**Chapter 11 Exercises: Solutions**

1. The between-school variance (τ00) is .966. ICC = τ00 / ( τ00 + π2/3) = .966 / (.966 + 3.29) = .227. This indicates that 22.7% of the total variance is accounted for by schools in level 2.

2.

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| . \* Random-intercept model. meologit Profmath i.gender cbyses cusecalc || SCH\_ID: Fitting fixed-effects model:Iteration 0: log likelihood = -22441.986 Iteration 1: log likelihood = -20863.917 Iteration 2: log likelihood = -20842.783 Iteration 3: log likelihood = -20842.733 Iteration 4: log likelihood = -20842.733 Refining starting values:Grid node 0: log likelihood = -20591.785Fitting full model:Iteration 0: log likelihood = -20591.785 (not concave)Iteration 1: log likelihood = -20511.131 Iteration 2: log likelihood = -20443.236 Iteration 3: log likelihood = -20442.872 Iteration 4: log likelihood = -20442.872 Mixed-effects ologit regression Number of obs = 14489Group variable: SCH\_ID Number of groups = 748 Obs per group: min = 2 avg = 19.4 max = 50Integration method: mvaghermite Integration points = 7 Wald chi2(3) = 1844.28Log likelihood = -20442.872 Prob > chi2 = 0.0000------------------------------------------------------------------------------ Profmath | Coef. Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- | 1.gender | -.2317172 .0319561 -7.25 0.000 -.29435 -.1690843 cbyses | .8857337 .025062 35.34 0.000 .8366131 .9348544 cusecalc | .2813883 .0133433 21.09 0.000 .255236 .3075406-------------+---------------------------------------------------------------- /cut1 | -3.15884 .0683451 -46.22 0.000 -3.292794 -3.024886 /cut2 | -.8336947 .0603729 -13.81 0.000 -.9520234 -.715366 /cut3 | .2931374 .060169 4.87 0.000 .1752083 .4110665 /cut4 | 1.894785 .061951 30.59 0.000 1.773364 2.016207 /cut5 | 5.795868 .1138533 50.91 0.000 5.57272 6.019017-------------+----------------------------------------------------------------SCH\_ID | var(\_cons)| .4887612 .038635 .4186124 .5706652------------------------------------------------------------------------------LR test vs. ologit regression: chibar2(01) = 799.72 Prob>=chibar2 = 0.0000. meologit, orMixed-effects ologit regression Number of obs = 14489Group variable: SCH\_ID Number of groups = 748 Obs per group: min = 2 avg = 19.4 max = 50Integration method: mvaghermite Integration points = 7 Wald chi2(3) = 1844.28Log likelihood = -20442.872 Prob > chi2 = 0.0000------------------------------------------------------------------------------ Profmath | Odds Ratio Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- | 1.gender | .7931704 .0253467 -7.25 0.000 .7450157 .8444377 cbyses | 2.424763 .0607695 35.34 0.000 2.308535 2.546843 cusecalc | 1.324968 .0176794 21.09 0.000 1.290766 1.360076-------------+---------------------------------------------------------------- /cut1 | -3.15884 .0683451 -46.22 0.000 -3.292794 -3.024886 /cut2 | -.8336947 .0603729 -13.81 0.000 -.9520234 -.715366 /cut3 | .2931374 .060169 4.87 0.000 .1752083 .4110665 /cut4 | 1.894785 .061951 30.59 0.000 1.773364 2.016207 /cut5 | 5.795868 .1138533 50.91 0.000 5.57272 6.019017-------------+----------------------------------------------------------------SCH\_ID | var(\_cons)| .4887612 .038635 .4186124 .5706652------------------------------------------------------------------------------LR test vs. ologit regression: chibar2(01) = 799.72 Prob>=chibar2 = 0.0000 |

3.

* OR for gender is .793, *p* < .001. This indicates that the odds of being above a particular math proficiency level for female students are .811 times as great as the odds for male students when holding other predictors constant.
* OR for cbyses is 2.425, *p* < .001. This indicates that for a one-unit increase in SES, the odds of being above a particular math proficiency level versus being at or below that level increase by a factor of 2.557 points.
* OR for cusecalc is 1.325, *p* < .001. This indicates that a one-unit increase in using calculators corresponds to a 1.335-point increase in the odds of being above a particular math proficiency level.

4.

