

KNOWLEDGE CHECK

23

MENTAL CALCULATIONS, FINDING A PERCENTAGE OF A QUANTITY

You should be able to handle questions such as the following, mentally, without the use of a calculator or any formal procedures.

- a) What is 40% of 55?
- b) A total of 83% of 600 schools participate in a regional athletics competition. How many schools do not participate?
- c) Interest at 17.5% is to be added to a loan of \$240. How much interest is to be added?
- d) A school's target for their upper primary children is that 65% of them should achieve at least a score of 105 in a standardized assessment of mathematics. If one year they have 180 children entering, how many must reach a score of 105 to achieve the target?

ANSWERS TO KNOWLEDGE CHECK 23

- a) 22. b) 102. c) \$42. d) 117 children.

DISCUSSION AND EXPLANATION OF KNOWLEDGE CHECK 23

Since $40\% = \frac{2}{5}$ and we can find a fifth of 55 very easily (11), the simplest approach with (a) is to think of the 40% as the equivalent fraction (two-fifths), so 40% of 55 is 22.

Otherwise, to handle percentage calculations mentally, start from what you know, or can work out easily, and then work your way gradually to the required percentage. A good starting point is often 10%, which, because it's equal to a tenth, is usually very simple to find. For example, in (b), because 83% participate and so 17% do not ($100\% - 83\% = 17\%$), we need to work out 17% of 600. Now 17% can be made up from 10%, 5%, 1% and 1%, all of which I can jot down instantly: 10% of 600 is 60; 5% is half of that, 30; 1% of 600 is just 6 (1 per 100). So 17% of 600 = $60 + 30 + 6 + 6 = 102$. To find the interest in (c) I would again start with 10%, then halve this to get 5%, and halve this to get 2.5%. Adding these will give 17.5%: $\$24 + \$12 + \$6 = \42 . In (d) we need 65% of 180. Here I would make up the 65% from 50%, 10% and 5%: $90 + 18 + 9 = 117$.

This ad hoc method is surprisingly efficient, even with percentage calculations that look unpromising. You can also involve subtraction if it helps. For example, to find 74% of 294 you do not really need a calculator, because you can work around 75%:

$$\begin{array}{rcl} 50\% \text{ of } 294 & = & 147 \quad (\text{half of } 294) \\ 25\% \text{ of } 294 & = & 73.5 \quad (\text{half of the } 50\%) \\ \hline 75\% \text{ of } 294 & = & 220.5 \quad (\text{adding the } 50\% \text{ and } 25\%, 147.0 + 73.5) \\ 1\% \text{ of } 294 & = & 2.94 \quad (\text{one hundredth}) \\ \hline 74\% \text{ of } 294 & = & 217.56 \quad (75\% - 1\%, \text{ i.e. } 220.50 - 2.94) \end{array}$$

Notice that to find 1% of a number you have to divide by 100. This results in all the figures moving two places to the right in relation to the decimal point. Notice also, when calculating a sum or difference involving decimals, it is safest to put in extra zeros to ensure that the numbers have the same number of figures after the decimal point (e.g. $147.0 + 73.5$, $220.50 - 2.94$).

SUMMARY OF KEY IDEAS

- To find a percentage of a quantity, if the percentage required is equivalent to a simple fraction (like $75\% = \frac{3}{4}$, $40\% = \frac{2}{5}$) that can be worked out easily, then use that fraction.
- Otherwise, for mental and informal calculations of percentages of a quantity, build up the required percentage from percentages that are easy to calculate, such as 10%, 5%, 1%, 2%, 50%, 25%).



FURTHER PRACTICE

Do these questions without using a calculator, by mental and informal, ad hoc methods.

23.1 What is (a) 12.5% of 160? (b) 30% of 220?

23.2 A secondary school's target for an examination in English Language is that at least 78% of its students should achieve grade C or above. How many students is that out of a cohort of 240?

23.3 In a year group of 125 children, 40% achieved a standardized score of 105 or above and 30% achieved a standardized score of 100 to 104 inclusive in an assessment of mathematics. How many children achieved a scaled score of less than 100?