With the surge of interest and sometimes confused interpretations of what is meant by Mastery in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged. However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom’s* theories of Mastery, the development of deeper conceptual understanding through a progression in Concrete-Pictorial-Abstract (CPA) experiences, first discovered by Bruner*, the realistic mathematics education of Freudenthal*, and the seminal Cockcroft Report*, particularly, its emphasis on the importance of practical experiences and problem-solving. More recently, Lo’s* research in the subject of Variation Theory has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop rich connections between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to deeper mathematical learning.

Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.
PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UKS2

5-6

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher’s own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children’s names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References


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NATURE OF THE ACTIVITIES SUGGESTED HERE

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]
Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]
There is also a file of resource sheets used in some of the activities, which may be reproduced freely. However, please include any source information on each copy.

<table>
<thead>
<tr>
<th>Related chapter, key learning &amp; rationale</th>
<th>Plan for teaching and learning</th>
<th>Crucial points &amp; barriers to understanding</th>
</tr>
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</table>
| 6. Numbers and Place Value                | **Place invaders** Demonstrate using an on-screen calculator, then children play against one another in pairs. They will need:  
• Simple (non-scientific) calculator for each pair;  
• Counters for scoring.  
*Space Invaders* was an arcade video game from the 1970s. The aim was to shoot down as many alien spacecraft as possible. Versions of this game are available on the internet today. **Place invaders** is based on this idea, to ‘shoot’ specified digits in a number displayed on a calculator, by making them zero (or blank for the most and least significant digits in the number) whilst leaving the other digits unchanged. Here is an example:  
• Meena clears the calculator and enters a number, say, with up to 6 integer digits and 2 decimal digits, for example 763981.72;  
• Charlie selects a digit for Meena to shoot, e.g.: ‘Take out the 3, Meena’;  
• Meena has to enter the correct subtraction into the calculator to do this. She enters: ‘–’  ‘3000’  ‘=’;  
• They both check that the calculator display has been correctly changed to 760981.72, and Meena gets 1 point;  
• Meena selects a target digit: ‘Shoot the 7 on the right, Charlie’  
• Charlie has to enter the correct subtraction. He enters: ‘–’  ‘0.7’  ‘=’  
• The calculator now shows 760981.02 and Charlie gets 1 point;  
• They continue alternating turns until the display is 0.  
If the display is incorrect after a subtraction, the player does not get a point, but they can continue to work with the changed number.  
If needed, simplify the game by reducing the number of digits, and/or removing decimal places, or extend it further with more digits in the integer and/or decimal parts.  
Does the child correctly recognise the actually place value of a specific digit from its position within the number?  
It is important that the children have already had sufficient teaching and experience in how to manipulate and interpret the calculator.  
Can the child use the calculator correctly to enter the number and press the keys in the correct order to carry out the subtraction?  
Can the children be further challenged, by giving an additional point to the target-setter if they can describe the target in terms of its actual place value? For example: ‘Take out the three thousand, Meena’, or ‘Shoot the seven tenths, Charlie.’ |