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| . \* Log likelihood ratio test comparing the unconditional model and random-intercept model. lrtest null ranintLikelihood-ratio test LR chi2(3) = 1867.25(Assumption: null nested in ranint) Prob > chi2 = 0.0000 |

The log likelihood chi-square test χ2(3) = 1857.05, *p* < .001. This indicates that the random intercept model fits the data better than the unconditional model.

5.

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| . \* Contextual model with level 1 and level 2 variables. meologit Profmath i.gender cbyses cusecalc i.urban || SCH\_ID: cusecalc, cov(uns) Fitting fixed-effects model:Iteration 0: log likelihood = -22441.986 Iteration 1: log likelihood = -20853.975 Iteration 2: log likelihood = -20832.558 Iteration 3: log likelihood = -20832.507 Iteration 4: log likelihood = -20832.507 Refining starting values:Grid node 0: log likelihood = -21362.35Fitting full model:Iteration 0: log likelihood = -21362.35 (not concave)Iteration 1: log likelihood = -21051.667 (not concave)Iteration 2: log likelihood = -20939.599 (not concave)Iteration 3: log likelihood = -20779.609 (not concave)Iteration 4: log likelihood = -20657.133 (not concave)Iteration 5: log likelihood = -20569.284 (not concave)Iteration 6: log likelihood = -20517.62 (not concave)Iteration 7: log likelihood = -20481.761 Iteration 8: log likelihood = -20433.162 (backed up)Iteration 9: log likelihood = -20422.648 Iteration 10: log likelihood = -20420.438 Iteration 11: log likelihood = -20420.427 Iteration 12: log likelihood = -20420.427 Mixed-effects ologit regression Number of obs = 14489Group variable: SCH\_ID Number of groups = 748 Obs per group: min = 2 avg = 19.4 max = 50Integration method: mvaghermite Integration points = 7 Wald chi2(4) = 1705.51Log likelihood = -20420.427 Prob > chi2 = 0.0000------------------------------------------------------------------------------------ Profmath | Coef. Std. Err. z P>|z| [95% Conf. Interval]-------------------+---------------------------------------------------------------- | 1.gender | -.2357592 .0322226 -7.32 0.000 -.2989143 -.1726042 cbyses | .8913306 .0252638 35.28 0.000 .8418144 .9408467 cusecalc | .2886735 .0158996 18.16 0.000 .2575109 .319836 1.urban | -.1822923 .0648597 -2.81 0.005 -.309415 -.0551697-------------------+---------------------------------------------------------------- /cut1 | -3.236959 .0791749 -40.88 0.000 -3.392139 -3.081779 /cut2 | -.8791664 .0719524 -12.22 0.000 -1.020191 -.7381423 /cut3 | .2612091 .0717853 3.64 0.000 .1205126 .4019057 /cut4 | 1.875847 .0734941 25.52 0.000 1.731801 2.019893 /cut5 | 5.787493 .1210132 47.83 0.000 5.550312 6.024675-------------------+----------------------------------------------------------------SCH\_ID | var(cusecalc)| .0399125 .0085619 .0262128 .0607724 var(\_cons)| .9712492 .1398702 .7324005 1.287991-------------------+----------------------------------------------------------------SCH\_ID |cov(\_cons,cusecalc)| -.1390014 .0319444 -4.35 0.000 -.2016112 -.0763916------------------------------------------------------------------------------------LR test vs. ologit regression: chi2(3) = 824.16 Prob > chi2 = 0.0000Note: LR test is conservative and provided only for reference.. meologit, orMixed-effects ologit regression Number of obs = 14489Group variable: SCH\_ID Number of groups = 748 Obs per group: min = 2 avg = 19.4 max = 50Integration method: mvaghermite Integration points = 7 Wald chi2(4) = 1705.51Log likelihood = -20420.427 Prob > chi2 = 0.0000------------------------------------------------------------------------------------ Profmath | Odds Ratio Std. Err. z P>|z| [95% Conf. Interval]-------------------+---------------------------------------------------------------- | 1.gender | .7899709 .0254549 -7.32 0.000 .741623 .8414706 cbyses | 2.438372 .0616025 35.28 0.000 2.320574 2.56215 cusecalc | 1.334656 .0212205 18.16 0.000 1.293706 1.376902 1.urban | .8333577 .0540513 -2.81 0.005 .7338762 .9463245-------------------+---------------------------------------------------------------- /cut1 | -3.236959 .0791749 -40.88 0.000 -3.392139 -3.081779 /cut2 | -.8791664 .0719524 -12.22 0.000 -1.020191 -.7381423 /cut3 | .2612091 .0717853 3.64 0.000 .1205126 .4019057 /cut4 | 1.875847 .0734941 25.52 0.000 1.731801 2.019893 /cut5 | 5.787493 .1210132 47.83 0.000 5.550312 6.024675-------------------+----------------------------------------------------------------SCH\_ID | var(cusecalc)| .0399125 .0085619 .0262128 .0607724 var(\_cons)| .9712492 .1398702 .7324005 1.287991-------------------+----------------------------------------------------------------SCH\_ID |cov(\_cons,cusecalc)| -.1390014 .0319444 -4.35 0.000 -.2016112 -.0763916------------------------------------------------------------------------------------LR test vs. ologit regression: chi2(3) = 824.16 Prob > chi2 = 0.0000Note: LR test is conservative and provided only for reference. |

