PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

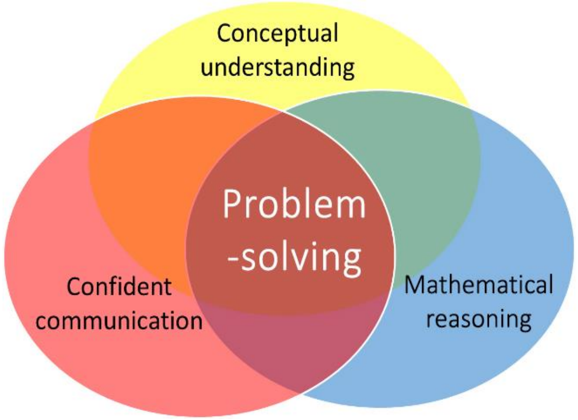
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

21. Concepts and Principles of Measurement

Estimate mass by comparison with some helpful common references.

Children are often given activities to estimate and then measure items to test their estimates. The discouraging thing about this for many children is that it does not take into account how we actually estimate in practice. Any time we make an estimate in real life, we do so by comparison with our *previous experience*. If we have little previous experience to draw upon, we have little confidence in our estimates, so often children will want to measure *first*, in order to make their estimates correct!

<u>**Estimating in practice**</u> Children work in groups of 3 or 4 to compare and agree estimates, and check one another's findings. They will need:

- An ad hoc class collection of different (labelled) objects of varying masses up to about 2 kilograms. These can be 'created' artificially by sealing masses inside discarded packets of different shapes and sizes, and can be one large collection to be used by the entire class;
- A prepared chart for recording the group's findings (see photocopiable resources):

Item	Estimate	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass
A				
B				
C				

- Helpful *reference* masses, such as:
  - 1 kg bag of sugar, or a litre bottle of water (effectively about 1 kg);
  - 400 g can of baked beans (effectively about ½ kg including the can);
  - 250 g packet of biscuits;
  - 150 g bag of crisps;
  - 30 g bag of crisps;

Do the children specify an accurate ***range*** for their estimate, even if it's not very precise?

Do they realise that our ability to estimate increases with experiences of weighing?

As they become more familiar with reference masses, can they estimate with increasing ***precision***?

When using analogue scales, have the children had sufficient practice in reading the divisions and interpreting the scale intervals accurately?

If they are using digital scales, can the children correctly interpret the digital reading?

If using balance scales, do children check their additions

	• Two identical supermarket carrier bags; • Balance scales and masses, and/or analogue scales. First of all Shelley, Rohan, Alice and Marek feel and become familiar with the actual masses of the reference objects. Next their job is to select and compare these reference masses with those labelled items in the collection to estimate the mass of each one and write this into their recording chart. As there is no predictable linear relationship between the shape or volume of the items and their mass, it can be difficult to compare them by handling them directly. The children may therefore choose to place the object and a comparison mass in each of the carrier bags so that the weight of both is similarly distributed in the children's hands. To help them in their estimating, allow the children to specify a range, e.g. 'from ½ kg to ¾ kg'. When they have done this, they should put their estimates in order from lightest to heaviest mass. The group compare and discuss their findings to agree a table of estimates. For example:	of the masses to ensure their measure is accurate?

Item	Estimate	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass
A	1 ½–2 kg	9th		
B	300–400 g	5th		
C	150–200 g	3rd		

Discuss the findings so far as a class. Where is there agreement or difference between groups?

Now allow Shelley, Rohan, Alice and Marek access to the scales to measure the actual masses and complete their charts. Again compare and discuss findings as a whole class.