APPENDIX 1

Introduction to concrete manipulative materials

Manipulative materials are any materials that allow pupils to physically touch, move and rearrange them. Some of the most commonly used materials are introduced and explored below.

Counters

Counters do not have to be small round plastic objects. I like to use nuggets made of coloured or iridescent glass that are sold as flower vase fillers and table decorations. Alternatives are small cubes, wooden bricks, lolly sticks, plastic shapes, games tokens, beans, beads, pebbles, buttons, shells, etc.

Changing the size and shape of counters from one activity to the next can be valuable, larger items being more suitable for younger children and for those with dyspraxia. However, it is best to avoid too much variety of size or colour. For example, counters with solid colours are easier to distinguish and count than rainbow-coloured or patterned ones, regular shapes are easier to distinguish than irregular shapes, and it is easier to see an overall pattern if each of the units within the pattern are the same size and shape as each other. Counters are very useful for work with smaller numbers but are not suitable for numbers much above 20.

Cuisenaire rods

Cuisenaire rods were invented by the Belgian educator Georges Cuisenaire in the 1930s and developed in the 1940s to teach arithmetic to primary school children. They consist of a set of rods made of wood, now also available in plastic, each with a cross-section measuring 1 centimetre square. The 10 different lengths of rods start at 1 cm long and increase by increments of 1 cm, the largest rod being the orange 10-rod. Cuisenaire rods are compatible with Dienes blocks thereby providing for larger denominations. Each size of Cuisenaire rod has its own distinctive and unvarying colour, which makes its length easy to identify without the need for measuring it against unit cubes and counting the units. This is, indeed, the main advantage of using continuous materials. Because the rods do not carry labels or unit markings, they encourage quantities to be seen as a whole, rather than a collection of single units. This, in turn, encourages the development of efficient calculation methods that do not depend on counting in ones.

Cuisenaire rods became popular in schools as a result of the work of Dr Caleb Gattegno who founded the Cuisenaire company in England in the 1950s. Cuisenaire rods are making a welcome comeback into our classrooms and are increasingly being mentioned in government publications on numeracy and maths.

Professor Sharma is one of the most distinguished American maths educators to champion the use of Cuisenaire rods as an aid to constructing sound and robust cognitive models, especially for children who experience difficulties with maths. In addition to writing papers and articles about maths teaching and maths difficulties, Professor Sharma has produced a variety of teaching videos in which he can be seen using the rods with pupils of various ages. (Sharma's publications and teaching videos are available in the UK from Berkshire Mathematics.)

In Appendix 2 you will find a leaflet written for parents, teachers and pupils who are not yet familiar with Cuisenaire rods. It summarises over 20 ideas for using rods in a mathematical way. To produce the leaflet, print both the given pages in landscape format, back to back on a single A4 sheet of paper. If correctly aligned, a single fold will transform the page into a 4-page A5 leaflet, which can be trimmed a little to fit inside a mini-box of Cuisenaire rods. Cuisenaire rods can be obtained from the Reading-based Cuisenaire company in the UK (www.cuisenaire.co.uk) or from ETA/Cuisenaire in the USA (www.etacuisenaire.com).

Dienes blocks

Zoltan Dienes produced his 'attribute blocks' not long after Cuisenaire produced his rods. The idea has been copied many times since then and Dienes blocks are now often known generically as 'base-ten materials'. They consist of wooden or plastic blocks, all of the same colour, and are based on 1 cm cubes formed into single cubes (1), longs (10) and square flats (100). Base-ten sets nowadays also offer large cubes to represent 1000. Unlike Cuisenaire rods, base-ten materials have scored surfaces to highlight the 1 cm cube units from which they are built. Unlike the rods, there are no blocks to represent the numbers between 1 and 10, which means that the numbers below 10 have to be represented by discrete cubes and counted out one by one. For this reason, I prefer to use Cuisenaire rods, supplemented by the larger Dienes blocks.

