Chapter 23 - Angle

Angle sum of pentagon 6 minutes and 33 seconds

Here's a pentagon just any old pentagon, it's not a regular pentagon or anything special and we're going to investigate the sum of the angles, in the pentagon. What I'm going to do is to tear of the five angles, imagine we're doing this so think of this as a paper pentagon. Tearing off the five corners in that way and we're going to piece them together to find the sum of the angles. Now you've probably done this before with triangles and discovered that the sum of the three angles in a triangle is a hundred and eighty degrees or two right angles. You may have done it with a quadrilateral and discovered that the four angles in a quadrilateral fit together to make three hundred and sixty degrees or four right angles, one complete rotation. One complete turn.

Well what about the five angles in a pentagon, have you ever tried piecing those together, this is how you could do it. So here are the five corners I've torn off, I've coloured them, so we can keep track of them. So, when we piece them together and starting in that direction, so I put the first piece the orange piece with one of the edges pointing in that direction, ok. So, this is the angle we have so far, I'm going to add now the green angle to that and this is the total angle now. You can see that that is a reflex angle, that's more than a 180 degree already. Now add the next piece and now the angle is round to there we've now got a total angle of something like three hundred and thirty degrees I should think. Getting on towards a complete rotation. Add the next angle, we've now gone past three hundred and sixty degrees and that much more we're pointing up there ok. Three hundred and sixty degrees and nearly another right angles worth. Now let's piece the last one into place. There it goes, and the total angle now brings us round to there now look there, isn't that remarkable that's the opposite direction to the direction we were facing in to begin with. So, all together, when we put these five angles together we have a complete three hundred and sixty degree turn, plus half a turn, plus another hundred and eighty degrees. So, the total angle, is five hundred and forty degrees or six right angles. So that was two...two right angles for the sum of

the triangle, four right angles for the sum of the angles in the quadrilateral, six right angles for the sum of the angles in a pentagon.

Try tearing off the corners for a six-sided figure, a hexagon and see what the sum of the angles is in that case. Now what we've just done there statically by sticking the angles together, we can also use…we can also do using the dynamic idea of angle and for this I'm going to take the idea of moving an arrow around the pentagon around the edges but when it gets to one of the corners I'm going to rotate the arrow in a clockwise direction round to the next edge and we're going to do that all the way round. Five angles and so, we're going to find the sum of these five rotations by seeing what angle my blue arrow rotates through. I'll show you what I mean, here we start from this point now have a good look at where the arrow is to begin with, we'll need to remember that ok, we slide it up to the next corner and now I'm going to rotate it about that vertex through that angle in a clockwise direction. Here it goes, ok, right it's now moving backwards but that doesn't matter, we'll slide it along to the next vertex and now rotate it in a clockwise direction through that angle, it's now lying along that edge. Move it up to the next vertex again rotate it in a clockwise direction, now slide it down to the next vertex. I'm going to rotate it through this angle in a clock dir…clockwise direction. On the way the arrow will be pointing in that direction. Now that is roughly the same direction as it was pointing in when it started its journey. So, this stage, the arrow seems to have rotated through a complete turn three hundred and sixty degrees four right angles.

Well let us continue with that rotation, there it is, and we slide it down now to the final vertex and add the fifth rotation and that takes our arrow to there. And look it's back where it started but pointing in the opposite direction. So, it's…we know it's turned more than one rotation and it's turned another half of a complete turn. So, all together, it's turned through one and a half complete turns, which is six right angles or five hundred and forty degrees. There you are, that's how to take an arrow for a walk around a pentagon. Try doing it with a hexagon, see if it will work out with a hexagon, what the total rotation is if you apply the same process. Thank you.