APPENDIX 1

Introduction to concrete manipulative materials

Manipulative materials are any materials that allow pupils to physically touch, move and rearrange them. Some of the most commonly used materials are introduced and explored below.

Counters

Counters do not have to be small round plastic objects. I like to use nuggets made of coloured or iridescent glass that are sold as flower vase fillers and table decorations. Alternatives are small cubes, wooden bricks, lolly sticks, plastic shapes, games tokens, beans, beads, pebbles, buttons, shells, etc.

Changing the size and shape of counters from one activity to the next can be valuable, larger items being more suitable for younger children and for those with dyspraxia. However, it is best to avoid too much variety of size or colour. For example, counters with solid colours are easier to distinguish and count than rainbow-coloured or patterned ones, regular shapes are easier to distinguish than irregular shapes, and it is easier to see an overall pattern if each of the units within the pattern are the same size and shape as each other. Counters are very useful for work with smaller numbers but are not suitable for numbers much above 20.

Cuisenaire rods

Cuisenaire rods were invented by the Belgian educator Georges Cuisenaire in the 1930s and developed in the 1940s to teach arithmetic to primary school children. They consist of a set of rods made of wood, now also available in plastic, each with a cross-section measuring 1 centimetre square. The 10 different lengths of rods start at 1 cm long and increase by increments of 1 cm, the largest rod being the orange 10-rod. Cuisenaire rods are compatible with Dienes blocks thereby providing for larger denominations. Each size of Cuisenaire rod has its own distinctive and unvarying colour, which makes its length easy to identify without the need for measuring it against unit cubes and counting the units. This is, indeed, the main advantage of using continuous materials. Because the rods do not carry labels or unit markings, they encourage quantities to be seen as a whole, rather than a collection of single units. This, in turn, encourages the development of efficient calculation methods that do not depend on counting in ones.

Cuisenaire rods became popular in schools as a result of the work of Dr Caleb Gattegno who founded the Cuisenaire company in England in the 1950s. Cuisenaire rods are making a welcome comeback into our classrooms and are increasingly being mentioned in government publications on numeracy and maths.

Professor Sharma is one of the most distinguished American maths educators to champion the use of Cuisenaire rods as an aid to constructing sound and robust cognitive models, especially for children who experience difficulties with maths. In addition to writing papers and articles about maths teaching and maths difficulties, Professor Sharma has produced a variety of teaching videos in which he can be seen using the rods with pupils of various ages. (Sharma's publications and teaching videos are available in the UK from Berkshire Mathematics.)

In Appendix 2 you will find a leaflet written for parents, teachers and pupils who are not yet familiar with Cuisenaire rods. It summarises over 20 ideas for using rods in a mathematical way. To produce the leaflet, print both the given pages in landscape format, back to back on a single A4 sheet of paper. If correctly aligned, a single fold will transform the page into a 4-page A5 leaflet, which can be trimmed a little to fit inside a mini-box of Cuisenaire rods. Cuisenaire rods can be obtained from the Reading-based Cuisenaire company in the UK (www.cuisenaire.co.uk) or from ETA/Cuisenaire in the USA (www.etacuisenaire.com).

Dienes blocks

Zoltan Dienes produced his 'attribute blocks' not long after Cuisenaire produced his rods. The idea has been copied many times since then and Dienes blocks are now often known generically as 'base-ten materials'. They consist of wooden or plastic blocks, all of the same colour, and are based on 1 cm cubes formed into single cubes (1), longs (10) and square flats (100). Base-ten sets nowadays also offer large cubes to represent 1000. Unlike Cuisenaire rods, base-ten materials have scored surfaces to highlight the 1 cm cube units from which they are built. Unlike the rods, there are no blocks to represent the numbers between 1 and 10, which means that the numbers below 10 have to be represented by discrete cubes and counted out one by one. For this reason, I prefer to use Cuisenaire rods, supplemented by the larger Dienes blocks.

Stern materials

Dr Catherine Stern, herself a Montessori kindergarten principal in Germany, first showed her materials to other European kindergarten practitioners in 1934. After the Second World War, she emigrated to the USA and began to develop her larger sized materials for teaching children arithmetic. Stern materials are based on 2 cm wooden cubes, with rods for the numbers up to 10 designed to look as if they are made from individual cubes stuck together, each length of rod having a distinctive colour (but unfortunately different colours to those found on Cuisenaire rods). The cubes and rods are used with specially designed wooden counting boards, pattern boards and number boxes, which all make the materials a pleasure to use with young learners, but rather expensive to buy. (The materials are available in the UK from Maths Extra Limited in Wiltshire.)

Hybrid materials

Bead strings, Unifix cubes and various types of abacus all attempt to bridge the divide between discrete and continuous materials. They are all useful in their own way, though none are as versatile as the combination of Cuisenaire rods and Dienes blocks.

Of all the different types of bead strings, the most useful is a string of ten beads with a colour change after five beads, because it models the all-important complement pairs that add up to 10. See Section 1 of my book *The Dyscalculia Toolkit* (2007) for more details and for instructions on making and using bead strings.

Of the different kinds of abacus, I recommend the Slavonic abacus, which is arranged as a field of 100 beads with a colour change after five beads and after five rows of beads. Ideas for using a Slavonic abacus with pupils who have difficulties with maths can be found in Eva Grauberg's *Elementary Mathematics and Language Difficulties* (1997).

Why and how to use concrete materials

Concrete materials are powerful tools for the teaching and learning of numeracy because they can be used to model operations on numbers as well as modelling the numbers themselves. This allows learners to explore ideas, patterns and relationships in a concrete, rather than an abstract, way.

Concrete materials are multi-sensory in that they can be appreciated by sight and by touch. They promote learning by visual, spatial and kinaesthetic routes. Teachers who ensure that all work with concrete materials is accompanied by lots of talk and discussion also cater for the auditory route to learning.

There is a variety of concrete material available to maths teachers these days, but too much variety can create problems. Presenting new models to illustrate new procedures can leave pupils with an incoherent view of maths as a series of isolated topics. For example, using a spike abacus for demonstrations of place value but for nothing else, encourages pupils to compartmentalise place value thinking quite separately from thinking about mental calculations. Another common example is teaching division as grouping or as repeated subtraction, but later explaining fractions through shaded pictures of pizza slices, which does nothing to help pupils see the interconnection between the two concepts. This kind of fragmentation is particularly detrimental to pupils who are dyscalculic or those with little number sense or natural 'feel' for numbers. Pupils with difficulties benefit from having the kind of coherent model that highlights the patterns and connections within the field of mathematics.

By far the best and most versatile apparatus to use with pupils who experience difficulties with maths are continuous base-ten materials such as Cuisenaire rods and Dienes blocks. They are the most robust materials, in the sense of being capable of modelling many different situations and procedures at many different levels. Naturally, discrete materials such as counters will precede work with rods, especially for younger children, but overuse of discrete material tends to encourage pupils to cling to inefficient counting-in-ones strategies. The ideas in this book are designed to take pupils beyond such immature strategies.

Concrete materials should be carefully introduced and demonstrated by the teacher, who should explain to pupils what they are for. Concrete materials are not intended to be a primitive alternative to calculating machines and should never be used in a mechanical way, simply to find an answer.

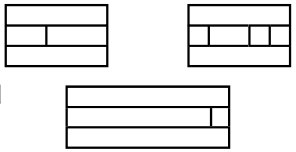
Concrete materials should not be used only for demonstration purposes, nor should they be used only for very basic work. They are not for the teacher to use but for pupils to handle and explore. They are most useful when the same materials are used at different stages, for different topics and at different levels of difficulty. Teachers must keep reminding their pupils that the actual mathematics is not what happens to numerals on paper, but what happens to numbers that are subjected to mathematical operations. Paper and pencil are just useful ways to record what happens, or to support our memory while we engage in mental calculation and abstract thinking.

Concrete materials make maths principles visible. This helps pupils to develop insight and intuition. Appropriate concrete materials allow learners to make meaning for themselves and to create a model for understanding numeracy that they can internalise. Increasingly sophisticated and secure cognitive models will supports pupils' progression to abstract high-level thinking. At the same time, working with the right concrete materials and explicitly building connections between topics helps to foster a cohesive view of mathematics as a rational subject whose components are interrelated and interdependent.

APPENDIX 2 Cuisenaire rods

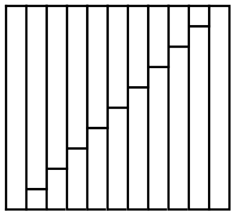
Making sandwiches

Sandwich two rods of the same colour with a 'filling' of any two rods that fit exactly. Can you make a different filling for the same sandwich? What if you were allowed more than two rods for the filling? Look at the 5-sandwich and say some of the different ways of making 5. Will it be the same if the sandwich is turned upside down? How many ways can you make a 6-sandwich or a 7-sandwich, etc. Can you cover the rods and still say what numbers build up to 6 or 7, etc?



