Online Resource

# Chapter 9: Measurements and uncertainty

## Logarithmic scales

Mathematics has an important tool, a function called the logarithm (usually shortened to log), that we can use occasionally to good use. The log function takes a number and a constant called the base and produces a new number. In mathematical terms, the log of a number is the new number to which the base must be raised to equal the old number. When the base is 10 we have two equations that say the same thing:





None of that really matters for our purposes. There is one property of this log function, however that is helpful: it compresses the differences between large numbers compared to small numbers. Look at this table which shows some numbers and their logarithms (to the base 10):

|  |  |
| --- | --- |
| **x** | **log(x)** |
| 1 | 0 |
| 10 | 1 |
| 100 | 2 |
| 1000 | 3 |

This table shows us that equal sixed steps in log(x) correspond to increasingly large unequal steps in x itself. The range of log(x) between 0 and 1 covers 1 to 10 in x itself; the range 1 to 2 in log(x) covers 10 to 100 in x and so on.

If we have a scale, like the scale for p-values, where most of the interesting things happen at very small numbers, but we also need to be able to see quite large numbers, then using a log scale is helpful. In the example table above, if we have a graph with a log scale covering the range of log(x) between 0 and 3, then the first quarter of the range is given to x values between 1 and 10 and the last quarter of the range is given to x values between 100 and 1000.

## Transformations for skew

If the distribution of values for a variable is skewed and it is desirable, for some reason, to remove the skew, then there is a family of transformations that can be applied to the data. The general plan is this:

1. Distribution of x is skewed

2. Apply a transformation xnew=transform(x)

3. So that the distribution of xnew is not skewed

The transform to use depends on the degree and direction of the skew. The choice of transform is a little bit like trial and error: it is a case of just finding the transform of choice from a wide library of possible transforms.

We have already seen how the log transformation compresses the high end of a scale. That means that the log transformation will also draw inwards the righthand tail of a distribution and so it could be a useful remedy for positive skew.

There is a more general family of transformations that can be used with some success. The basic principle is that

1. Positive skew: compress the high values of a distribution and expand out the low values

2. Negative skew: expand the high values outwards and compress the low values

Now look at the table which shows two transformations of a number. Both of these involve familiar maths.

|  |  |  |
| --- | --- | --- |
| **x** | **sqrt(x) = x½**  | **x2** |
| 1 | 1 | 1 |
| 2 | 1.414 | 4 |
| 3 | 1.732 | 9 |
| 4 | 2 | 16 |

We can see that the sqrt(x) produces successively smaller steps (0.414, 0.318, 0.268) as we go down the table. The square of x produces successively larger steps as we go down the table (3, 5, 7). So the first is compressing the high values and the second is expanding the high values.

Mathematicians call the square of x “x to the power of 2” and by a nice feature of maths, the square root is “x to the power of ½”. So we can think of a family of transformations:





where *b* is a positive number greater than 1. The first transformation is used to reduce negative skew and the larger the value of *b* the larger the effect. The second transformation can be used to deal with positive skew in the same way. This family of transforms is called the Box and Cox transformation.