PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN KS1

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

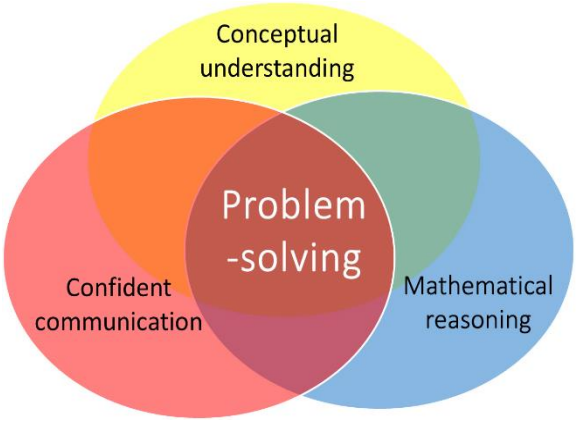
With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

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Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

16. Decimal Numbers and Rounding **The use of rounding in the context of money.** **Combine amounts to make a particular value.** **Find different combinations of coins that equal the same amounts of money.** To enable children to develop a proper understanding of rounding, they will need to understand that the nearest number to round towards depends on the context of the problem. They can begin to see this in real contexts at an early age, for example when paying for things they wish to buy. In this problem, they have only a few coins from which to choose and they need to offer the smallest amount necessary to pay for the intended item.	**Pay the least amount** Children to work in groups of 4, possibly with some adult support. They will need: • A number of items labelled with different prices: 3p, 6p, 8p, 11p, 13p, 17p, 22p, 26p; • Three purses or bags each containing one each of the following coins only: 2p, 5p, 10p and 20p; • One small tray of mixed coins including 1p pieces, and/or a number line to assist in calculating the change; • Counters to award to the 'winners' of each round. First model how we may not have the exact money when we pay for something, and how to find the smallest value combination of coins, for example, to offer for an item costing 1p, 4p and 7p. In each case discuss different coins which could be used from the purse or bag and which would provide the smallest amount. Then set the children working in their groups, e.g.: • Emily is the shopkeeper with the items for sale. Starting with the cheapest of these, she places the 3p item in front of her. Luke, Kasia and Nathan select the smallest amount that they would need to offer to pay for this item. All should all choose 5p. Those who offer the smallest amount from their coins are winners and may take one counter each. • Emily works out the change that would be given for the smallest amount, in this case 2p. The other children discuss and help her if she has difficulty and there is no adult present. If the change is the correct amount, Emily takes one counter. • Note that the children do not hand over the coins, only show what they would offer. They keep the coins to use again for subsequent items. No change is actually given so that the coins in each purse or bag remain the same. • Luke passes his bag to Emily, and he now becomes the shopkeeper. The 6p item is next, so the children should offer 7p (5p + 2p) as the smallest amount, and Luke should find 1p in change. For higher attainers extend the amounts up to 50p. Vary the coins and prices to enable other combinations to be explored.	Do the children correctly recognise the *values* of different coins and add these *values* correctly? Do the children realise that it is always possible to offer an amount more than the price required, but we are trying to find the ***nearest*** possible amount to the cost of the item? Do children understand that we cannot *round down* to find the amount we need? We cannot offer an amount less than the price of the item (as we are not usually able to haggle with the shopkeeper!).