Make a wall and find number bonds to 10

Start with a staircase. Find the rod to fit on top of each step to make 10. Practise lots of questions, first looking at the wall, and later just 'seeing' it in your head: What and two makes ten? Ten is five and what? What more must we add to seven to get ten? Ten minus eight is . . . ? etc.



Number lines

Put 5 orange rods side by side. Throw lottery dice or pick random numbers below 50. Where on the orange number line will you find this number? (Lay out the number in rods, on top, if necessary.) How far is it from the next round number? What is that round number called?

Make and read equations

a) Take any two rods and put them end to end. Find the rod that is the same length. For this example put a red and a light green next to a yellow rod (like an open sandwich). Read the equation in four different ways, pointing at the relevant rods as you speak:

e.g. **2 + 3 = 5**, **3 + 2 = 5**, **5 - 3 = 2**, **5 - 2 = 3**.

When you read '**+**' you can say 'and', 'add', 'plus'. When you read '**-**' you can say 'minus', take away', 'subtract'. When you read '**=**' you can say 'equals', 'is', 'is the same as'. Try using a variety of these words - don't always stick to the same ones. Sometimes start with the answer: **5=3+2**, **5=2+3**.

b) Put some rods into a bag, take out two rods without looking, then make and read an equation for them. Now shut your eyes, visualise the rods in your mind and say the four equations again.

Ronit Bird 2002

Some things you can do with your Cuisenaire rods

Play around with them

The rods can only be used to help mathematical thinking when the relationship between the sizes and colours are very well known. The best way to become really familiar with them is to spend plenty of time playing around with them. Flat patterns are best. Do not allow younger siblings to join in - Cuisenaire rods are not toys.

Name the Colours

The convention is to use these names: white, red, light green, purple, yellow, dark green, black, brown, blue, orange.

Put them back in the box

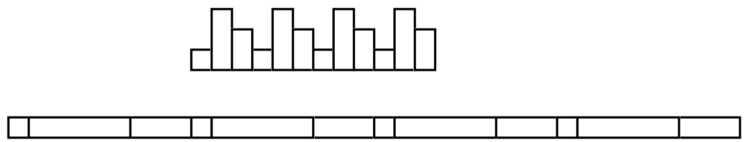
Putting the rods back gives an opportunity to match colours and sizes and to talk/think about different sizes and what fits.

Build flat designs

Use the rods flat on the table to make colourful pictures (a house, a car, etc) or designs (abstract patterns). Talk about the designs: What did you make? What rods did you use? etc.

Make sequences and play 'Hide the Rod'

Pick three rods at random and put them side by side. Repeat this pattern three more times until you have a sequence. While one player looks away, the other takes one rod out of the sequence and closes up the gap. The other player works out which colour is missing. Later try sequences of four or more repeated rods.



Matching designs

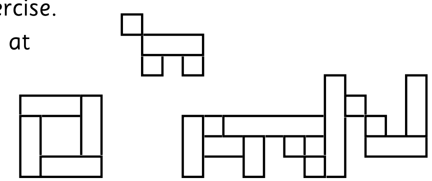
Take a **small** handful of rods and put them in a heap on the table. The second player must find the same number and colours to put in a heap of their own. The first player arranges the rods in a pattern or design while the other looks away. The second player looks at it and copies the pattern.

Copying designs onto paper

Make a simple pattern on squared paper. (The rods are in units of one cm, so find squares of 1 cm if possible). Using coloured pencils in Cuisenaire colours, copy the design and colour it.

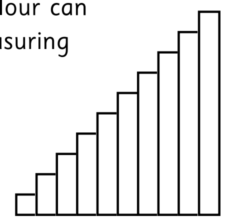
Match designs from a picture

At another session, try to reproduce the designs made in the exercise above, from the coloured picture. Alternatively, draw some new designs on paper for this exercise. As you and your child get better at this, you can make the designs more and more complicated.



Build a staircase

A staircase is built from one of each colour of rod, arranged in order of size, flat on the table. With practice, a staircase can be built in less than 30 seconds. Count the steps, and associated each colour with its number. The number for each colour can be found either by counting the steps or by measuring how many ones (white) fit into the same length.



See the staircase in your mind

Make a staircase then cover it with a sheet of paper. Say the colours in order. Name a colour for the other player to say the number, and vice versa. Throw a die, then see who can be first to pick out of the box the right rod to match the number. Pick a rod at random and ask the other player to say the number and colour of the next size bigger (or smaller).

Play 'Hide the Rod' from the staircase

Players take turns to hide one of the rods out of a staircase closing the gap afterwards. The other player must say both the colour and the number of the missing rod.

Race to build a staircase with 10-sided dice

Two players take turns to throw a 1-10 die and take a rod to match the throw. Put the rod down in roughly the place it will be in your final staircase, leaving spaces for the rods still to come. Who can complete their staircase first?

Measuring the rods

Every rod can be measured in white rods. How many white rods? Can every rod be measured in red rods? Why not? Talk about odds and evens. Find which rods can be measured exactly in light greens, or yellow.

Estimating and measuring with rods

First guess and then measure how many orange rods will fit into the length of a ruler, that book, your shoe, a box, etc. Note that an orange rod is 10 cm long, so after you find that something is, say, a little longer than 2 orange rods, you can also say it is just more than 20 cm.

Estimating and measuring bigger numbers with rods

Take a handful of mixed rods and put them end to end in a 'train'. Estimate roughly how long the train is by saying how many orange rods you think will fit into the length, therefore how many centimetres long the train is. Measure in orange rods first, then with a metre rule.

Finding doubles

Which rods can be measured exactly by two other identical rods? So, double two is . . . , double five is . . . , half of eight is . . ., etc.

Doubles up to 20

Find, say, double 7: Put two black rods end to end, then measure against an orange. How much more than one orange rod is it? So, how much is double 7? What is half 14? Repeat for the other numbers.

Overcoming Difficulties with Number

Supporting Dyscalculia and Students who Struggle with Maths

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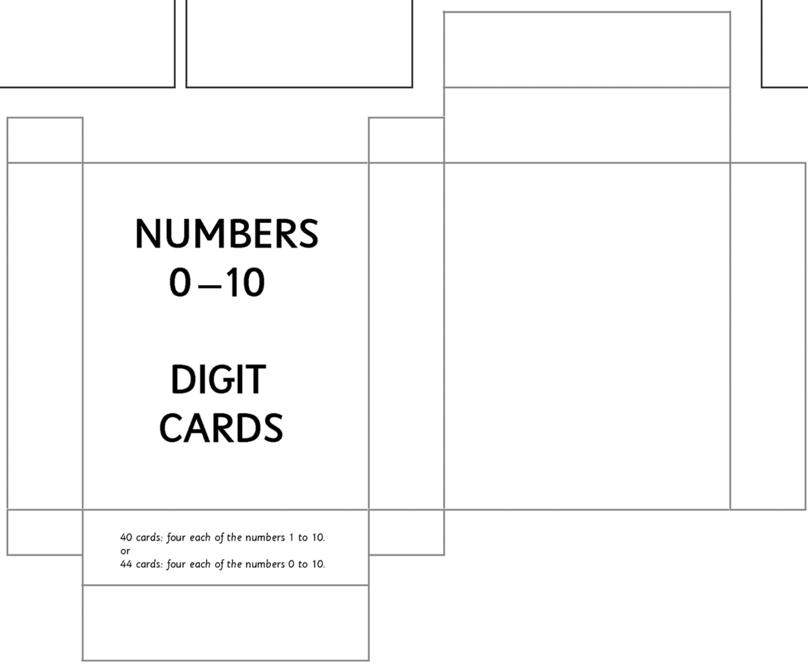
Contents of CD

DIGIT CARDS AND BOX

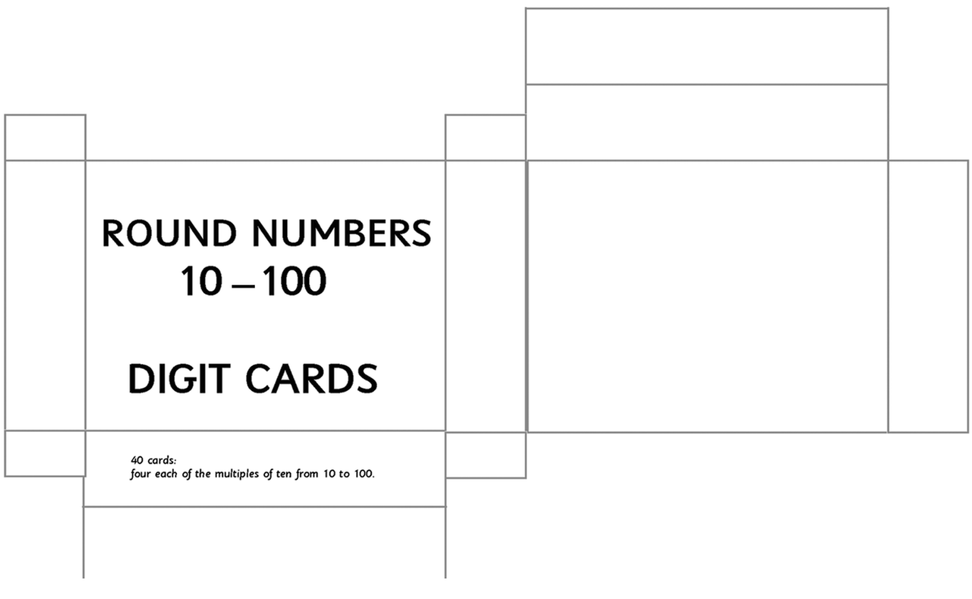
Enlarge as required. Make 4 cards for each digit per box.

