PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UKS2

5-6

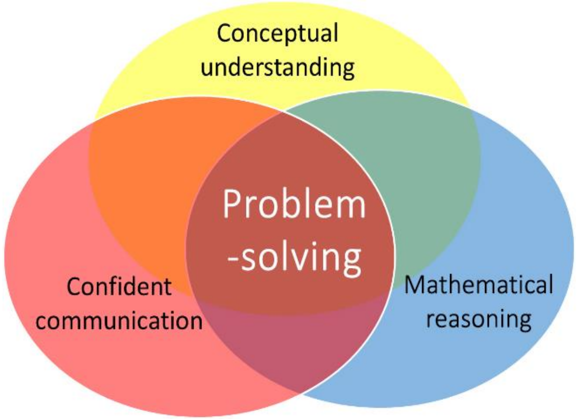
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

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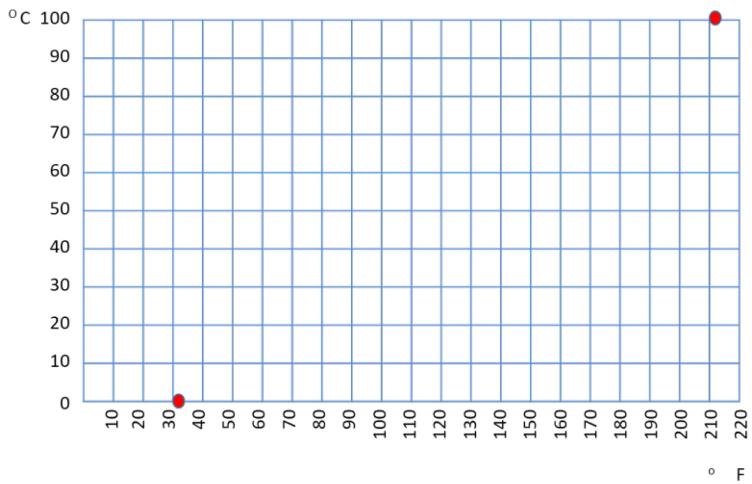
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

20. Coordinates and Linear Relationships **To understand a real-life linear relationship in a practical example.** This is a practical exercise for exploring the linear relationship between the Celsius and Fahrenheit scales of temperature. Although most countries now use the Celsius scale, the Fahrenheit scale used to be used in the UK, and is still more familiar to some older citizens. The Fahrenheit scale continues to be used in a few countries, most notably the United States of America.	**Some like it hot!** Children work in pairs, to discuss and share ideas. They will need: • A4 Squared paper (1 cm^2); • Classroom thermometer in both °C and °F (optional). Start by explaining that in the United States they measure temperature using a different scale to us, and find out what the children may already know. Establish that we are going to make a 'ready reckoner' to convert temperatures easily between the two countries' scales. Display a squared grid with the longer axis horizontal, and label this axis Fahrenheit. Label the vertical axis Celsius. Now mark the grid lines on each scale as 10° intervals. Do any children know the equivalent values in each scale for the freezing point and boiling point of water? (At standard atmospheric pressure, of course!) Establish that these are respectively 0°C: 32°F and 100°C: 212°F, and explain that these values are co-ordinates to mark the points where they are represent the same temperature on each scale. Mark these (approximately) on the display: Explain that both temperature scales measure in regular intervals so that a difference of 1 degree on one scale always represents same interval in temperature on that scale. Could they use this to work out the approximate equivalent values between the scales for other temperatures? How would they do this?	Do the children see that the graph shows the relationship between two variables? One variable is the temperature measure in degrees Celsius, and the other variable is the temperature measured in degrees Fahrenheit. Do they see that a straight line shows there is a ***linear relationship*** between two variables? Do they understand that this means the values of the Celsius and Fahrenheit scales vary proportionally to one another?

	The children can discuss this between them. They may conclude from other experiences that joining the two known points with a straight line will show the intersection of the two scales for any other temperature. If not, make this point and draw the line on your display.  Charlie and Meena now create their own conversion graphs. Insist that they position the axes so that they use the **uppermost 10** grid squares for the Celsius temperatures and the **rightmost 22** grid squares for Fahrenheit. Tell them that this will be useful for some later questions. Then ask them to plot the freezing and boiling points. They can use mm measures on their rulers to make a more accurate point for each one on the Fahrenheit axis. They connect the two points with a straight line and they are ready to find at least approximate answers to the following questions:	

	• What is 20ºC in Fahrenheit? • What is 180ºF in Celsius? • If an American tells you it is 110º, what temperature would a European think this is? • … and so on. Challenge Charlie and Meena to think about how to use the conversion graph for other values: • What is -10ºC in Fahrenheit? • What is 0ºF in Celsius? • What is -20ºF in Celsius? To answer these questions, the children will need to extend the graph into the 3rd and 4th quadrants. There should be room on an A4 sheet of $1cm^2$ paper to extend the axes below zero on both scales and to extend the line of the relationship. A further challenge could be to calculate the conversion between Fahrenheit and Celsius. *(Hint: Start by equating the difference between boiling point and freezing point in both scales: 100ºC = 180ºF.)* A few children may even be able to work out that, if *n* is the temperature in Celsius and *m* is the equivalent in Fahrenheit, then $n = (m - 32) \times \frac{5}{9}$	