PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UKS2

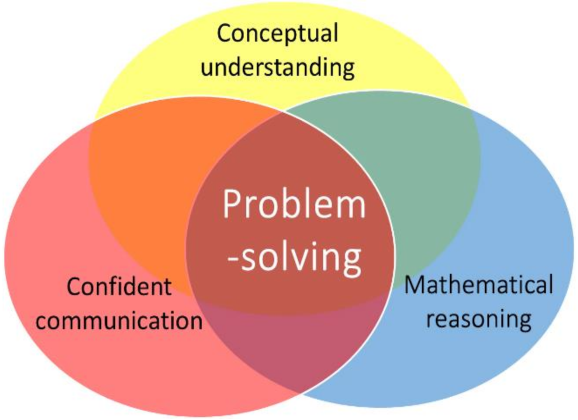
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

23. Angle **To use a protractor to measure angles accurately for a purpose.** This is a practical investigation into the sum of the internal angles of different irregular polygons. It provides opportunities for measuring many different angles, as well as multiple additions.	**<u>How many degrees</u>?** Children work in pairs, to discuss and compare findings. They will need: • Protractors (try to ensure these are 360°, for measuring angles greater than 180°); • Recording chart (see photocopiable resources); • Scissors. With the whole class ask them if they know the sum of all the internal angles of a triangle. (They should: 180°.) Ask them *how* they know? Have they checked? Some children may know the activity for tearing off the vertices and placing them adjacently to make a straight line. Ask them if this works to find the sum of the internal angles for a quadrilateral? Get the children to draw an irregular quadrilateral and cut it out, then tear off the angles to try it. They should find that they create a complete circle (360°). Now ask them to see if they can work out the sum of the internal angles for a pentagon? Again, get the children to try this, drawing and cutting out an irregular pentagon. They should find that the angles overlap one another. Suggest that we test these by measuring the angles and adding them up. Check that the children can use the protractor correctly.	Do the children align the protractors accurately upon the angles and read and interpret the scale correctly? Do they check they have measured every internal angle? How can they do this systematically? Do they re-measure and revise their results if they find differences between polygons with the same number of angles? Do they realise that all measurements are approximate and that even though we try to be as accurate as possible a small imprecision in measuring is unavoidable.

Charlie and Meena draw different (irregular) pentagons, measure the angles and add these up. As they do so they complete a recording chart:

Shape	Angles	Sum of angles
Pentagon(1)	35º, 72º, 69º, 123º, 243º	542º
Pentagon(2)	161º, 88º, 117º, 146º, 31º	543º
Pentagon(3)	97º, 182º, 152º, 46º, 30º	538º

After several examples, ask the class whether it is about the same every time, and if there may be a reason for values to differ? (Some inaccuracies in measuring the individual angles.)

Ask the children to suggest what they think is ***sum of the angles*** for a ***pentagon*** (540º).

This exercise can be extended to ***hexagons*** (720º), ***heptagons*** (900º), ***octagons*** (1080º), and so on.

Ask the children if they can see any pattern in the relationship between the number of sides and the sum of the angles? *(Hint: how many 180s are there in each total?)*

Children do not need to know the formula, but this is further opportunity to develop their algebraic reasoning.