Stern materials

Dr Catherine Stern, herself a Montessori kindergarten principal in Germany, first showed her materials to other European kindergarten practitioners in 1934. After the Second World War, she emigrated to the USA and began to develop her larger sized materials for teaching children arithmetic. Stern materials are based on 2 cm wooden cubes, with rods for the numbers up to 10 designed to look as if they are made from individual cubes stuck together, each length of rod having a distinctive colour (but unfortunately different colours to those found on Cuisenaire rods). The cubes and rods are used with specially designed wooden counting boards, pattern boards and number boxes, which all make the materials a pleasure to use with young learners, but rather expensive to buy. (The materials are available in the UK from Maths Extra Limited in Wiltshire.)

Hybrid materials

Bead strings, Unifix cubes and various types of abacus all attempt to bridge the divide between discrete and continuous materials. They are all useful in their own way, though none are as versatile as the combination of Cuisenaire rods and Dienes blocks.

Of all the different types of bead strings, the most useful is a string of ten beads with a colour change after five beads, because it models the all-important complement pairs that add up to 10. See Section 1 of my book *The Dyscalculia Toolkit* (2007) for more details and for instructions on making and using bead strings.

Of the different kinds of abacus, I recommend the Slavonic abacus, which is arranged as a field of 100 beads with a colour change after five beads and after five rows of beads. Ideas for using a Slavonic abacus with pupils who have difficulties with maths can be found in Eva Grauberg's *Elementary Mathematics and Language Difficulties* (1997).

Why and how to use concrete materials

Concrete materials are powerful tools for the teaching and learning of numeracy because they can be used to model operations on numbers as well as modelling the numbers themselves. This allows learners to explore ideas, patterns and relationships in a concrete, rather than an abstract, way.

Concrete materials are multi-sensory in that they can be appreciated by sight and by touch. They promote learning by visual, spatial and kinaesthetic routes. Teachers who ensure that all work with concrete materials is accompanied by lots of talk and discussion also cater for the auditory route to learning.

There is a variety of concrete material available to maths teachers these days, but too much variety can create problems. Presenting new models to illustrate new procedures can leave pupils with an incoherent view of maths as a series of isolated topics. For example, using a spike abacus for demonstrations of place value but for nothing else, encourages pupils to compartmentalise place value thinking quite separately from thinking about mental calculations. Another common example is teaching division as grouping or as repeated subtraction, but later explaining fractions through shaded pictures of pizza slices, which does nothing to help pupils see the interconnection between the two concepts. This kind of fragmentation is particularly detrimental to pupils who are dyscalculic or those with little number sense or natural 'feel' for numbers. Pupils with difficulties benefit from having the kind of coherent model that highlights the patterns and connections within the field of mathematics.

By far the best and most versatile apparatus to use with pupils who experience difficulties with maths are continuous base-ten materials such as Cuisenaire rods and Dienes blocks. They are the most robust materials, in the sense of being capable of modelling many different situations and procedures at many different levels. Naturally, discrete materials such as counters will precede work with rods, especially for younger children, but overuse of discrete material tends to encourage pupils to cling to inefficient counting-in-ones strategies. The ideas in this book are designed to take pupils beyond such immature strategies.

Concrete materials should be carefully introduced and demonstrated by the teacher, who should explain to pupils what they are for. Concrete materials are not intended to be a primitive alternative to calculating machines and should never be used in a mechanical way, simply to find an answer.

Concrete materials should not be used only for demonstration purposes, nor should they be used only for very basic work. They are not for the teacher to use but for pupils to handle and explore. They are most useful when the same materials are used at different stages, for different topics and at different levels of difficulty. Teachers must keep reminding their pupils that the actual mathematics is not what happens to numerals on paper, but what happens to numbers that are subjected to mathematical operations. Paper and pencil are just useful ways to record what happens, or to support our memory while we engage in mental calculation and abstract thinking.

Concrete materials make maths principles visible. This helps pupils to develop insight and intuition. Appropriate concrete materials allow learners to make meaning for themselves and to create a model for understanding numeracy that they can internalise. Increasingly sophisticated and secure cognitive models will supports pupils' progression to abstract high-level thinking. At the same time, working with the right concrete materials and explicitly building connections between topics helps to foster a cohesive view of mathematics as a rational subject whose components are interrelated and interdependent.