6. Level 1 and level 2 equations for the contextual model are as follows:

Level 1: logit [kij(*Y* ≤ *k*)] = k  β0j + β1jgenderij + β2jcbysesij + β3jcusecalcij)

Level 2: β0j = γ00 + γ01urbanj + *u*0j

 β1j = γ10

 β2j = γ20

 β3j = γ30 + *u*3j

7. OR for urban is .833, *p* < .001. This indicates that the odds of being above a particular category of math proficiency for students in urban schools are .856 times as great as the odds for students in suburban or rural schools when holding other predictors constant.

8. Comparing the random intercept model (Model 2) and the contextual model (Model 3), the log likelihood chi-square χ2(3) = 44.89, *p* < .01. This indicates that the contextual model fits the data better. Therefore, among all three models, the contextual model fits the data best.

9.

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| . \*Model comparisons using AIC and BIC statistics. estimates stats null ranint ranslopAkaike's information criterion and Bayesian information criterion----------------------------------------------------------------------------- Model | Obs ll(null) ll(model) df AIC BIC-------------+--------------------------------------------------------------- null | 14489 . -21376.5 6 42765 42810.48 ranint | 14489 . -20442.87 9 40903.74 40971.97 ranslop | 14489 . -20420.43 12 40864.85 40955.83----------------------------------------------------------------------------- Note: N=Obs used in calculating BIC; see [R] BIC note |

Among the three models, the AIC statistic for the contextual model is 40864.84, which is the smallest; the BIC for the model is 40955.83, which is also the smallest. The results support the finding from the log likelihood ratio tests.

10. See the following output.

Stata 13 output

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| . \*margins and marginsplot. \*Stata 13. margins gender, atmeans predict(fixedonly outcome(2)) vsquishAdjusted predictions Number of obs = 14489Model VCE : OIMExpression : Predicted mean (2.Profmath), fixed portion only, predict(fixedonly outcome(2))at : 0.gender = .4949962 (mean) 1.gender = .5050038 (mean) cbyses = -.6583912 (mean) cusecalc = 3.39071 (mean) 0.urban = .6696114 (mean) 1.urban = .3303886 (mean)------------------------------------------------------------------------------ | Delta-method | Margin Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- gender | 0 | .2525066 .0047051 53.67 0.000 .2432848 .2617284 1 | .2669462 .0045831 58.25 0.000 .2579636 .2759289------------------------------------------------------------------------------ |

Stata 14 output

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| . \*Stata 14. margins gender, atmeans predict(outcome(2)) vsquishAdjusted predictions Number of obs = 14,489Model VCE : OIMExpression : Marginal predicted mean (2.Profmath), predict(outcome(2))at : 0.gender = .4949962 (mean) 1.gender = .5050038 (mean) cbyses = -.6583912 (mean) cusecalc = 3.39071 (mean) 0.urban = .6696114 (mean) 1.urban = .3303886 (mean)------------------------------------------------------------------------------ | Delta-method | Margin Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- gender | 0 | .2340777 .0042181 55.49 0.000 .2258105 .242345 1 | .244455 .0042256 57.85 0.000 .2361731 .252737------------------------------------------------------------------------------ |

11. See the following output.

Stata 13 output

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| . margins gender, atmeans predict(fixedonly outcome(5)) vsquishAdjusted predictions Number of obs = 14489Model VCE : OIMExpression : Predicted mean (5.Profmath), fixed portion only, predict(fixedonly outcome(5))at : 0.gender = .4949962 (mean) 1.gender = .5050038 (mean) cbyses = -.6583912 (mean) cusecalc = 3.39071 (mean) 0.urban = .6696114 (mean) 1.urban = .3303886 (mean)------------------------------------------------------------------------------ | Delta-method | Margin Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- gender | 0 | .0042686 .0004397 9.71 0.000 .0034068 .0051304 1 | .0033751 .0003499 9.65 0.000 .0026894 .0040609------------------------------------------------------------------------------ |

