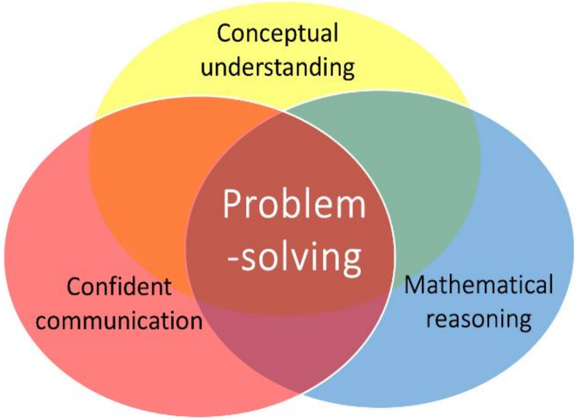
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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<td>12. Written Methods for Multiplication and Division

Divide numbers up to 4 digits by a two-digit number (leading to understanding and using the formal methods of long and short division).

Solve problems involving multiplication and division using their knowledge of factors and multiples.

Both long and short division formal methods rely on the structure of repeated subtractions (inverse of multiplication) and both these methods partition numbers into their powers of ten: H, T, U and so on. In order to work, at every step it is important to find the largest subtraction (chunk) possible within each power of ten (or order of magnitude), and</td>
<td><u>The Meatiest Chunks!</u> Individually, then peer-assessment in pairs, and exchange solutions with other groups.

The teacher tells the children that a friend has been doing division by subtracting chunks, but finds that each has taken a long time and used up a lot of paper. Show one of your friend's divisions. Could they write a shorter solution? Can the children see where the friend could have found bigger chunks to subtract, and have less to write out?

For example, to divide 544 by 17, the friend has subtracted ten 17s (to leave 374), another ten 17s (to leave 204), another ten 17s (to leave 34) and then two 17s to complete the division, giving the answer as 10 + 10 + 10 + 2 = 32. This is set out as shown below. Explain how this could be done quicker, for example, by starting with a chunk of twenty 17s (340), or even a chunk of thirty 17s (510), as shown below. It can also be written more simply, as shown on the right.

32
17) 544
10 x 17 | 170
374
10 x 17 | 170
204
10 x 17 | 170
34
2 x 17 | 34
32 x 17 | 0

Could be simplified to:
32
17) 544
30 x 17 | 510
34
2 x 17 | 34
32 x 17 | 0

or written more simply as:
32
17) 544
30 | 510
34
2 | 34
32 | 0

Provide a number of such examples of division set out using ad hoc subtraction and challenge the children to find bigger chunks to subtract.</td>
<td>Does the child recognise the distributive law being applied to division?

Do they see that in effect the number is being partitioned into subgroups which are helpful multiples of the divisor?

When there is more than one subtraction for any power of ten, does the child see a quicker way of estimating the highest multiple?

Can the children quickly work out some helpful multiplication 'facts' for a divisor, to find the largest chunk to subtract from the dividend?</td>
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<td>then repeating this with the remainder of the number.

By the time children are using large two digit divisors it is inefficient and error-prone to model this concretely with p.v. counters, so it is not surprising to see children writing out several multiplications to find the largest chunk!

Ofsted (2011) identified that schools had difficulty in teaching the formal methods of division and that those who taught written division by chunking as a step towards this often did not successfully help children learn to look for the bigger, more efficient chunks. This activity gives children practice in doing this.</td>
<td>Meena and Charlie each try to simplify the division calculations. They aim to use the shortest number of steps, each time trying to find the biggest *chunk* that can be subtracted from the *thousands, hundreds, tens* and *ones* in turn. They assess each other's calculations, to check that they are correct and to see who has the most efficient solution.
For example:

117
8)936
50 x 8 | 400
536
50 x 8 | 400
136
10 x 8 | 80
56
5 x 8 | 40
16
2 x 8 | 16
117 x 8 | 0

73
13)949
20 x 13 | 260
689
20 x 13 | 260
429
20 x 13 | 260
169
10 x 13 | 130
39
2 x 13 | 26
13
1 x 13 | 13
73 x 13 | 0

48
27)1296
20 | 540
756
20 | 540
216
5 | 135
81
2 | 54
27
1 | 27
48 | 0

32
34)1598
20 | 680
918
20 | 680
238
5 | 170
68
2 | 68
47 | 0

Then they compare their solutions with other children's.
To simplify, provide calculations with single-digit divisors. Extend by introducing divisions which produce quotients with fractional parts to be simplified. E.g. 935 ÷ 12 = 77 $^{11}/_{12}$
N.B. It is more appropriate to develop the most compact standard form of long division before applying to numbers which produce decimal places.</td>
<td>E.g. when dividing by 17:
2 x 17 = 34 *double 17*
4 x 17 = 68 *double 34*
10 x 17 = 170
5 x 17 = 85 *halve 170*

From these, other multiples of 17 could be found easily from adding or multiplying combinations of the above.</td>
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