PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

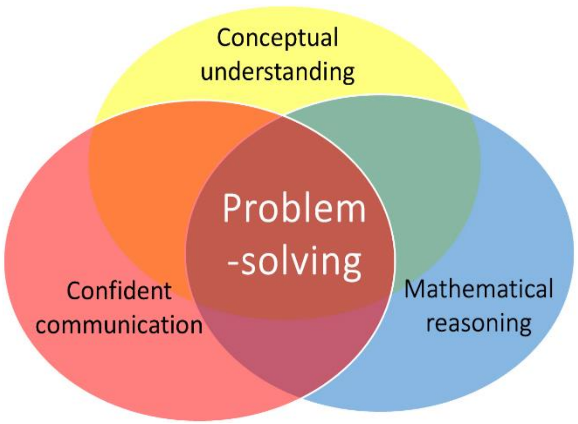
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

There is also a file of resource sheets used in some of the activities, which may be reproduced freely. However, please include any source information on each copy.

Related chapter, key learning & rationale	Plan for teaching and learning	Crucial points & barriers to understanding
6. Numbers and Place Value **Recognise the place-value of each digit in a three-digit number (hundreds, tens, and ones).** **Add and subtract numbers mentally, including a three-digit number and tens.** The children play this game to practise mental/informal addition and subtraction, emphasising the place values of the digits in a number, through physically exchanging between the *hundreds, tens* and *ones* of the *base-10* concrete equipment. (The lower target number of 100 is chosen so that the game does not involve negative numbers.)	<u>**King of the castle**</u> Demonstrate the game, then children play in pairs. They will need: • One set of place-value (p.v. or 'arrow') cards representing *tens* and *ones* for each child; • *Base-10* or *Dienes' apparatus* (up to 3 *hundreds*, 19 *tens* and 19 *ones*); • Place value (p.v.) mat (for 3 digits); • Counters for scoring points. Shelley and Rohan separate the p.v. cards into two separate, shuffled piles, face down: a *tens* pile and a *ones* pile. Start with 200 (two *hundred base-10* 'flats') in the *hundreds* column of the p.v. mat: Shelley intends to subtract *base-10* pieces to make the pile less than 100; Rohan intends to add *base-10* pieces to make the pile bigger than 300. The winner of the game is the first to reach their target. Here is an example: • Shelley turns over the top p.v. card from each pile. She combines these to assemble a two-digit number, in this example 37 and writes down the calculation she is going to carry out: 200 – 37 = • Shelley carries out the physical subtraction, by exchanging one base-10 *hundred* block for 10 *tens*, and one of those *tens* for 10 *ones* as needed. At the end of her turn, she writes the result of the calculation which is represented on the p.v. mat: 200 – 37 = 163.	Do the children actively recognise that 300 is equivalent to 3 of the *hundred pieces,* and so on? Do the children recognise that '*10 of these is 1 of those*', and vice versa? Do the use the terms ***exchange*** and ***regroup*** correctly? Can the children use these pieces to add/subtract mentally? If not, can they model it the calculation informally on an empty number line?

	• Rohan watches carefully and checks that Shelley's answer is correct. • Rohan turns over the next two p.v. cards, and combines these to assemble a two-digit number, say 56. He writes down the calculation he is to carry out given the present value represented on the mat: 163 + 56 = • He carries out the physical addition, exchanging 10 *ones* for 1 *ten*, and 10 *tens* for 1 *hundred* as needed, whenever he has more than 9 of the pieces for a particular place. At the end of Rohan's turn, he writes the result of the calculation, which is represented on the p.v. mat: 163 + 56 = 217. • Shelley watches carefully and checks that Rohan's answer is correct. • They continue to alternate turns, Shelley subtracting and then Rohan adding until either Shelley arrives at a number below 100 or Rohan gets to a number greater than 300. When this happens the winner gains an extra 5 points, and they begin again, swapping the subtract/add roles. • When they have used all the p.v. cards, they shuffle them and place them again to continue the game. • An extra point is awarded to Rohan if he shows Shelley that she made an incorrect calculation (and vice versa). • An extra point is awarded to Shelley if she spots that Rohan has left more than nine pieces in one of the *ones* or *tens* column of the p.v. mat (and vice versa). To simplify the game, work with addition and subtraction of single-digit numbers starting with a value of, say 50, and aiming for targets of below 10 and above 90.	