1	2	3	4
5	6	7	8
9	10		0

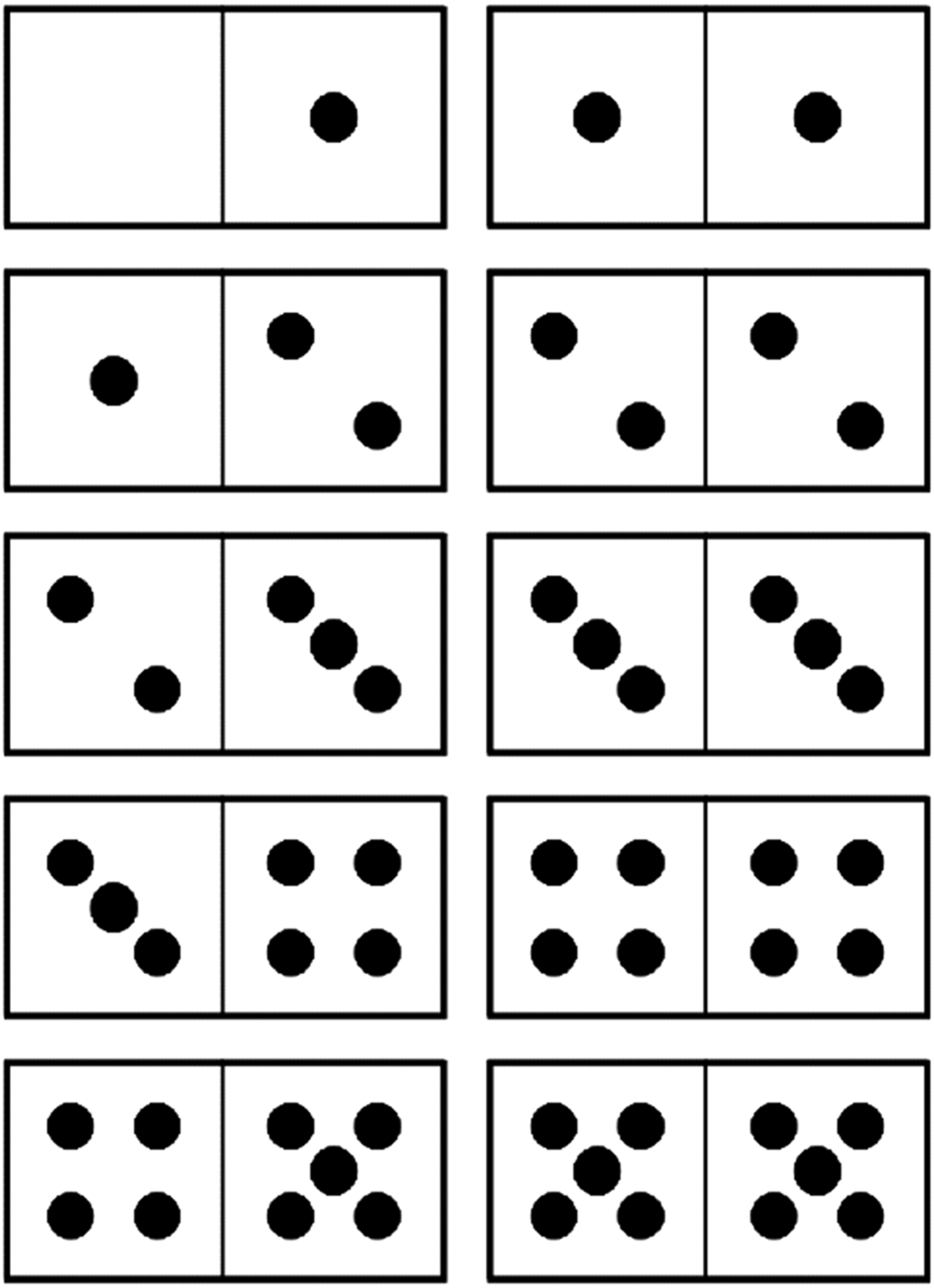


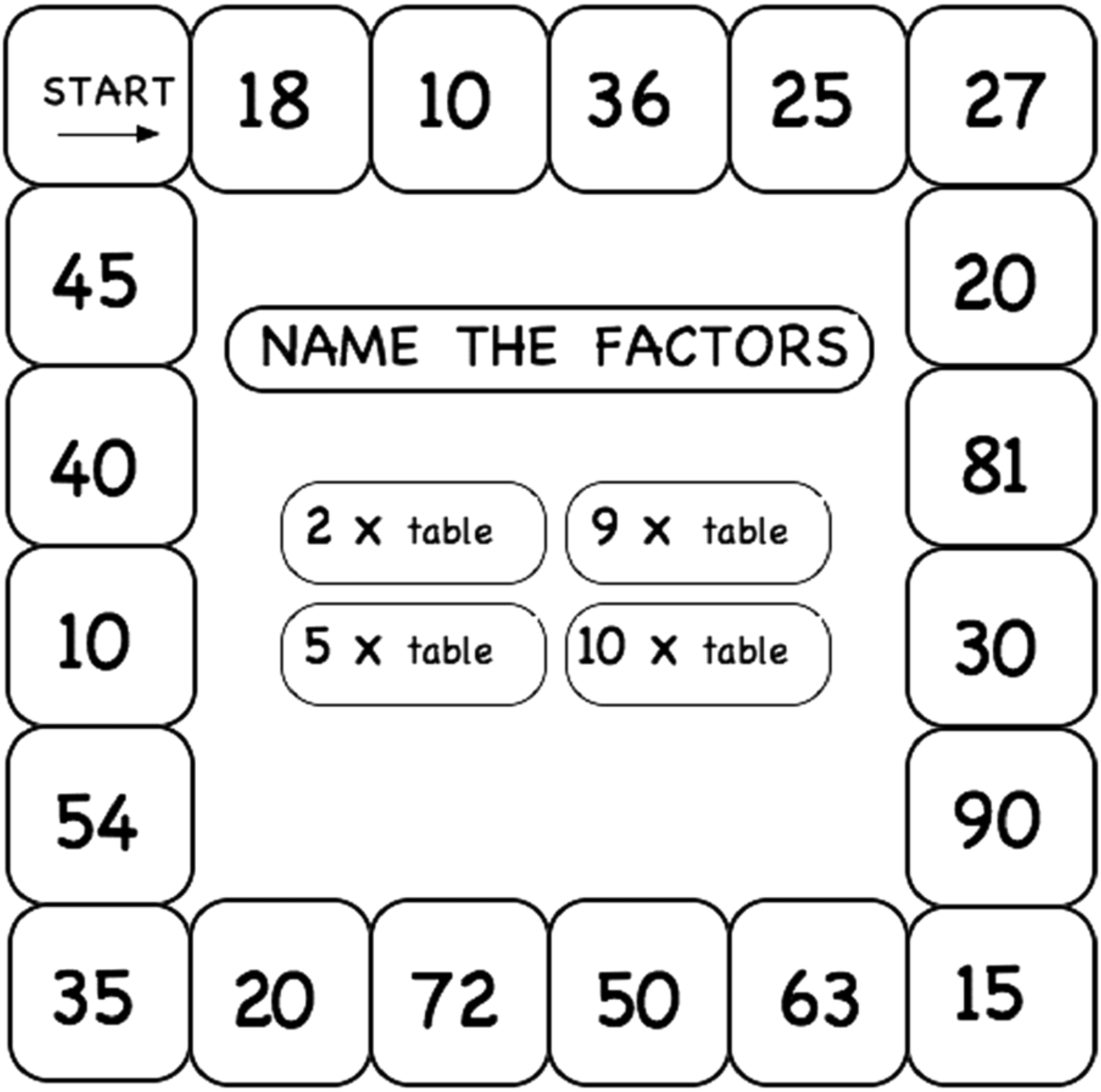
DIGIT CARDS AND BOX

10	20	30
40	50	60
70	80	90
100		



A set of 10 domino cards





<u>RULES</u>: This game is for 2 or 3 players.

