PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UKS2

5-6

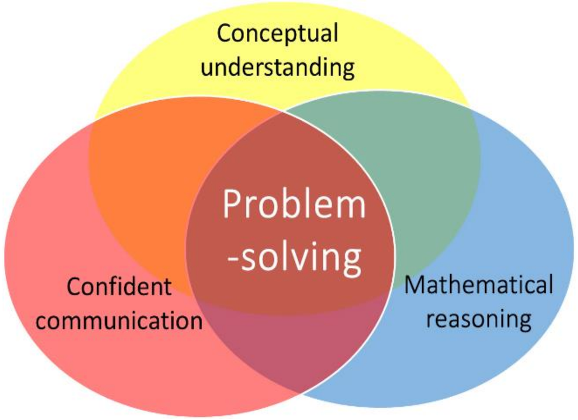
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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NATURE OF THE ACTIVITIES SUGGESTED HERE

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

17. Calculations with Decimals **Multiply one digit numbers with up to two decimal places by whole numbers.** **Solve problems which require answers to rounded to specific degrees of accuracy.** This activity requires some additions and multiplications with decimal numbers, in a typically real context.	**<u>The new carpet</u>** Working in pairs, children will need the following: • Metre stick or long tape measure. This activity is to calculate the (wall-to-all) area of (non-patterned) carpet needed for the classroom. It is reasonable to make some adjustment for the actual shape of the room, so that an approximate rectangle can be calculated. This is natural, as the carpet will be cut as rectangular pieces from a roll. However, it is unlikely that the room will be small enough to carpet the floor from just one complete piece. It is likely to require two or more pieces joined together side by side. First model how the carpet would be laid. Use a roll of kitchen towel to represent the carpet and use the top of a table to represent the floor. Show how the 'carpet' cannot cover the 'floor' unless more than one piece is cut from the roll and laid down side by side. To find out how much carpet is needed add up the combined length ***L*** of all the pieces. ***L*** is the total length of carpet to cut from the carpet roll to cover the floor. The area of carpet to buy will be this length ***L*** multiplied by the width ***W*** of the carpet roll. Note that the actual dimensions of the room will mean that some carpet is wasted, as it is unlikely to fit an exact number of carpet widths from wall to wall. The price of the carpet is calculated per square metre, so what is the area of carpet that has to be bought? Meena and Charlie use the tape measure or metre stick to measure the length (say, 10.5 m) and width (say, 7 m) of the room. Given a *standard* carpet width of 3.7 m, they need to decide how many carpet widths are needed to cover the width of the room, hence the number of pieces (2) and the total length ***L*** of carpet needed (21 m). They calculate the area as 3.7 × 21 = 77.7 square metres.	Do children align, add and multiply decimal numbers correctly when calculating? Do they estimate an answer to any calculation using an appropriate rounding of the numbers involved? Do they use an appropriate calculation method correctly? Do they realise that multiplying by a number which is less than one will make a product which is smaller than the original multiplicand? Do they understand that area is the product of length and width?

	If different carpet widths are available, for example 3.9 m or 4.6 m, what area of carpet would be needed? Would the joins need to be aligned with the room length or room width to minimise the waste? For lower attainers, this activity could be carried out with rounded measurements or simplified decimals, for example, to the nearest 0.5 m. Extend the activity by giving a price per square metre, which may vary with the different carpet widths and the children calculate the cost of the different options.	