PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

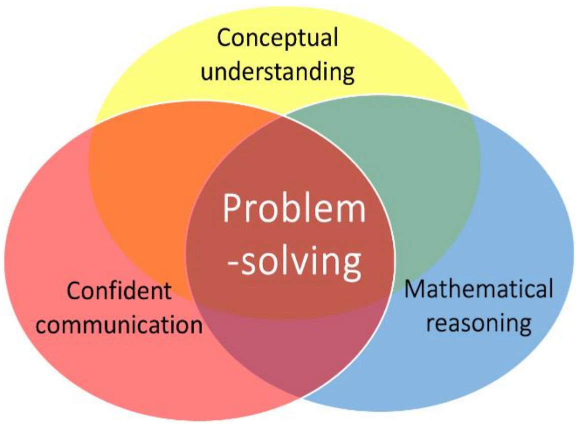
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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NATURE OF THE ACTIVITIES SUGGESTED HERE

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

15. Fractions and Ratios **Recognise and show families of common equivalent fractions.** **Understand that equivalent fractions all express the same constant ratio.** This activity gives children experiences of fractions in 3D, informally learning how to combine fractions of different denominations.	<u>**Creative Cuboids**</u> Children work in pairs. They will need: • Access to a class set of (e.g. 1000) multilink cubes with a number of different colours from which to choose. This activity invites Shelley and Rohan to make cuboids using a given number of cubes, with various fractions of the cubes used being in different colours. To ensure that it is possible to see all the pieces to work out each fraction, ask the children to make their cuboids just one multilink cube deep to begin with. Both Shelley and Rohan construct their own cuboids then check and agree they have the correct fractions of each colour. For example: • 24 cubes: $^1/_3$ red, $^1/_4$ blue, $^1/_6$ green and remainder yellow: each cuboid should comprise 8 red cubes, 6 blue, 4 green. They describe the fraction that is yellow: $^1/_4$ (6 cubes). • 36 cubes with the same fractions of each colour: $^1/_3$ red, $^1/_4$ blue, $^1/_6$ green and remainder yellow. What do they notice? They should find that the number of each colour has increased proportionally: 12 red, 9 blue, 6 green and that the fraction that is yellow is still $^1/_4$ (9 cubes). • 48 cubes with the same fractions of each colour: $^1/_3$ red, $^1/_4$ blue, $^1/_6$ green and remainder yellow. What do they notice this time? • 24 cubes: $^1/_3$ red, $^1/_4$ blue, $^3/_8$ green and remainder yellow: Each cuboid should comprise 8 red cubes, 6 blue, 9 green and yellow: $^1/_{24}$ (1 cube). • 32 cubes: $^1/_2$ red, $^1/_5$ blue, $^3/_{10}$ green and remainder yellow: 10 red, 6 blue, 9 green; $^1/_6$ yellow (5 cubes). • 40 cubes: $^1/_2$ red, $^1/_5$ blue, $^3/_{10}$ green and remainder yellow: 20 red, 8 blue, 12 green; No yellow cubes. Simplify, if needed, by using simpler fractions, smaller numbers of cubes and fewer colours. Challenge children to construct cuboids that have a depth of *two* multilink cubes. All the cubes will be visible, but the children will need to examine all faces of the cuboids to work out the fractions.	Do children see why $^1/_8$ of 24 = 3 and how to work out, say, $^3/_8$ of 24 by first finding $^1/_8$? In the first three examples do they recognise that the increase in the number of each colour does not change the fraction of this colour in the whole cuboid? Do the children realise that they do not need to arrange the colours in contiguous blocks, though that is likely to be their initial inclination? The fraction of a particular colour can be distributed in different parts of the cuboid, and helps the children to realise it is the number of cubes, not their arrangement that determines the fraction.