You need a die, a token for each player, paper and pencils and coins.

Take turns to throw a die and move around the board. When you land on a number, write down all the factors of that number **<u>except</u>** 1 and the number. For example, all the factors of 10 are 1, 2, 5 and 10, but for this game only 2 and 5 will count.

Read out your list, using the word 'factor' each time. For example, if you land on 10, say: '2 and 5 are **factors** of 10.' If your opponents agree with your list, take coins to match the value of each factor you have written down correctly.

A tip to help you play: notice that all the numbers on this board come from the four multiplication tables named in the middle.

The game ends when all players have been round the board twice. Sort your coins into stacks that are each worth 10 to find who has the most.

MARCHING ON

A game for 2 players

10	20	30	40	50	60	70	80	90	100	110	120

RULES: Each player has their own game board. The game uses a pack of digit cards built from four cards each of the numbers 5–9 and eight cards each of the numbers 1–4.

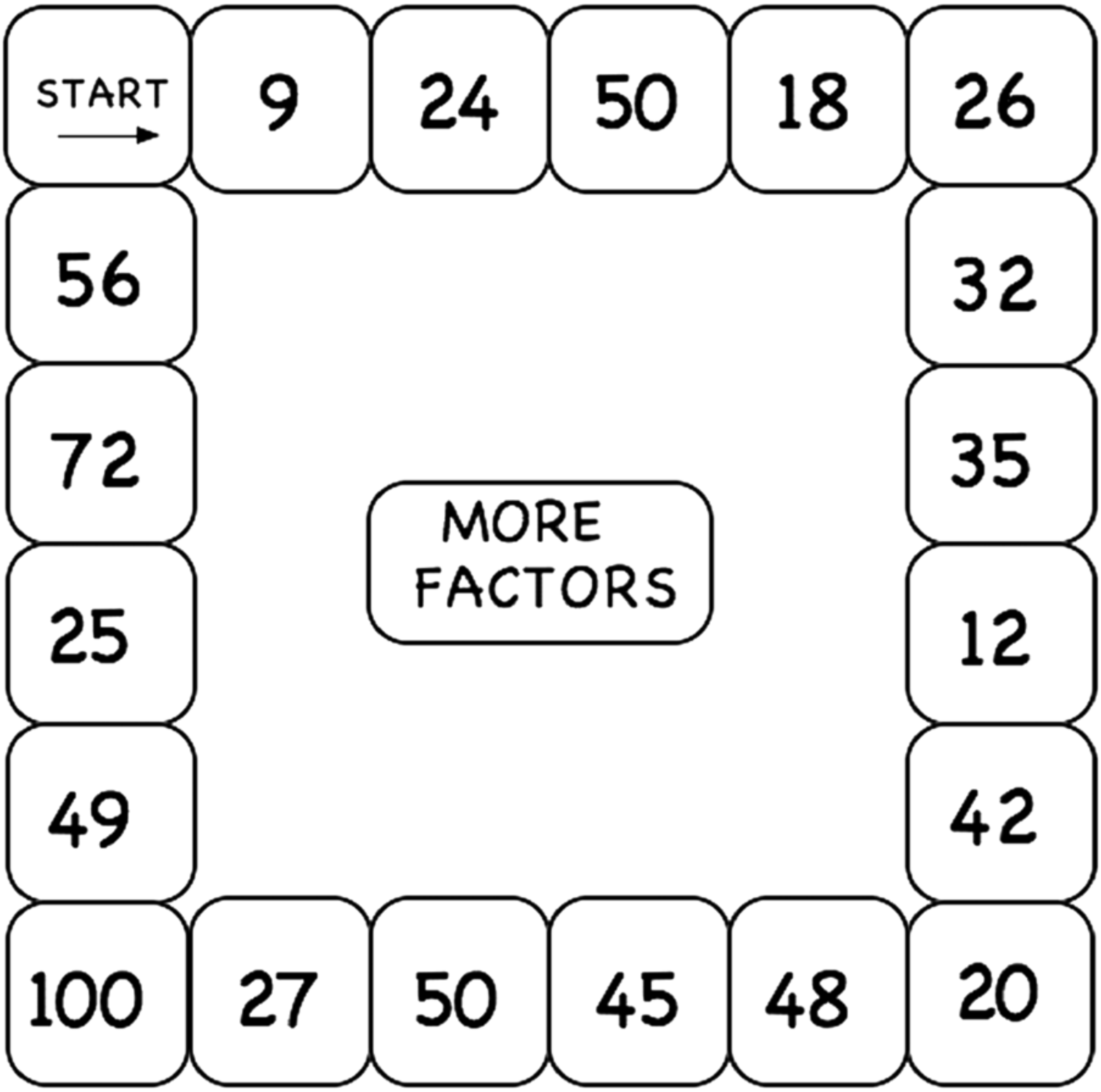
Players take it in turn to spin a spinner and to take between 1 and 5 cards, according to the spin. If the numbers on any combination of cards add up to 10, the player displays them, then puts them face down on top of the number track, blocking out the numbers that have already been reached. The player must announce aloud the movement along the track, e.g. ‘I can move from zero to 10’, or ‘I have put together another 10, so now I move from 20 to 30’. The same player can go as many times as the cards allow, but can only move in steps of ten, and keeps any remaining cards for combining with new cards on subsequent turns. The winner is the first person to reach the end of the track.

MARCHING ON

A game for 2 players

<u>RULES</u>: Each player has their own game board. Players agree on any twelve consecutive multiples of 10 for the game and write one in each square of the number track. The game uses a pack of digit cards built from four cards each of the numbers 5–9 and eight cards each of the numbers 1–4, and a spinner showing the numbers 1–4 or 1–5.

Players take it in turn to spin the spinner and to take the number of cards that is directed by the spinner. If the numbers on any combination of cards add up to 10, the player displays them, then puts them face down on top of the number track, blocking out the numbers that have already been reached. The player must announce aloud the movement along the track, e.g. ‘These cards make ten. This ten will take me from 80 to 90’. The same player can go as many times as the cards allow, but can only move in steps of ten. Any remaining cards are kept for combining with new cards on subsequent turns. The winner is the first person to reach the end of the track.



<u>RULES</u>: This game is for 2 or 3 players.

You need a die, a token for each player, paper and pencils and coins.

Take turns to throw a die and move around the board. When you land on a number, write down all the factors of that number **<u>except</u>** one and the number. Read out your list, using the word ‘factor’ each time (e.g., if you land on 26, say: ‘2 and 13 are **factors** of 26’). If your opponents agree with your list, take coins to match the value of each factor.

The game ends when all players have been round the board twice. Sort your coins into stacks worth 10 to find the winner.

The Multiples Game

Multiples from the 1 to 6 times tables

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

RULES: This game is for 2 players.

You will need a 1–6 die and counters in two colours.
Players take turns to throw the die and to put one of their counters on any number that is a multiple of the number shown on the die. For example, if you throw a 2, cover any multiple of 2, which means any numbers from the 2 times table.
The winner is the first person to have four counters in a row.

The Multiples Game

Multiples from the 4 to 9 times tables

24	25	27	28	30
32	35	36	40	42
45	48	49	50	54
55	56	60	63	64
65	70	72	80	90

<u>RULES</u>: This game is for 2 players.

You will need a 4–9 die and counters in two colours.

Players take turns to throw the die and to put one of their counters on any number that is a multiple of the number thrown.

The winner is the first person to have four counters in a row.

Multiplication and Division Word Problems

Word problems involve a story set-up: scenes from real life in which a calculation needs to be made.

A simple number sentence (an equation) involves three numbers, e.g. $x + y = z$, or $a \times b = c$.

In a simple word problem, two of the three numbers from an equation are given in the set-up and the question directs you to find the third number. For the work on this page, rewrite each given set-up as a word problem, including only two numbers, and write the question that will produce the third number.

Example:
Set-up: 6 wheels on a lorry, 8 lorries.
Write a word problem for: (i) $6 \times 8 =$ (ii) $48 \div 6 =$ (iii) $48 \div 8 =$

(i) $6 \times 8 =$ A garage has 8 lorries, each of which has 6 wheels. All the tyres need replacing. How many tyres will the garage need to replace?
(ii) $48 \div 6 =$ Tom counts the wheels on lorries. All the lorries he sees have 6 wheels each. After 10 minutes, he has counted 48 wheels. How many lorries has he seen?
(iii) $48 \div 8 =$ How many wheels has each lorry, if eight lorries have a total of 48 wheels?

