PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

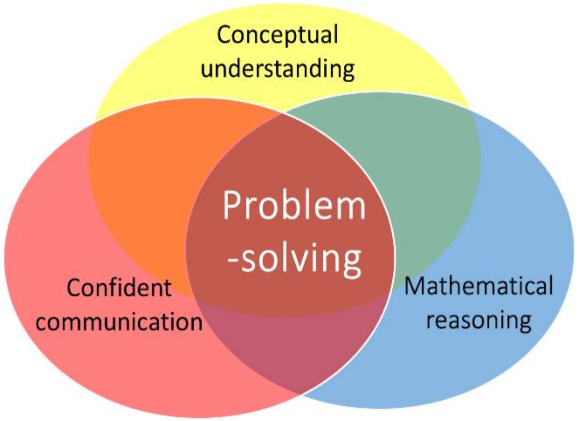
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

13. Natural Numbers: Some Key Concepts

Recall and use multiplication facts up to 12 × 12, recognising factors and multiples.

Understanding and identifying prime numbers, by elimination of other multiples.

Solve problems involving multiplication, and reinforce multiplication as repeated additions.

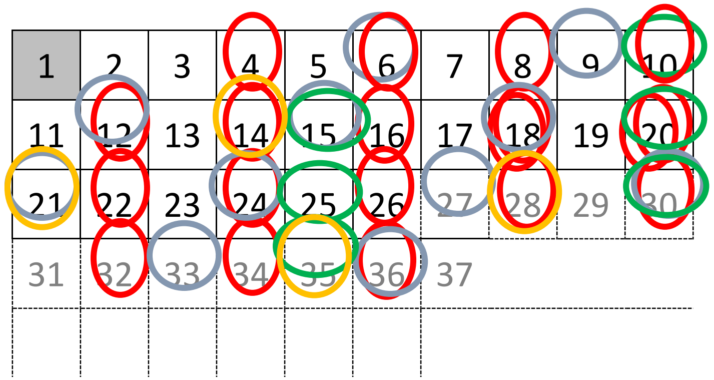
This activity provides practical concrete experience of multiples and reveals prime numbers in a very visual way. It is an activity known as the Sieve of Eratosthenes, after the Greek mathematician to whom it has been attributed.

<u>**The Greek sieve**</u> Children work in pairs. Each pair will need:

- Paper 100-square (numbered 1–100); See photocopiable resources;
- Various coloured pencils.

Starting with the number 2 and taking each successive number in turn, Shelley and Rohan use a coloured pencil and, skipping the 2, they circle every subsequent number that is a multiple of 2. They use a different coloured pencil and, skipping 3, they circle every successive multiple of 3. Where a number has already been circled as a multiple of 2, it is circled again if it is a multiple of 3. Thus, numbers that have several prime factors will be circled several times.

Shelley and Rohan then start with the next available *blank* number after 3 and find multiples of this. As 4 is already circled (because it is a multiple of 2) they move on to 5, then 7, 11, 13 and so on. For example:



If children do not already recall which numbers are multiples of others, can they accurately count on by their chosen number along the number square?

Do they recognise these multiples from their recall of multiplication facts (times tables)? Do they see that every times table is endless?

Do children recognise that nearly half of all the natural numbers are sieved out as multiples of 2?

Do the children visually recognise which numbers are multiples of others through the layout colour patterns created on the number square?

This approach to 'sieve' out multiples leaving just the prime numbers is one of the methods that computers use to calculate very much larger prime numbers.	When they have done this they will see that there are several numbers that have not been circled at all. Why is this? Apart from the number 1, It is because all of these numbers have only two factors: the number itself and 1, e.g. 2 × 1, 3 × 1, 5 × 1, etc. As the number 1 only has only 1 factor it does not fit that pattern. All the others that have not been circled, starting with the number 2, have just two factors and are what we call ***prime numbers***. Just like Eratosthenes, Shelley and Rohan have just discovered all the prime numbers less than 100! A simpler introduction would be to find multiples of each number on a separate sheet from all the others, so that they can more easily see the visual pattern produced on the number square for each times table.	Do they realise that the number of times a number is circled records how many factors it has (other than the number itself and 1)?