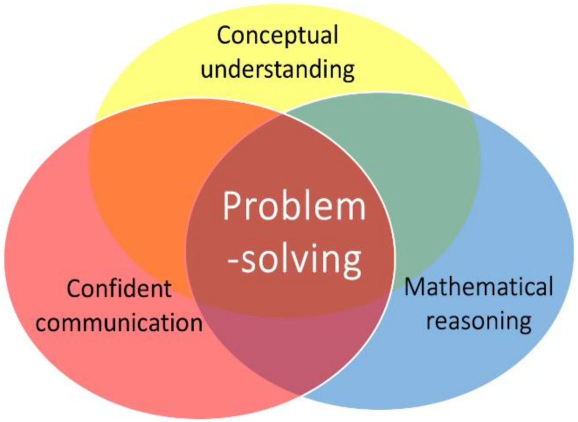
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

10. Multiplication and Division Structures

Use the inverse-of-multiplication structure with ad hoc strategies to solve a division.

The aim here is to reinforce the children's understanding of the relationship between multiplication and division, showing how they can use multiplication to carry out a division. They also develop strategies to find unknown multiplication facts from those facts that they have already established.

Division by multiplying! The children will need to have had experience of finding some multiplications they don't know using number facts they do know in ad hoc ways.
For example, from 6 × 3 = 18, they can work out:
6 × 6 = 36 (by doubling 6 × 3);
6 × 30 = 180 (by multiplying 6 × 3 by 10);
6 × 60 = 360 (by multiplying 6 × 6 by 10), and so on.

Now for the problem: the children have been asked by the car manufacturer 'Nissota' to work out the sizes of petrol tanks needed for different models of its new cars. Nissota wants each car to be able to drive at least 500 miles between visits to filling stations. The company needs to know the minimum size each car's fuel tank must be in litres. Here is a table of the number of miles per litre of petrol each new model can drive:

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Miles/litre	6	8	11	12	14

(See photocopiable resource.)

The teacher models an example with the children: Rival car manufacturer 'Kionda' has a car which goes 7 miles on 1 litre of petrol. To find how much petrol is needed to drive at least 500 miles, we will build up gradually to that figure. This can be done in a number of ways, depending on the child's confidence with mental multiplication by 7. For example:
10 litres will cover 7 × 10 = 70 miles;
so 20 litres will cover 7 × 20 = 140 miles (doubling the result for 10 litres);
and 40 litres will cover 7 × 40 = 280 miles (doubling the result for 20 litres);
but 80 litres would cover 7 x 80 = 560 miles (doubling again, but this is too large);
so 60 litres will cover 140 + 280 = 420 miles (adding results for 20 litres and 40 litres);
so 70 litres will cover 420 + 70 = 490 miles (adding results for 60 litres and 10 litres);
and 2 litres will cover 7 × 2 = 14 miles.

Are the children confident in adapting and developing new calculations from those they already know?

Can they do this methodically to develop a progressive sequence of facts?

With successive calculations, do the children use what they already know to find the largest multiples as soon as possible wherever they can? This will help when developing vertical written methods of division too.

Children need to be confident in scaling multiplication facts by 10 and 100.

	So if we add the results for 70 litres and 2 litres we pass the 500 miles target: 390 + 14 = 504 miles. So to travel at least 500 miles the Kionda car's petrol tank needs to hold at least 70 + 2 = 72 litres. Meena and Charlie now use the same approach each work out the minimum size of each Nissota model's fuel tank in litres. They compare their strategies and check each other's calculation. To simplify this use only single-digit mileage/litre figures or reduce the minimum driving distance to, say, 200 miles. To make it more of a challenge, add some more economical models, for example with fuel consumption at 19 or 23 miles/litre or extend the driving range to, say, 550 miles or 720 miles.	