1) Set-up: 12 eggs in an egg box, 5 boxes.
Write a word problem for: (i) $12 \times 5 =$ (ii) $60 \div 5 =$ (iii) $60 \div 12 =$

2) Set-up: 6 glassfuls per bottle, 7 bottles.
Write a word problem for: (i) $7 \times 6 =$ (ii) $42 \div 7 =$ (iii) $42 \div 6 =$

3) Set-up: 4 chairs around a table, 9 tables.
Write a word problem for: (i) $4 \times 9 =$ (ii) $36 \div 4 =$ (iii) $36 \div 9 =$

4) Set-up: 56 hours worked over 7 days.
Write a word problem for: (i) $8 \times 7 =$ (ii) $56 \div 8 =$ (iii) $56 \div 7 =$

5) Set-up: 36 athletes, 3 teams.
Write a word problem for: (i) $3 \times 12 =$ (ii) $36 \div 3 =$ (iii) $36 \div 12 =$

In the following questions choose your own numbers for each set-up before making up the three word problems.

6) .... plants in a row .... rows of plants
Write a word problem for: (i) $\times$ $=$ (ii) $\div$ $=$ (iii) $\div$ $=$

7) .... biscuits .... packets of biscuits
Write a word problem for: (i) $\times$ $=$ (ii) $\div$ $=$ (iii) $\div$ $=$

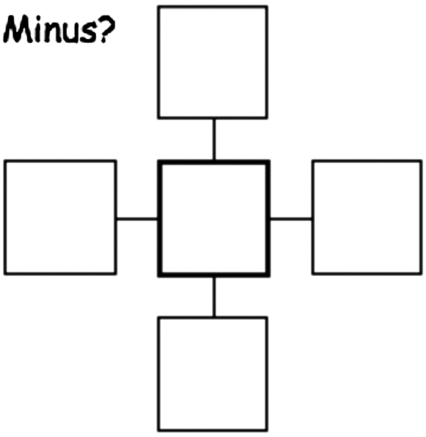
8) .... people .... coaches
Write a word problem for: (i) $\times$ $=$ (ii) $\div$ $=$ (iii) $\div$ $=$

9) .... books .... bookshelves
Write a word problem for: (i) $\times$ $=$ (ii) $\div$ $=$ (iii) $\div$ $=$

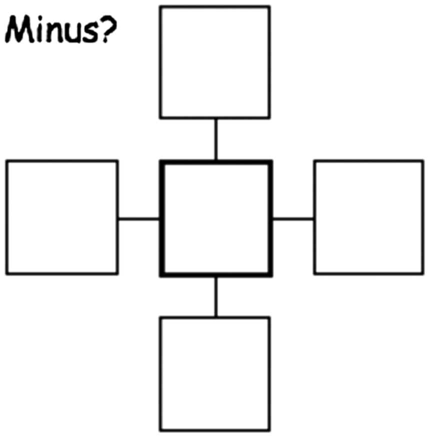
10) .... items bought each costs ....
Write a word problem for: (i) $\times$ $=$ (ii) $\div$ $=$ (iii) $\div$ $=$

Blank templates for the Plus or Minus game (Chapter 1)

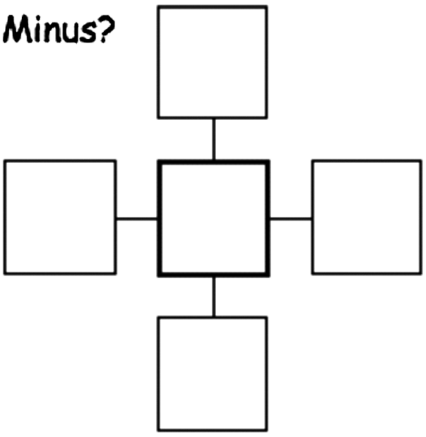
Plus or Minus?



Plus or Minus?



Plus or Minus?



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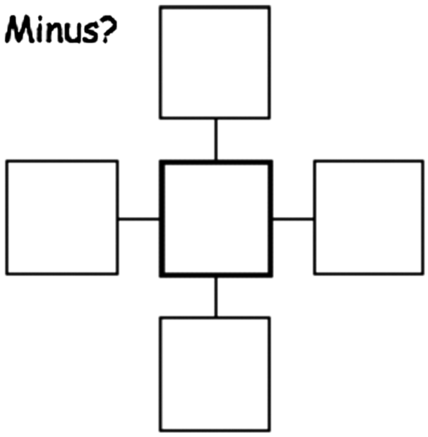
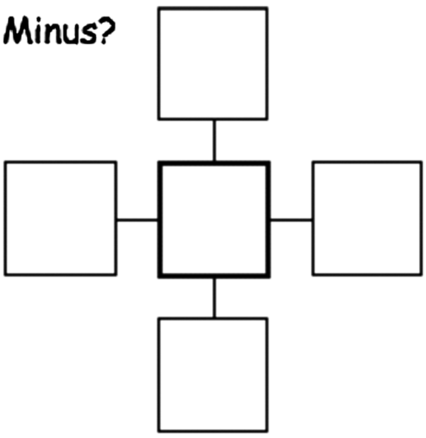


Plus or Minus?



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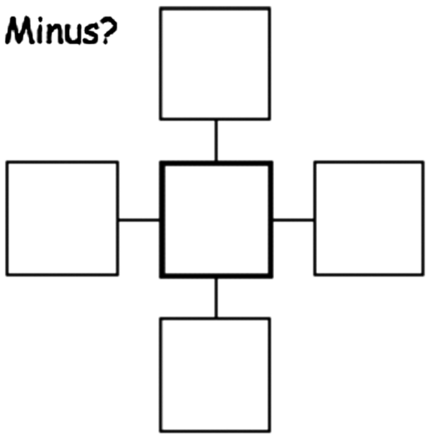
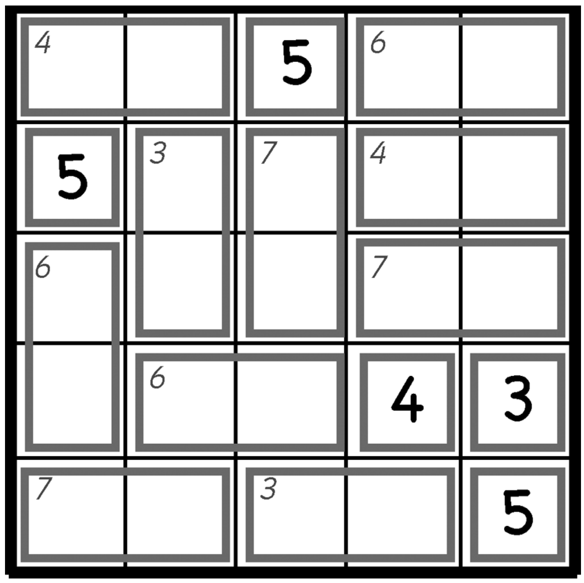


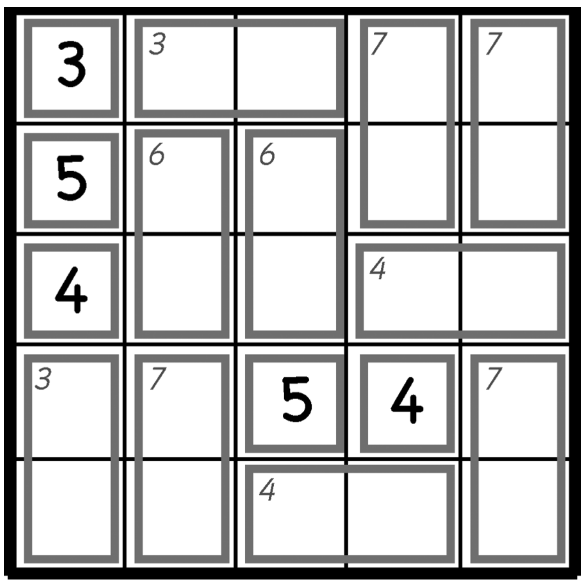
SU DOKU COMPONENT PUZZLES

Moderate 1 & 2

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

Puzzles at this level can be solved by elimination and by thinking about all the possible component pairs of the numbers, especially the numbers 3 and 4.

