

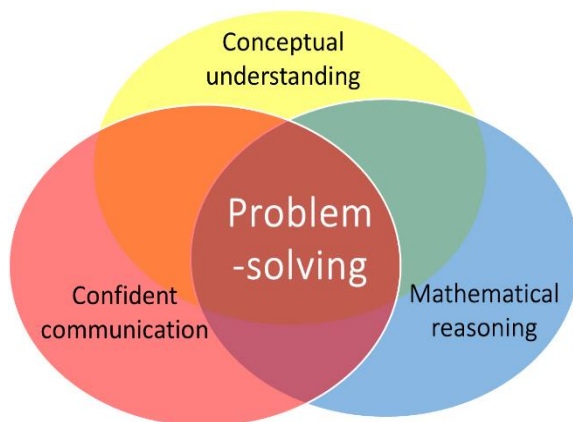
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by **Mastery** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery*, the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report**, particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

11. Mental Strategies for Multiplication and Division

Divide numbers of up to 4 digits by 1 digit, using mental strategies.

The ability to partition numbers is key to the distributive law, which is the underlying strategy for the written methods of division (long and short). This activity is for children to practise solving divisions by 1-digit numbers using mental strategies based upon ad hoc partitioning of the dividend.

This reinforces the inverse relationship with multiplication

Division by partitioning

The teacher models some examples, asking how a number, the dividend could be partitioned (broken up) to make it easier to divide? For example:

$$\begin{array}{r}
 70 \quad + \quad 5 \\
 5 \quad \boxed{\begin{array}{|l} 350 \\ \hline 25 \end{array}} \\
 375 \div 5 = (350 + 25) \div 5 \\
 \text{(chosen because we know } 35 \div 5 = 7\text{)} \\
 = (350 \div 5) + (25 \div 5) = 70 + 5 = 75
 \end{array}$$

$$\begin{array}{r}
 60 \quad + \quad 3 \\
 441 \quad \boxed{\begin{array}{|l} 420 \\ \hline 21 \end{array}} \\
 7 \\
 \div 7 = (420 + 21) \div 7 \\
 \text{(chosen because we know } 42 \div 7 = 6\text{)} \\
 = (420 \div 7) + (21 \div 7) = 60 + 3 = 63
 \end{array}$$

$$\begin{array}{r}
 120 \quad + \quad 20 \quad + \quad 4 \\
 6 \quad \boxed{\begin{array}{|l} 720 \\ \hline 120 \\ \hline 24 \end{array}} \\
 864 \div 6 = (720 + 120 + 24) \div 6 \quad \text{(chosen because we know } 72 \div 6 = 12\text{)} \\
 = (720 \div 6) + (120 \div 6) + (24 \div 6) = 120 + 20 + 4 = 144
 \end{array}$$

Can children adapt the strategy of using the distributive law to division, and see how each part expresses a subset of repeated subtractions of the divisor?

Can children use sketches to illustrate the partitions?

Children need to be confident in scaling multiplication facts by 10 and 100.

	<p>Charlie and Meena each try the following then compare how they each chose to partition the numbers and check each other's calculation: $552 \div 4$; $276 \div 6$; $660 \div 8$; $747 \div 9$; $1750 \div 7$; $2568 \div 4$; Encourage children to see if there are other ways to partition the same numbers: e.g. $441 \div 7 = (350 + 91) \div 7$. Ask them to explain which partitions they found to be the most helpful or the most efficient and why. Simplify or challenge with simpler/larger numbers for the dividends. Challenge higher attainers with some straightforward 2-digit divisors, for example: $1001 \div 11$; $3756 \div 12$; $2775 \div 25$; or some divisions that yield remainders.</p>	
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