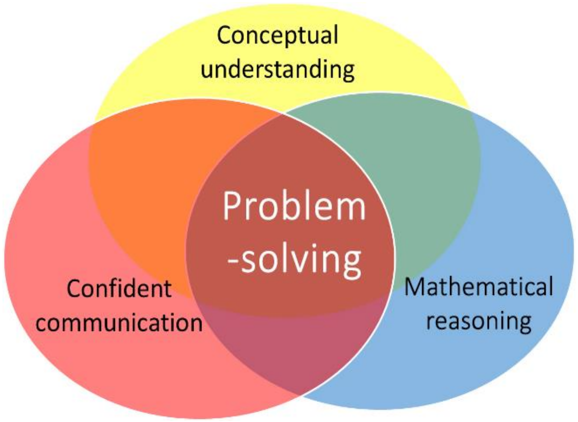
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

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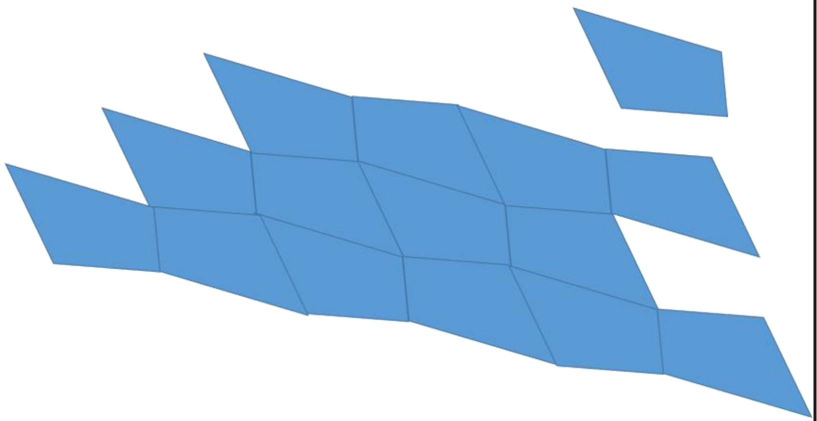
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

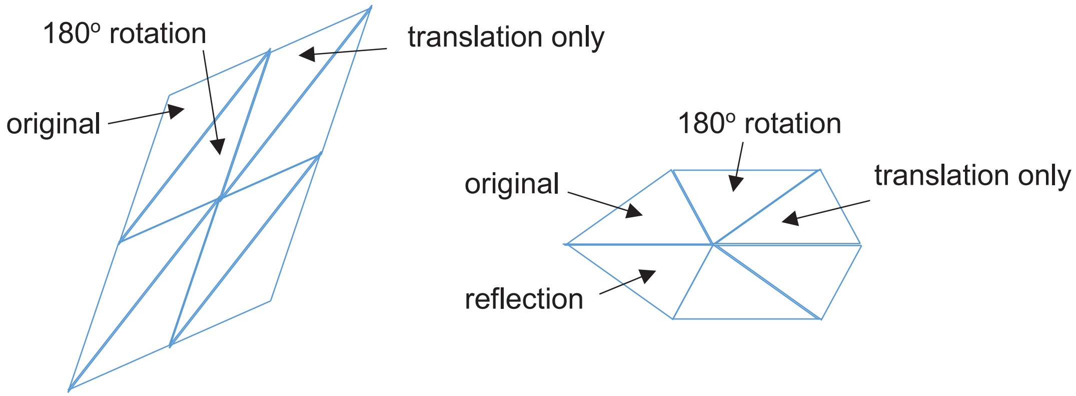
Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

24. Transformations and Symmetry **To understand how transforming some 2-dimensional shapes enables them to tessellate.** **To use the vocabulary: *orientation*, *rotate*, *translate*, *reflect*, and *tessellate*.** A practical activity for children to explore how irregular shapes can be transformed (but remain congruent) in order to tessellate. It is a particularly helpful use of technology with which children can rapidly replicate and manipulate shapes.	**Irregular tessellations** Children will work in pairs. Each pair will need • PC/tablet with a simple drawing program/app. Introduce/revise the drawing program. Show how an *irregular* polygon can be drawn, how it can be *moved in the same orientation* ***(translated)***, ***rotated*** *about the centre of the shape*, and *flipped over* ***(reflected).*** Show that it is still the same shape – the same number and lengths of sides and the same angles. Through copying and pasting the created shape, show the children how to make several duplicate *tiles* of the same shape. Show how you can transform (***reflect, rotate*** and/or ***translate***) the tiles so that some of the sides of separate tiles can be made to touch one another. Remind children how some shapes can ***tessellate***, so that they can be arranged to touch one another without leaving any gaps. Demonstrate tessellation with a rectangle, then with an irregular quadrilateral. For example: 	Do the children understand the difference between each of the different transformations? Can they identify where two or more transformations have been made and what these are? Can the children see that tessellating polygons also create larger, intrinsically tessellating polygons with more sides? For tessellating quadrilaterals create tessellating octagons, and tessellating triangles create tessellating quadrilaterals and hexagons?

Using their PC/tablet, Meena and Charlie create an *irregular* triangle. Ask them to create duplicate tiles. Challenge the children to see if they can transform and arrange the tiles so that the tiles will ***tessellate***.

- Will this work for an irregular triangle? (Yes)
- In what ways have they transformed the original triangle? As shown in the examples below, the tessellation may involve just ***rotations*** through 180° and ***translations***; or it might involve ***reflections*** as well. For example:



	Meena and Charlie go on to investigate the question 'Will any quadrilateral shape tessellate?' (Yes) In what ways do they have to transform the original shape to do this? Discuss the children's findings as a class. Challenge higher attainers to find some pentagons and hexagons that will tessellate. The house-shaped' pentagon and 'arrow-shaped' hexagon shown below are examples that do tessellate, but most irregular and regular pentagons and most irregular hexagons don't.  Children can also explore various websites which enable them to create very complex shapes which will tessellate. Where shapes do tessellate, can the children describe any conditions which enable this? E.g. sides of same length and orientation.	