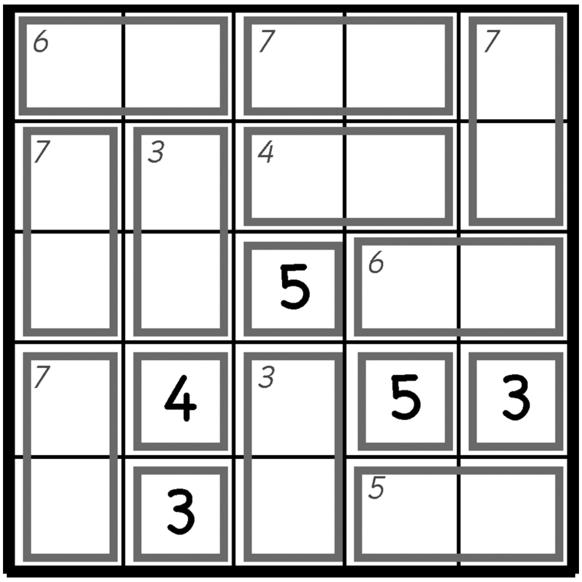


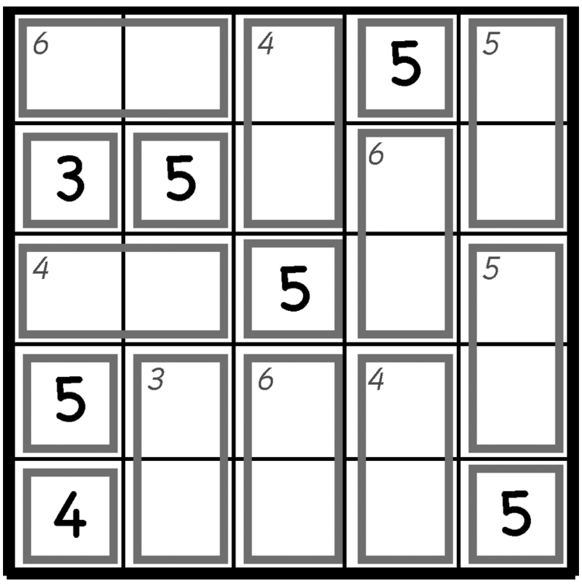


SU DOKU COMPONENT PUZZLES

Moderate 3 & 4

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

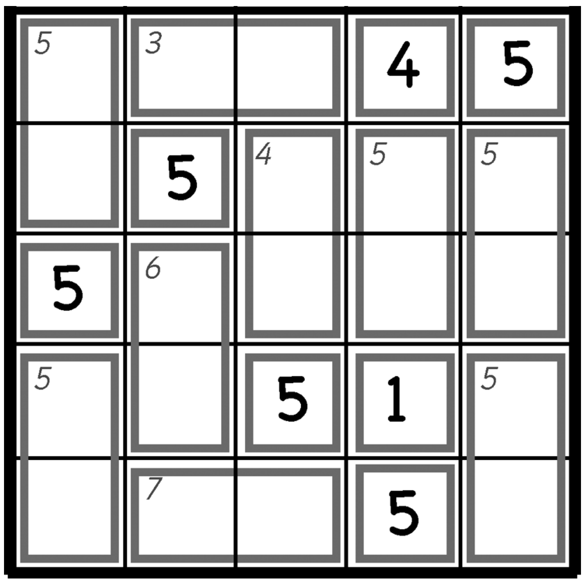


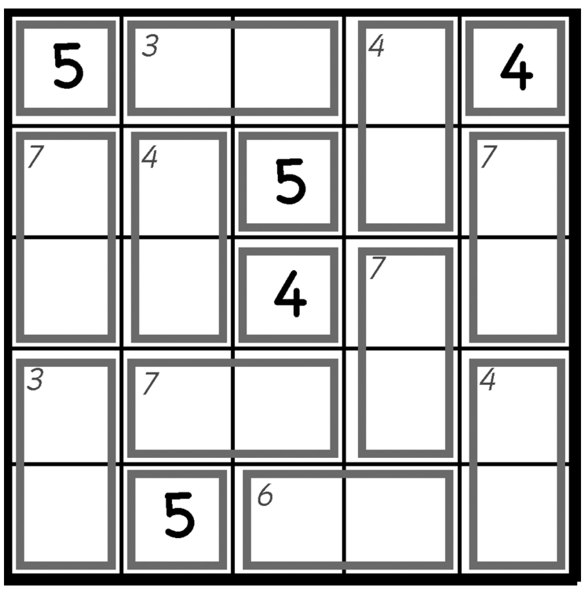


SU DOKU COMPONENT PUZZLES

Moderate 5 & 6

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.



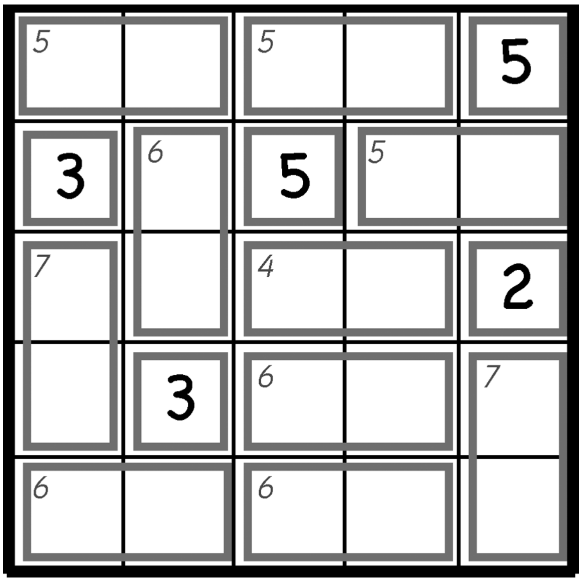


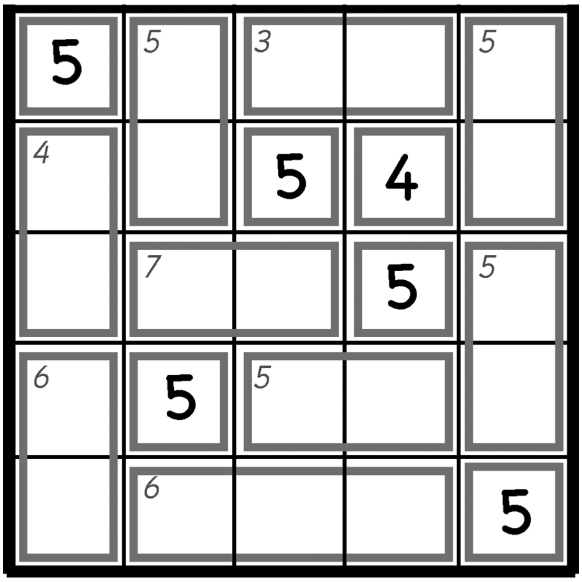
SU DOKU COMPONENT PUZZLES

Intermediate 7 & 8

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

Puzzles at this level can be solved by using all the same techniques as for the moderate puzzles as well as thinking about the total value of each row and column.

