Chapter 19 - Squares, Cubes and Number Shapes

Trial and improvement 8 minutes and 31 seconds.

Many problems in mathematics can be solved by a process of trial and error. To make it sound more respectable, we usually call it trial and improvement. This is perfectly acceptable mathematics involving good mathematical reasoning if you want to make it sound even more impressive, you can say that you're using an iterative method. Here's an example, umm….a fraction question that you might give to year six children. What number goes into the boxes-the same number in both boxes-to make these two fractions equal? I can't think of a better way of doing this than trying a few numbers, let's try three. Does two thirds equal three eighteens? A little bit of checking shows that the one on the left is much larger than the fraction on the right. Let's try a larger number, four, is two quarters equal to four eighteenths? No it's not and a bit of checking, the number on the left is again larger than the one on the right. Let's try a bigger number, eight. These are not equal, but now the one on the left is the smaller of the two fractions. So perhaps, we need something in between the four and the eight, so let's try six and that works. Two sixths is a third, six eighteenths is a third, by trial and improvement, we have found the solution to the problem.

Another example, somebody says, I'm thinking of a number. Add twenty to my number, multiply the result by my number and the answer is three thousand eight hundred and sixty nine. What is my number? Well, we're going to try some numbers and see if we can gradually improve our guess. I'm going to record my results in a table, starting with umm….anything at random, let's try ten. What if the number was ten? Add twenty you get thirty, multiply them together you get three hundred. Much too small so let's try something quite a bit bigger. A hundred, now the result is twelve thousand which is much too large so I try something in between. Eighty. Add twenty is a hundred and multiply them, eight thousand, that's still too large, something smaller, forty. Add sixty, the product is two thousand four hundred now it's too small I want to be somewhere in between forty and eighty. And do you see how we're gradually improving

our trials. I think I'll try fifty, add seventy, sorry add twenty makes seventy, multiply those together- three thousand five hundred, we're getting closer, a bit larger than fifty I'll try fifty-five. That's too large so somewhere between fifty and fifty five umm…let's try fifty-two add twenty is seventy-two, multiply them together is three thousand seven hundred and forty four, still too small there so a little bit larger than fifty-two, let's try fifty-three and that's it. The missing number is fifty-three. Now the mathematical thinking involved here is that it's very like solving an equation in algebra. The number we're trying we could think of as being x add twenty to that and we have x add twenty the product of those is x multiplied by x add twenty and effectively we're trying to solve this equation. X multiplied by x plus twenty equals three thousand eight hundred and sixty-nine. This is very good experience to help children make sense of algebraic thinking.

Here's another example, I want to design a garden, with a square lawn and a patio. And the patio is going to be five meters wide. I want to find the umm…appropriate dimensions for the square lawn in order to produce a total are of a hundred square meters. Well I can set up a table to solve this using trial and improvement, I'm just going to try something for the length of the side of the square. Add five to it, to find the length of the rectangle and then multiply the length and the width of the rectangle together to see how close the total area is to a hundred. Look, this is what I'm thinking. Let's try a square of side ten metres then the total length of the rectangle would be fifteen metres and the area would be a hundred and fifty. Now that's too large so we try something a bit less than ten. Let's try five metre what about a five metre square lawn. That would be a rectangle of length ten five times ten is fifty- too small. Try something in between, seven metres, the length of the rectangle would be then twelve. Seven twelves are eighty-four, that's not enough I need something larger than that, try eight multiplied by thirteen gives a hundred and four square metres for the total area. We're getting very close now, it's clearly just a bit less than eight for the side of the square lawn. We'll try umm…seven point nine, in which case when we add five to that the length of the rectangle would be twelve point nine. I use a calculator now and I find the product is a hundred and one point nine one, just a little bit more than we need umm….one square,

nearly two square metres more than we need. We're heading for a hundred square metres. So I'll try smaller than seven point nine, seven point eight so the length of the rectangle would be twelve point eight, multiply them together. Ninety-nine point eight four, that's very close indeed and I could if I like go on and get it even more exact. Try seven point eight one add five to that, multiply them together and that's so close to a hundred square metres that for all practical purposes, I have solved this problem. The length of the square lawn, the length of the side should be seven point eight one metres. That's probably to the nearest centimetre, which for practical purposes is clearly good enough.

Notice again that what we've actually been doing here is solving an equation because if our trial is x and we add five to that to get the length of the rectangle, we have x plus five and the area is x multiplied by x plus five and we're solving the equation x multiplied by x plus five equals a hundred, this kind of experience is really valuable in building up children's understanding of the way in which algebra works. (Long pause) I hope I've convinced you that solving problems by trial and improvement is respectable mathematics.