PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

3-4

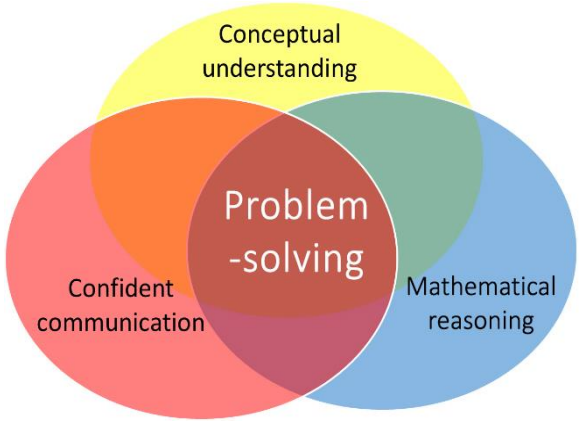
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

14. Integers: Positive and Negative **Interpret negative numbers in context.** **Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.** This activity is to help children understand how negative numbers are used to represent a debt which is owed to someone else, the reward for steady savings and the potential impact that borrowing brings.	**Pocket money overdraft** The teacher demonstrates to the class. Then children work individually and compare solutions in pairs. Model an example: Shelley wants to download some music, which she sees is on sale at half-price just for this week at £9. In her piggy bank she has £4. Her parents will lend her just enough money to enable her to download he music this week. How much would this need to be? She needs to borrow at least £9 – £4 = £5, so she will owe her parents £5. Write down her pocket money balance as – £5. Explain that this negative value means Shelley does not have any money in her piggy bank and that she owes a debt of £5 to her parents. She will need to repay all of the debt from her future pocket money before she has money to spend again. Shelley receives £2 each week in pocket money. How many weeks' before she will have a positive amount in her piggy bank again, and what (balance) will she have in her piggy bank then? Add £2 in the first week: – £5 + £2 = -£3 Note that this does not actually put any money into her piggy bank, but her parents keep the pocket money she would have received and that simply reduces the debt she owes her parents. … in the second week: – £3 + £2 = - £1 … in the third week: – £1 + £2 = +£1 Note that this has now cleared her debt and she will receive £1 actual cash to put in her piggy bank. Show that this could be more quickly worked out as the number of £2 amounts that will give Shelley more than £5 she owes. Write 3 × £2 = £6; Then –£5 + £6 = +£1.	Can the children explain their addition and subtractions using an empty number line? Can the children use the vocabulary of balance and know why we use a negative value for a debt we owe? Do they picture a balance of – £2 as: 'We have nothing, yet we still need to pay £2?

	Now set several more of these problems for the Rohan and Shelley to solve individually and assess with each other. For all of these problems: • How much do they need to borrow? • How many weeks to repay? • What would be the first positive balance afterwards? For example: 1. Rohan has £2 in his piggy bank. He wants to buy a present for his brother's birthday which costs £9. How much would he need to borrow? He receives £3 pocket money each week. 2. Alexi has £1, and wants to spend £6. She receives £3 per week. 3. Woljca has £5, and wants to spend £11. He receives £2 per week. 4. Danielle has £14, and wants to spend £22. She receives £4 per week. 5. Harley has £18, and wants to spend £27. He receives £3 per week. Simplify or challenge some by reducing or increasing the amounts involved.	