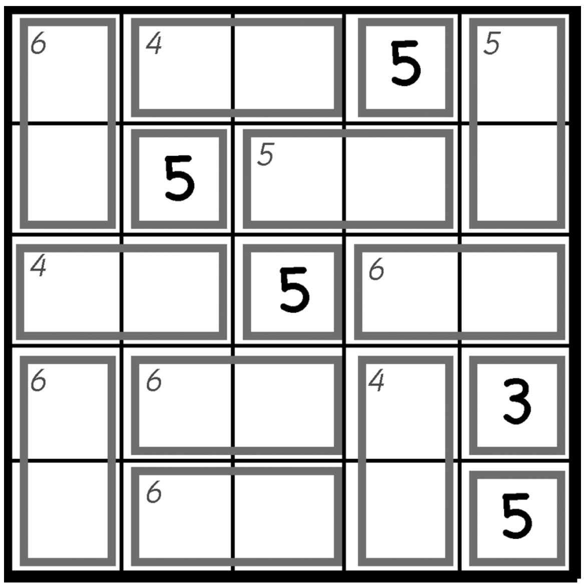


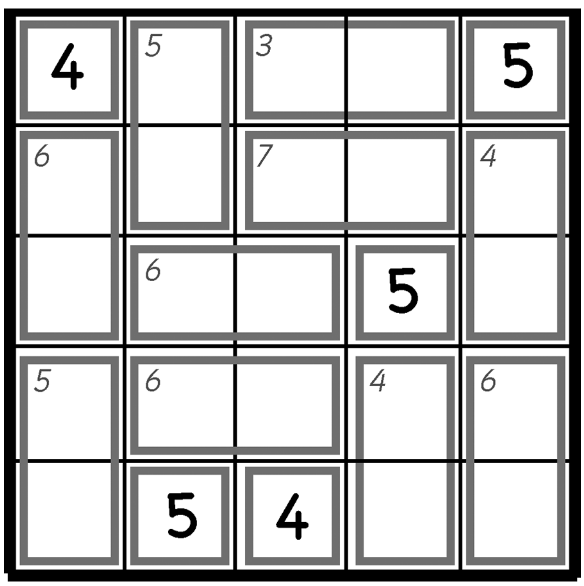


SU DOKU COMPONENT PUZZLES

Intermediate 9 & 10

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

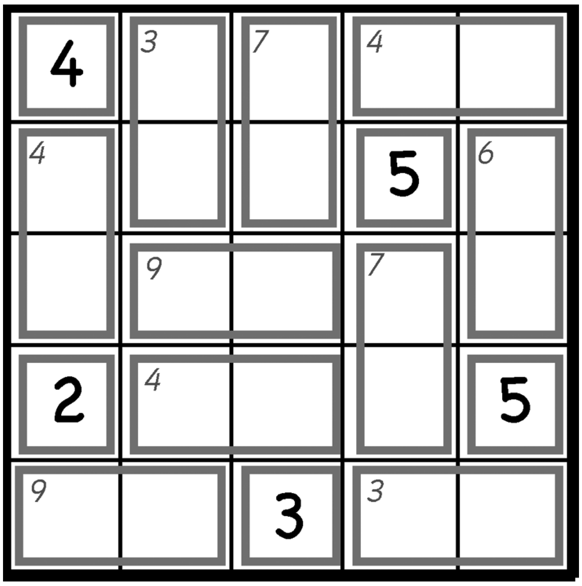


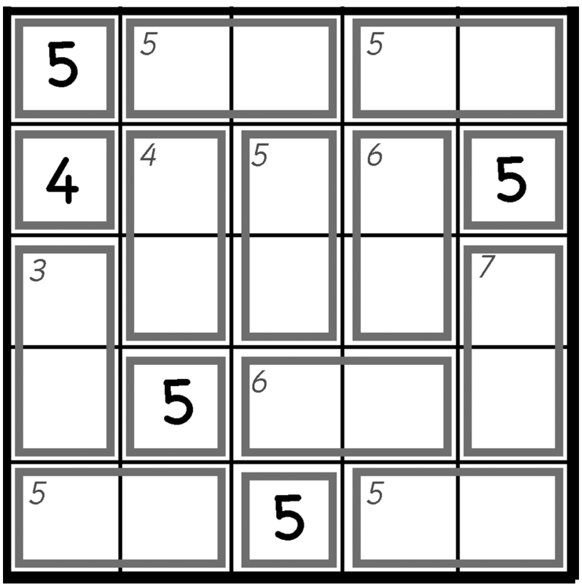


SU DOKU COMPONENT PUZZLES

Intermediate 11 & 12

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

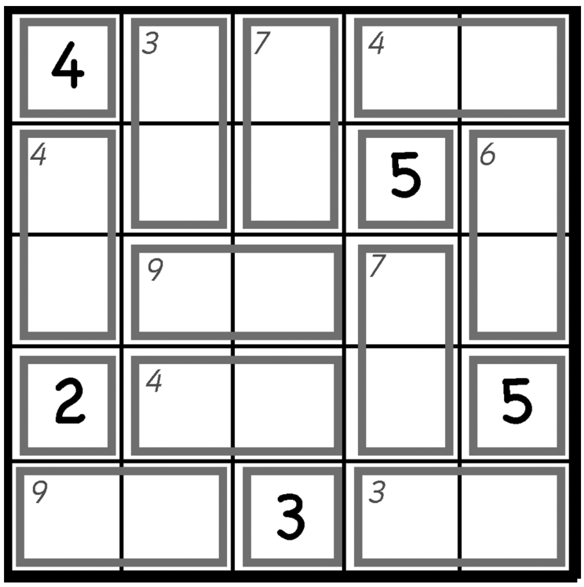


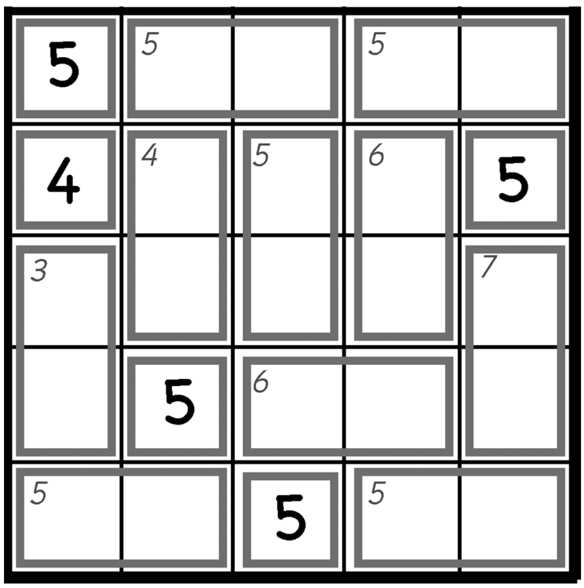


SU DOKU COMPONENT PUZZLES

Difficult 13 & 14

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

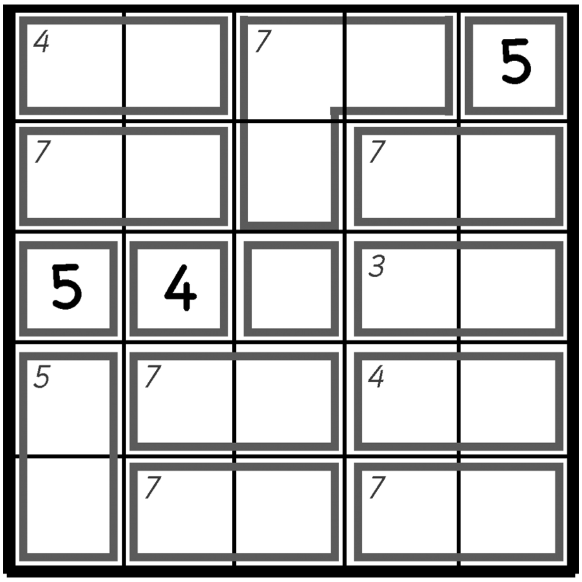


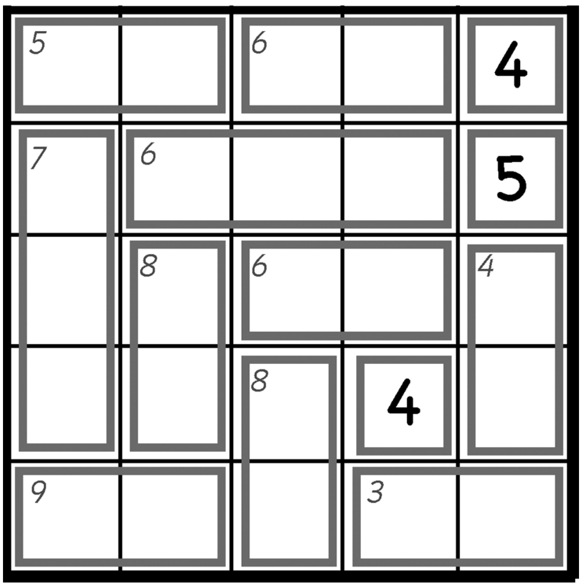


SU DOKU COMPONENT PUZZLES

Difficult 15 & 16

The numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.

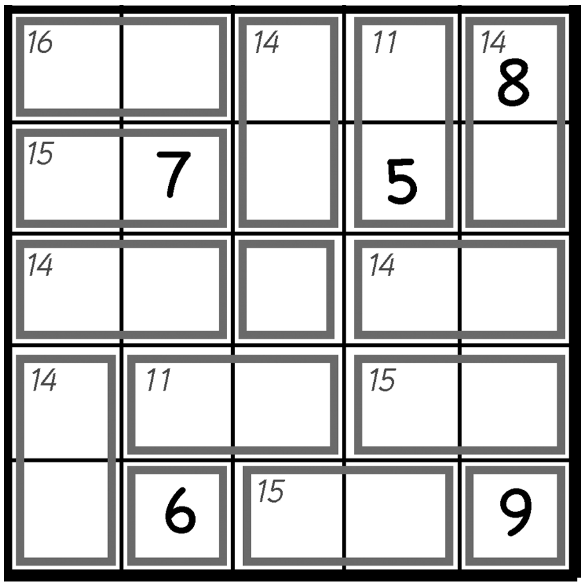


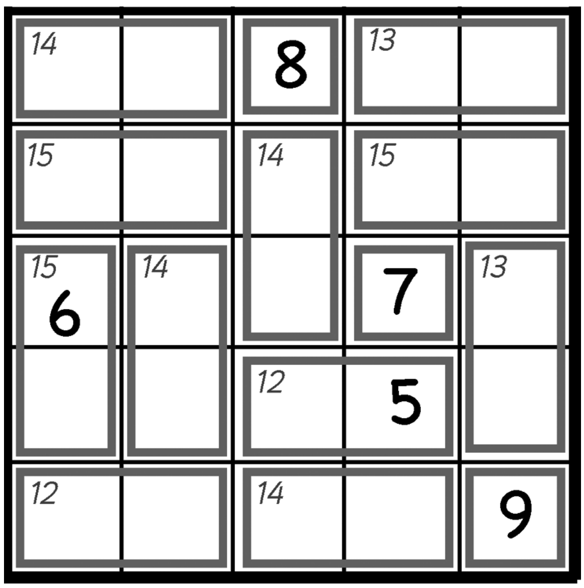


SU DOKU COMPONENT PUZZLES

Difficult 17 & 18

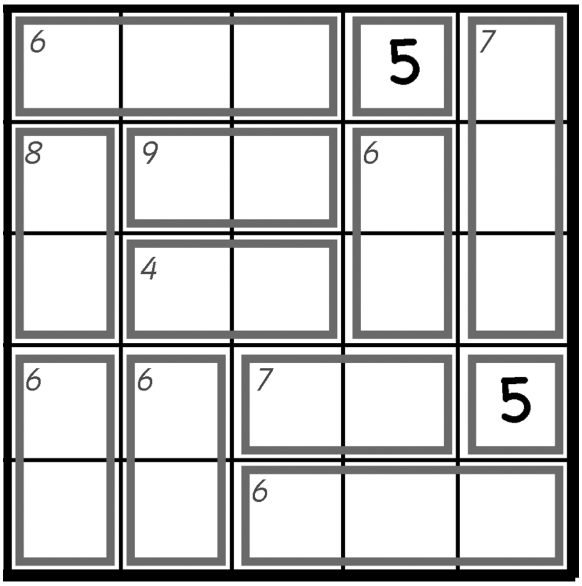
The numbers from 5 to 9 must appear once in each row and in each column. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.



