PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

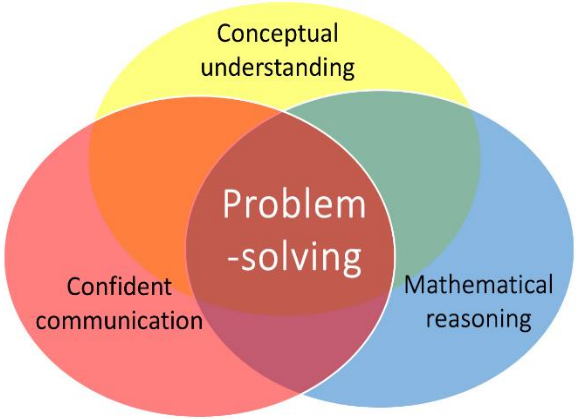
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*<u>References</u>

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

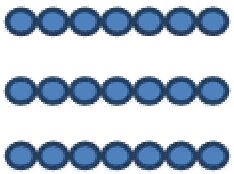
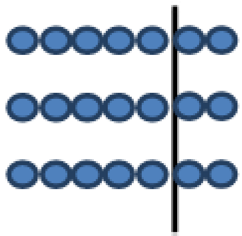
NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

11. Mental Strategies for Multiplication and Division **Solve problems involving multiplying and adding, using the distributive law.** This activity provides practical concrete experience of the distributive law. It is important for children to understand how the distributive law works when they encounter written methods of multiplication – and subsequently in algebra.	<u>**The any-times table**</u> The teacher demonstrates using a multiplication table that some may find challenging but others think they are getting to know, say, 7-times. Set out 'counters' in several rows of 7 on the teacher's 'maths mat' (for example, by using paper cups on a table). Write down the number sentence for the total each time another row is added:  1 × 7 = 7 2 × 7 = 14 3 × 7 = 21, … Explain that if you did not know the 7-times table you could work it out another way using tables you do know. Partition the columns into 5s and 2s. Then write the number sentences for the total each time another row is added like this:  1 × (5 + 2) = 7, 2 × (5 + 2) = 14, 3 × (5 + 2) = 21 (and so on).	The children can arrange a multiplication *m x n* as a rectangular array. Children understand that partitioning the array does not change the total, but restructures the array into two smaller component arrays. The total can be expressed as the sum of these arrays. Children can adapt the strategy of using the distributive law to other multiplications, and hence it is possible to construct any times table from multiplication facts they already know.

	Then discuss how the pattern here could also be written as: (1 × 5) + (1 × 2) = 5 + 2 = 7, (2 × 5) + (2 × 2) = 10 + 4 = 14, (3 × 5) + (3 × 2) = 15 + 6 = 21, … So it is possible to work out any fact for the 7-times table from knowing the 5-times and the 2-times tables! Shelley and Rohan now calculate, say, 9 × 7 from their knowledge of 9 × 5 and 9 × 2; and then 12 × 7 from 12 × 5 and 12 × 2. They go on to explore how to use this approach to work out other multiplication tables, for example: 8-times, 12-times, 13-times, 14-times, 15-times, 17-times. This activity could be simplified by starting with the illustration that the 5-times table is made up of the 3-times and 2-times tables.	Children need to write the distributed calculation for each multiplication table fact, to establish the connection between the number sentence and its concrete representation. Ensure the language is also reinforced, for example: '3 × 7 is the same as 3 × 5 added to 3 × 2'.