Chapter 13 - Multiples, Factors and Primes

Prime factorisation 6 minutes and 59 Seconds

Prime factorisation. A fundamental theorem of arithmetic is that any positive whole number greater than one can be expressed as a product of prime numbers uniquely. There's only one way of doing it. Now here I've written down for us the first eight prime numbers, two, three, five, seven, eleven, thirteen, seventeen and nineteen. I'm going to show you how you find the prime factorisation of any particular number. So let's take a simple example to start with, two hundred and forty. We're going to express that as the product of prime numbers. So, what we do is to take each of the prime numbers in turn, starting with the smallest, two and then three and then five and check whether these, this number two hundred and forty can be divided by them. So, starting with the two, can two hundred and forty be divided by two well yes it can, its two multiplied by a hundred and twenty. Is there another two in the hundred and twenty as a factor? Yes there is so, that's two multiplied by sixty. Can we get another two out of the sixty as a factor? Yes, we can that's two multiplied by two multiplied by thirty. And the thirty, well there's another two as a factor in there. Two multiplied by two multiplied two multiplied by two multiplied by fifteen. And then finally can we get another two out of the fifteen? No, we can't, two is not a factor of fifteen. So, we got to the next prime number-three. Is that a factor of fifteen? Yes, it is. It's three times five so you now have two multiplied by two multiplied by two multiplied by two multiplied by three multiplied by five and five itself is a prime number. We can see that in our list of prime numbers at the top. So, we now have expressed two hundred and forty as a product of prime numbers. And that is the only way it can be done.

Let's take another example, one thousand and one. We're going to try to express that as the product of prime numbers. Now is two a factor of a thousand and one? No clearly it isn't. Because that's an odd number. Is three a factor of a thousand and one? No, because the digital sum of a thousand and one is two and that's not a multiple of three, that's a pattern that we discover in digital sums for multiples of three. Is five a factor a thousand and one? No because it doesn't end in a zero or a five. So, what about seven, is seven a factor of a thousand and one? Well if we divide a thousand and one by seven, we get a hundred and forty-three. So yes, seven is a factor. We now try seven again, is seven a

factor of a hundred and forty-three? Clearly not. Seven goes into fourteen yes but then we're left with a three. So, let's try the next prime number. That's ele…that's eleven, is eleven a factor of a hundred and forty-three? Yes it is, it's eleven multiplied by thirteen and thirteen is a prime number, so we've come to the end here, we've expressed a thousand and one as the product of the three prime numbers seven, eleven and thirteen.

One more example, three thousand eight hundred and fifty. We start with two again. Is two a factor of that? Yes, it is because it's and even number, so divided by two, one thousand, nine hundred and twenty-five. So, this is err…equal to two multiplied by one thousand, nine hundred and twenty-five does two go into the one, nine, two five? No because it's an odd number. Is it a multiple of three? That's the next prime number ummm…adding up the digits, ten, eleven, twelve, seventeen that's not a multiple of three, ok. Now try five. Yes, clearly it is a multiple of five because it ends in a five. So that's two multiplied by five multiplied by something. How many fives are there in one thousand, nine hundred and twenty-five? Do the division, three hundred and eighty-five. Ok, so we have two times five times, now can we get another five out of the three hundred and eighty-five as a factor? Yes, we can. It's five multiplied by umm…seventy-seven. So, now we have two multiplied by five multiplied by five multiplied by seventy-seven. Now we can't get five anymore out of that, but we can get a seven that's the next prime number. It's seven multiplied by eleven which itself is a prime number so we finished we have expressed three thousand, eight hundred and fifty as the product of prime numbers. Two, five, five, seven and eleven.