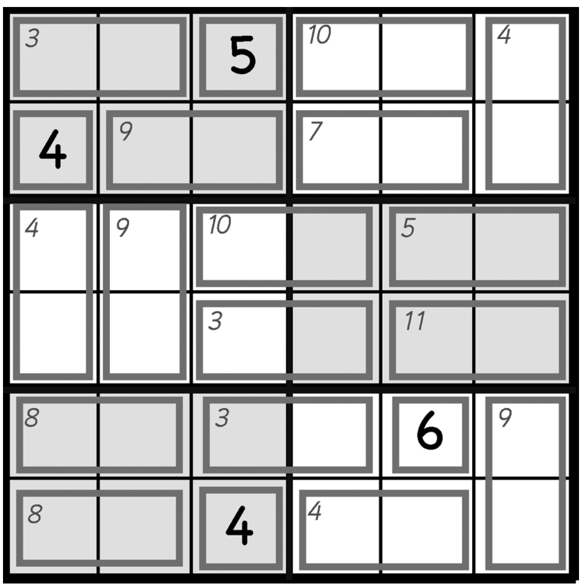


SU DOKU COMPONENT PUZZLES

In these puzzles the numbers from 1 to 5 must appear once in each row and in each column. The thicker lines enclose two different components that must add up to the number in the top left corner of the enclosure.



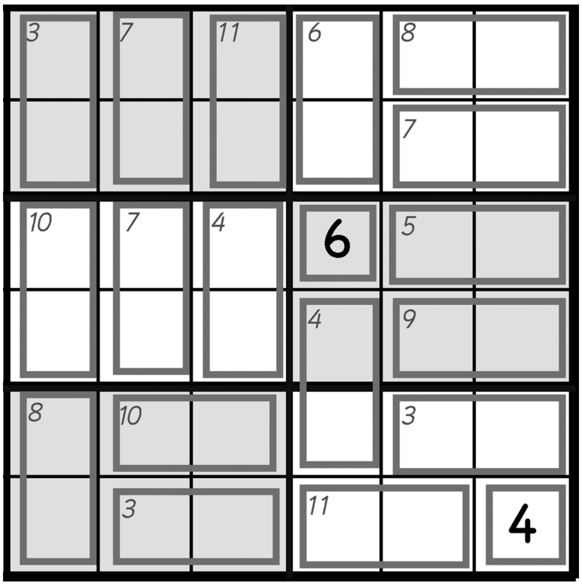
The numbers from 1 to 6 must appear once in each row and in each column. The shaded areas denote cells which must also contain each of the numbers 1–6. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.

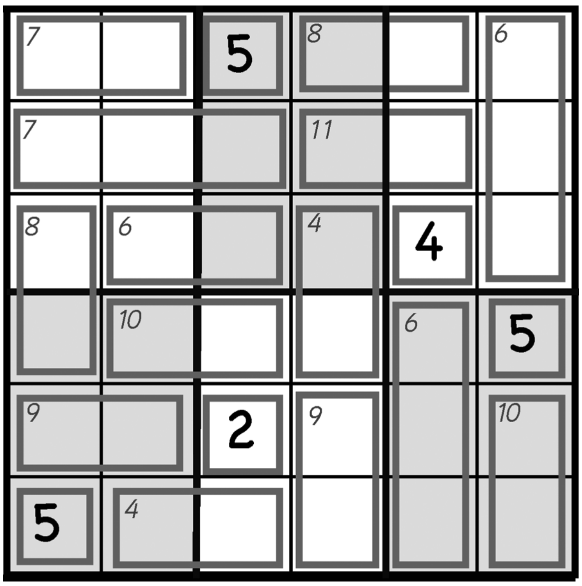


SU DOKU COMPONENT PUZZLES

Difficult 21 & 22

The numbers 1–6 must appear once in each column and row and once in each of the 3 × 2 cells defined by the background shading.

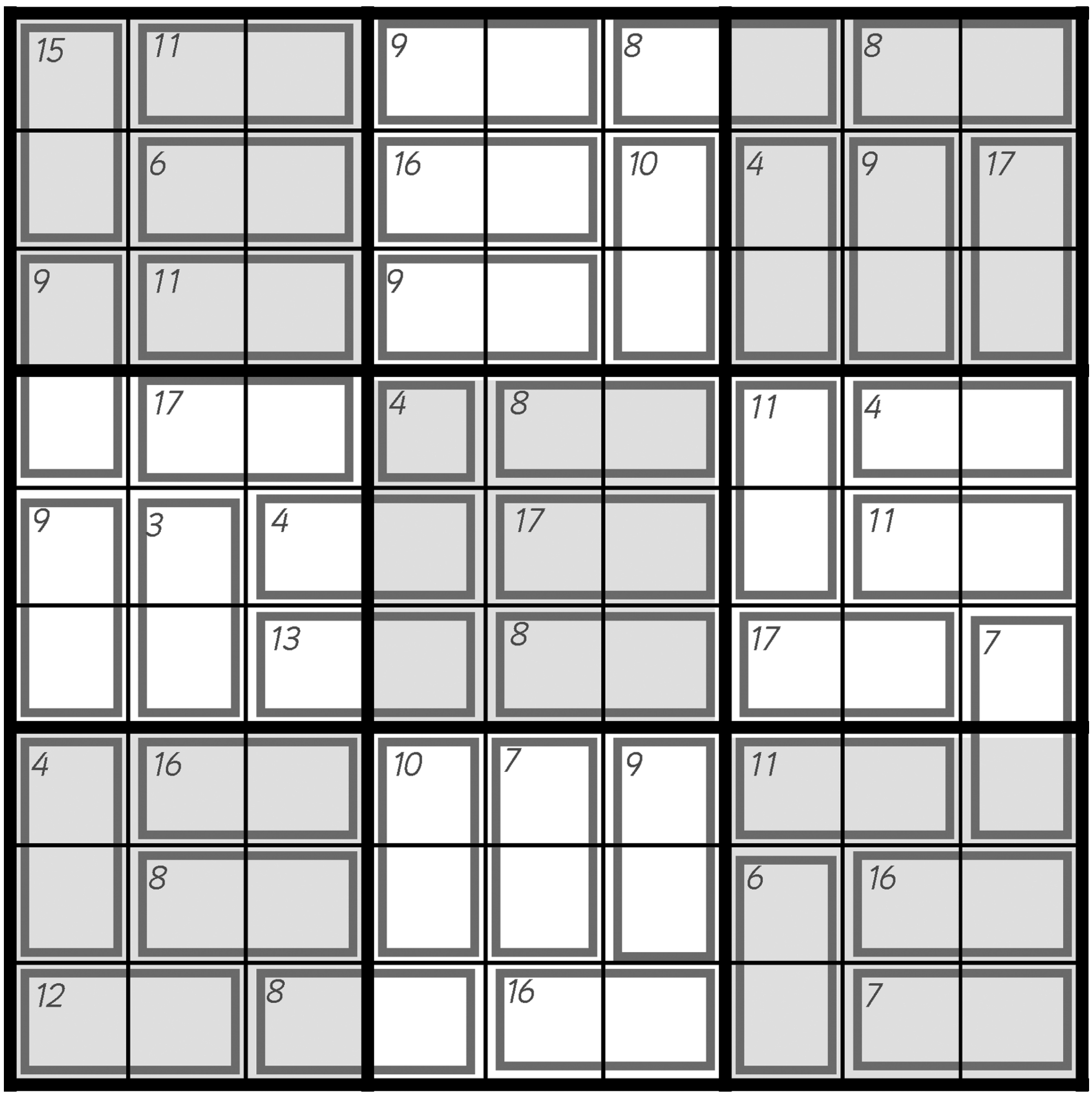




SU DOKU COMPONENT PUZZLES

Difficult No. 23

Insert the numbers from 1 to 9 once in each row and in each column. The shaded areas denote cells which must also contain each of the numbers 1–9. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.



SU DOKU COMPONENT PUZZLES

Difficult No. 24

The numbers from 1 to 9 must appear once in each row and in each column. The shaded areas denote cells which must also contain each of the numbers 1 to 9. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.