PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN KS1

NATURE OF THE ACTIVITIES SUGGESTED HERE

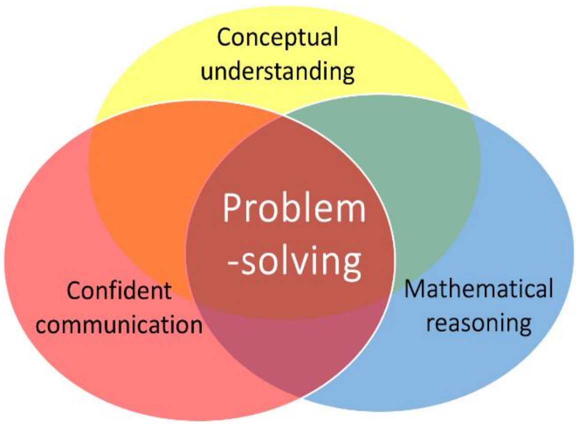
With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

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12. Written Methods for Multiplication and Division

Recall and use multiplication and division facts for the 2, 5 and 10-times multiplication tables, including recognising odd and even numbers.

This activity is a 'real-life' use of money, that of selecting and working out the value of a group of coins.

While it is inappropriate to introduce formal written methods for multiplication and division to children in KS1, it is helpful for children to see that money can be counted using multiplication and practise strategies of multiplication and division as repeated additions and subtractions.

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<u>**Counting up the coins**</u> Demonstrate to the whole class, then children work in pairs. They will need:

- Purse or wallet containing mixed coins of different denominations 1p, 2p, 5p, 10p;
- Tray of additional coins;
- Number of items labelled with different prices in multiples of 2p, 5p and 10p. For example: A pencil costing 16p, and eraser costing 35p and a book costing 60p, etc.

First Emily and Luke must sort the coins in the purse into their different denominations. After this, they each write down the value of the coins of each type as multiplication. For example:

- 6 × 1p = 6p; 9 × 2p = 18p; 5 × 5p = 25p; 7 × 10p = 70p.
- Luke and Emily now swap their purse with another pair and find the values of the coins in a different purse.
- After this they use the coin tray to help them count up and write down the calculation for each item they may buy, according to these rules:
    - If the price is an exact multiple of 10p, write the number of 10p coins needed, for example: for the book, write 6 x 10p = 60p.
    - If the price is an exact multiple of 5p, write the number of 5p coins, for example: the eraser is 7 x 5p = 35p and the book is 12 x 5p = 60p.
    - If the price is not a multiple of 5p or 10p, but it is an even number, write the number of 2p coins, for example the pencil would be 8 x 2p = 16p.

The activity can be simplified by reducing the numbers of coins or limiting the denominations.

To extend the children, when they have counted the coins from different purses, challenge them to work out the total values of each purse, for example by using mental or informal strategies to calculate, say, 70p + 25p + 18p + 6p (as in the example above).

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Are the children secure in understanding that a coin has a ***unitising*** or ***1:many relationship*** with other coin denominations?

Do they add these correctly according to their value, not simply counting the number of coins?

Can children discuss which purse may contain the largest sum of money and why they think this is, say by comparing the number and relative values of the different coins?

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