Chapter 13 - Multiples, Factors and Primes

Digital sums and roots 8 minutes and 42 seconds

Let me remind you what a digital sum is first of all, if you have a number like two hundred and fifty-six and add the digits, two add five add six which gives you thirteen that's the digital sum of two hundred and fifty-six. Seven hundred and ninety-two, the digital sum is eighteen. If you then continue…. continue adding the digits in the answer until you get down to a single digit number, that's called the digital root. So, the digital root of two hundred and fifty-six um….is the sum of the one and the three in thirteen which is four. Seven ninety-two we find the digital sum-eighteen. Add the digits in that until we get down to a single digit result, the digital root is nine. Well, what's this all about? Well exploring digital roots and digital sums gives children the opportunity to enjoy the pattern in numbers. To explore pattern, to make…. errr…conjectures about what's going to happen next. To articulate the patterns, they find in the form of generalisations.

So, let's give an example, multiplication tables in particular are a fertile area for exploring digital roots. Let's have a look at the nine times table to begin with. So here we go umm…we're up to eleven nine's there, twelve nines', thirteen nines' and so on. Let's find the digital sums first of all. Adding the digits, twenty-seven gives me nine, thirty-six gives me another nine. Another nine, here comes another nine. They're all nines so far, all the way up to ten nines, the digital sum is nine. Well that's an extraordinary result for a... for the…for a start. And certainly, something children should know because it helps them to remember the pattern in the nine times table. The next one, ninety-nine, eleven nines that has a digital sum of eighteen, which isn't nine but is a multiple of nine. A hundred and eight is nine, a hundred and seventeen that gives us nine again, and if you go on, what you discover is that all the digi…digital sums are either nine or a multiple of nine.

Later on for example, you get to eight thousand five hundred and sixty-eight and the digital sum there, eight add five add six add eight, that's twenty-seven, which is again, a multiple of nine. Now this means that all the digital roots turn out to be nine, those are nine already and those are. And the digital root of ninety-nine is the sum of one and eight in eighteen which is another nine and this eight thousand five hundred and sixty-eight, the digital root there, two add seven is nine. So, here's an amazing result, for every multiple of nine, the digital root is nine. Now that's only true of the nine times table but yes, it is a surprising result. And it means you can check whether a number is or is not a multiple of nine very easily-just add the digits. So, take this big number here, add up the digits to get thirty-five continue adding the digits till we get the digital root which is eight, that's not nine, so it's not a multiple of nine. Seven hundred and eighty-two thousand, five hundred and sixty-seven is not a multiple of nine. Six hundred and eighty-seven thousand, two hundred and thirteen has a digital sum of twenty-seven and a digital root of nine, so that one is a multiple of nine.

Well, we can explore patterns in the multiplication tables for other numbers. What about the multiples of six, here they come, I wonder if we'll get a similar kind of result, with these multiples. Let's see what happens, there's err…twelve sixes, thirteen sixes and so on. Well let's find the digital roots then. Err…so we're going to go from six we add up the six and get six. Twelve, that gives us one add two which is three, eighteen, one add eight is nine… (clears throat) …getting different answers here. Twenty-four, two add four is six again. Thirty, three add zero is three, thirty-six, three add six is nine. I think I can see a pattern emerging, six, three nine, six, three nine the next one is six, the next one should be three. Forty-eight, four add eight is twelve, add the one and the two in twelve to get the digital root and that gives us three, yes, and here's a nine, looks like it doesn't it. Six, is this a three? Sixty-six, six add six is twelve, one add two makes three yes. This is a nine, seventy-eight, what will that be? Should be a six. Seven add eight is fifteen, and one and the five in fifteen, add up to six. Now isn't that fantastic, there's another very strong pattern there. We can predict now what the next one will be should be another three let's see, we check that conjecture, eighty-four. Eight add four is twelve and the one and two in twelve gives us the result-three. So, we found the pattern in the six times table.

What about the two times table? Well, we'd expect something fairly straightforward with a simple multiplication table like this. Let's look again at the digital roots. Two of course, four, six, eight they're already single digits, ten, now we add up the one and the zero, ahh and we get one, what's going on here? Add up the one and the two in the twelve and we get three and then five and then seven, what would we expect next? Two, four, six, eight, those are the even single digit numbers. One, three, five, seven, those are the odd ones perhaps the next one is nine, yes, it is. And then what happens? We go back to the two again. And then the four. And then the six. And then the eight, so what are we expecting next? Will it be another one and then we start again on the odd numbers, one, three, five, seven, nine? Let's check. Twenty-eight, two add eight makes ten. And the one and zero in ten add up to one, yes, so there's an interesting pattern, in the multiples of two.

Well in the same way, we could explore the patterns in the multiples of three. You find the digital roots of the multiples of three follow this sequence. The same as the six times table, but starting with a three. Three, six, nine. Three, six, nine. Three, six, nine and so on.

You could look at the multiples of seven, now the digital roots of the multiples of seven begin like this, seven, then five, that's for fourteen, then three, that's for twenty-one, then one, eight, six, four, two. Can you predict what the next one will be? And the next after that? And the next after that? Can you find the sequence of digital roots for multiples of seven? Well go on investigate, try the multiples of five, the multiples of eight, the multiples of eleven. See what you can discover about the patterns in digital roots.