Stata 14 output

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| . \*Stata 14. margins gender, atmeans predict(outcome(5)) vsquishAdjusted predictions Number of obs = 14,489Model VCE : OIMExpression : Marginal predicted mean (5.Profmath), predict(outcome(5))at : 0.gender = .4949962 (mean) 1.gender = .5050038 (mean) cbyses = -.6583912 (mean) cusecalc = 3.39071 (mean) 0.urban = .6696114 (mean) 1.urban = .3303886 (mean)------------------------------------------------------------------------------ | Delta-method | Margin Std. Err. z P>|z| [95% Conf. Interval]-------------+---------------------------------------------------------------- gender | 0 | .005403 .0005536 9.76 0.000 .004318 .006488 1 | .004276 .0004409 9.70 0.000 .0034118 .0051403------------------------------------------------------------------------------ |

12.

Margins plot using Stata 13



Margins plot using Stata 14



13. See the following output.

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| . \*Fitting a Multilevel PO model using meglm. meglm Profmath i.gender cbyses cusecalc i.urban || SCH\_ID: cusecalc, family(ordinal) link(> logit) cov(uns)Fitting fixed-effects model:Iteration 0: log likelihood = -22441.986 Iteration 1: log likelihood = -20853.975 Iteration 2: log likelihood = -20832.558 Iteration 3: log likelihood = -20832.507 Iteration 4: log likelihood = -20832.507 Refining starting values:Grid node 0: log likelihood = -21362.35Fitting full model:Iteration 0: log likelihood = -21362.35 (not concave)Iteration 1: log likelihood = -21051.667 (not concave)Iteration 2: log likelihood = -20939.599 (not concave)Iteration 3: log likelihood = -20779.609 (not concave)Iteration 4: log likelihood = -20657.133 (not concave)Iteration 5: log likelihood = -20569.284 (not concave)Iteration 6: log likelihood = -20517.62 (not concave)Iteration 7: log likelihood = -20481.761 Iteration 8: log likelihood = -20433.162 (backed up)Iteration 9: log likelihood = -20422.648 Iteration 10: log likelihood = -20420.438 Iteration 11: log likelihood = -20420.427 Iteration 12: log likelihood = -20420.427 Mixed-effects GLM Number of obs = 14489Family: ordinalLink: logitGroup variable: SCH\_ID Number of groups = 748 Obs per group: min = 2 avg = 19.4 max = 50Integration method: mvaghermite Integration points = 7 Wald chi2(4) = 1705.51Log likelihood = -20420.427 Prob > chi2 = 0.0000------------------------------------------------------------------------------------ Profmath | Coef. Std. Err. z P>|z| [95% Conf. Interval]-------------------+---------------------------------------------------------------- | 1.gender | -.2357592 .0322226 -7.32 0.000 -.2989143 -.1726042 cbyses | .8913306 .0252638 35.28 0.000 .8418144 .9408467 cusecalc | .2886735 .0158996 18.16 0.000 .2575109 .319836 1.urban | -.1822923 .0648597 -2.81 0.005 -.309415 -.0551697-------------------+---------------------------------------------------------------- /cut1 | -3.236959 .0791749 -40.88 0.000 -3.392139 -3.081779 /cut2 | -.8791664 .0719524 -12.22 0.000 -1.020191 -.7381423 /cut3 | .2612091 .0717853 3.64 0.000 .1205126 .4019057 /cut4 | 1.875847 .0734941 25.52 0.000 1.731801 2.019893 /cut5 | 5.787493 .1210132 47.83 0.000 5.550312 6.024675-------------------+----------------------------------------------------------------SCH\_ID | var(cusecalc)| .0399125 .0085619 .0262128 .0607724 var(\_cons)| .9712492 .1398702 .7324005 1.287991-------------------+----------------------------------------------------------------SCH\_ID |cov(\_cons,cusecalc)| -.1390014 .0319444 -4.35 0.000 -.2016112 -.0763916------------------------------------------------------------------------------------LR test vs. ologit regression: chi2(3) = 824.16 Prob > chi2 = 0.0000Note: LR test is conservative and provided only for reference. |