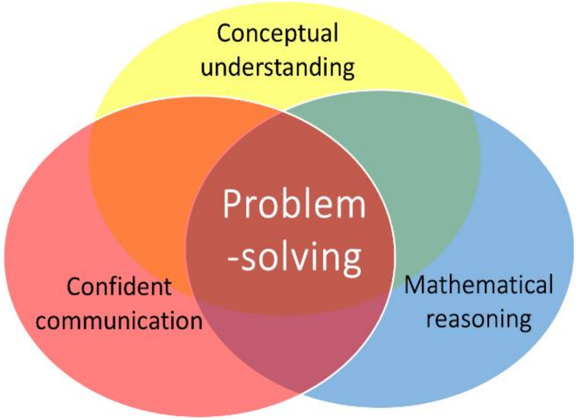
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

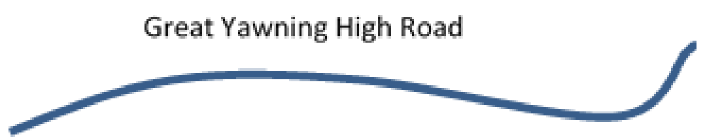
NATURE OF THE ACTIVITIES SUGGESTED HERE

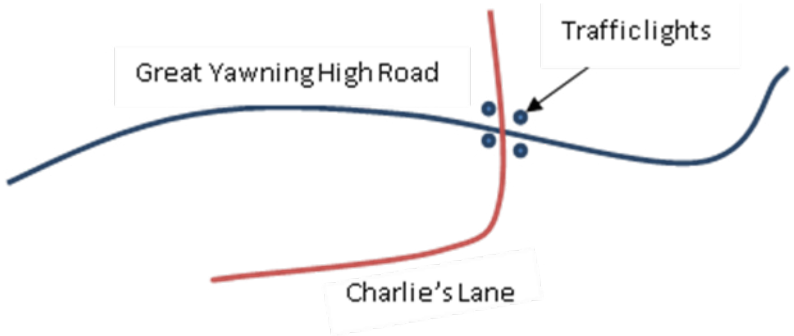
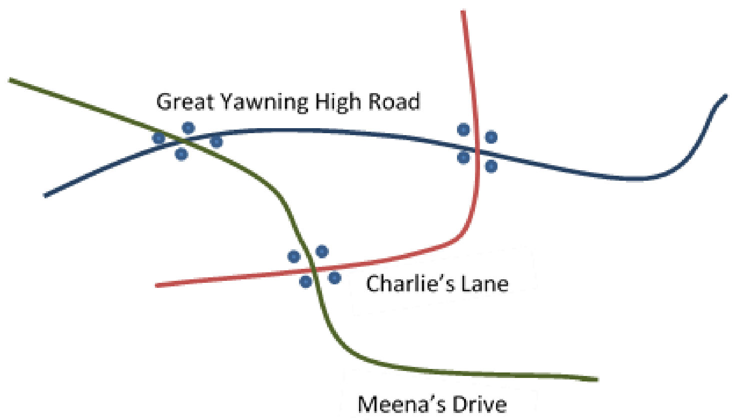
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

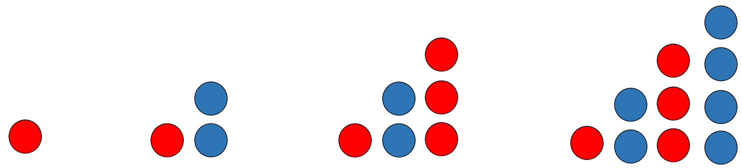
Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

13. Natural Numbers: Some Key Concepts **Explain, extend and infill other number sequences which follow a progressive sequence.** **To explore and recognise the sequence of triangular numbers.** This is a practical exercise for exploring number patterns to find the sequence of triangular numbers. N.B. It is not necessary for the children to arrive at a formula for the sequence, which is difficult to see unless they can double and factorise the pattern for the sequence.	<u>**Road planning**</u> After demonstration, children explore the activity in groups of 3 or 4, to discuss and share ideas. Resources needed for each group: • Sheet of A3 paper; • Different colour felt pens; • Counters to model the increasing sequence of intersections; • A construction manager's hard hat for the teacher (optional!). The children are town planners for *Great Yawning Development Corporation*. *Great Yawning* is going to be a new town, and the children's project is to design the road network. Wearing your 'hard hat', introduce the project to the class. *Great Yawning* is going to be built on land through which an existing main road already runs. This will be renamed *Great Yawning High Road.* Show them a picture of this road:  Explain that new roads need to be added to the plan to create the new town. The children can design any routes and shapes for the roads that they wish, but each time a new road is added, it must have ***one and only one intersection with every other*** road. It is also important for safety, and to reduce traffic congestion, that only two roads cross at any point. There will need to be sets of traffic lights at each intersection. How many sets of traffic lights are needed will depend on the number of intersections in the whole road network.	Do they ensure that each new road crosses every existing road once and once only? The may need to extend the length of some of the existing roads to do this. After they have added several roads, can they identify any patterns here? What is the difference in the number of intersections when each is added? (They should see that the difference increases by 1 with each new road they add.) Could they predict how many intersections there will be for any given number of roads in *Great Yawning* without drawing them?

	Model a design for the network with two roads:  How many intersections with just two roads? (1) Now add a third road: There are three roads, requiring 3 intersections. Now hand the project over to the road designers. Ask them to write how many intersections there are for the total number of roads every time they add another road.  1 road requires 0 intersections, 2 roads require 1 intersection, 3 roads " 3 intersections, 4 roads " 6 intersections, … and so on.	When arranging the sequence with counters, do they recognise the *triangular* shape of each number in the sequence?

Explore further this pattern of the increasing number of intersections. Ask the children to see if they can use counters to arrange a sequence of the number of intersections every time a new road is added:



No. of roads:	1	2	3	4	5
No. of intersections:	0	1	3	6	10

It is called a series of *triangular* numbers. Why might that be?