PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN KS1

NATURE OF THE ACTIVITIES SUGGESTED HERE

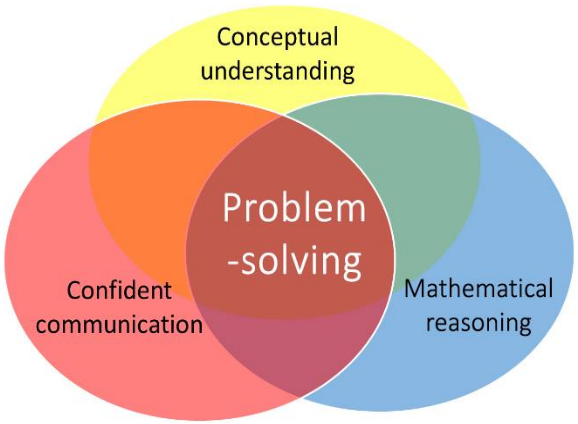
With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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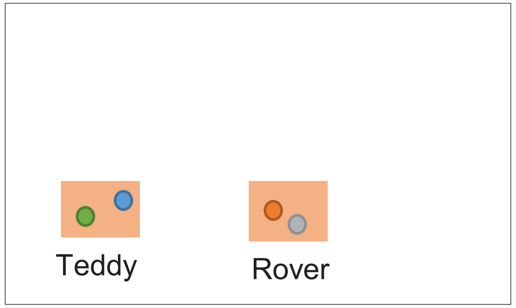
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

15. Fractions and Ratios **Recognise, find, name and write fractions $^1/_4$, $^1/_2$, $^2/_4$, and $^3/_4$ of a set of objects or quantity.** To enable children to develop a solid understanding of fractions, it is important for them to have as much practical experience as possible in which they associate the language and notation used to express fractions with concrete examples. This activity concentrates on finding fractions of a number, as many of children's experiences of fractions are likely to be with shape.	<u>**Sharing the sweets**</u> Children work in pairs. Each pair will need: • Counters (for *sweets*) in prepared paper bags of 4, 8, 12 and 16; • Four 'soft toys', say *Teddy, Rover, Froggy and Rabbit* to be the recipients of sharing the sweets; • *Post-it notes,* placed on a maths mat, one in front of each soft toy, used as 'plates' to collect the sweets. **Halves: Sharing equally between two toys:** • Starting with the bag of 4 sweets, show the children how to ***share** the sweets **equally between 2*** toys, *Teddy and Rover.*  The children pour out the sweets on to their maths mat, and at the bottom of the mat they stick one *post-it* for *Teddy* and another *post-it* for *Rover*. They then start ***sharing equally*** the sweets between *Teddy* and *Rover* until there are none left. At the end emphasise that '*Teddy* has ***half*** of the 4 sweets, and *Rover* has ***half*** of the 4 sweets. ***One half*** of 4 is 2.' You may write this symbolically for the children to see '$^1/_2$ of 4 = 2'.	Do the children connect the number who are sharing and the fraction this produces? Do they see that every fraction must be an equal share of the number of sweets? When finding quarters, do children realise how many quarters *Teddy, Rover* have together? (***two quarters***: if appropriate, you could write this as $^2/_4$), Taking this further, do children see how many quarters *Teddy, Rover* and *Rabbit* have together?' (***three quarters***: you could write this as $^3/_4$). Do the children recognise the equivalence between $^2/_4$ and $^1/_2$ of a number of sweets?

	• Repeat the demonstration with the bag of 8 sweets. This time ***one half*** is … Explain that between them *Teddy* and *Rover* actually have '***two halves***' and in each example ask the children 'How many sweets in ***two halves***?' • Now Emily and Luke ***share*** the bag of 12 sweets ***equally*** between *Teddy* and *Rover*. They need to agree what is ***one half*** of 12. • Repeat this with the bag of 16 sweets. **Quarters: Sharing equally between four toys:** • Now explain that *Froggy* and *Rabbit* have also come to play with *Teddy* and *Rover.* How many will need to share the sweets altogether now? If necessary, demonstrate sharing equally between 4, to find ***one quarter***, using $^1/_4$ of 4 and then $^1/_4$ of 8, before asking them to share the 12 and the 16.	