–125 and –10) first, then divided by 2, and finally subtracted that quotient from 450.)

3. a. This expression says to sum all x scores: $7 + 18 + 42 + 11 + 34 + 65 + 30 + 27 + 6 + 29 = 269$.

   b. Note the parentheses in this expression. It tells you to first sum all the x scores and then square the sum: $(7 + 18 + 42 + 11 + 34 + 65 + 30 + 27 + 6 + 29)^2 = (269)^2 = 72{,}361$.

   c. Following the order of operations, first square each x score and then sum these squared scores: $7^2 + 18^2 + 42^2 + 11^2 + 34^2 + 65^2 + 30^2 + 27^2 + 6^2 + 29^2 = 49 + 324 + 1{,}764 + 121 + 1{,}156 + 4{,}225 + 900 + 729 + 36 + 841 = 10{,}145$.

4. a. 118.95

   b. 65.19

   c. 156.15 (Round up because the number to the left of the 5 is an even number.)

   d. 87.91 (Round down because the number to the left of the 5 is an odd number.)

   e. 3.21

   f. 48.57

   g. 48.53