PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LKS2

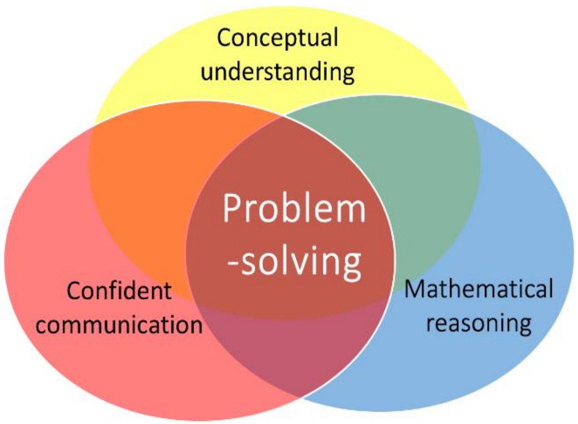
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

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There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

9. Written Methods for Addition and Subtraction

Add and subtract numbers with up to three digits, using (formal) written methods of vertical addition and subtraction.

Interpret data using tables.

Solve addition and subtraction multi-step problems in context.

It is, of course, important to spend some time practising some straightforward calculations when introducing each progressive step in developing vertical written methods. After that, it is more helpful to practise a few calculations more frequently than to have lessons intensively practising pages of 'sums'. For example, we have found it can be much more useful setting up just a couple of practice calculations twice each week for 'early day

<u>The Cheap Coach Company</u> The teacher explains that *The Cheap Coach Company* has just started operating some routes to provide cut-price travel between cities in the UK – any journey for the same fixed fare, no matter how many cities you need to change at during the route (see photocopiable resource).
Display the chart showing distances between cities for which the company runs coaches:

	Aberdeen	Birmingham	Bristol	Carlisle	Edinburgh	Exeter	Leeds	Liverpool	London	Manchester	Newcastle	Portsmouth	Southampton
Birmingham													
Bristol		88											
Carlisle													
Edinburgh	127												
Exeter			80										
Leeds			207	117	217								
Liverpool		99			221								
London		118	118	308			195	212					
Manchester		87	168	119			43	35	200				
Newcastle					120		98		283				
Portsmouth			121										
Southampton		134							80			22	
Swansea			84				271	243					

Do the children interpret the table correctly, with cells containing known distances between two points (cities)?

Can they translate their understanding of the data as the shortest road distances between those two cities ***on the map***?

Do they realise that a blank cell, does not represent a 'zero' distance, but simply that there is no direct coach connection between those two cities?

Do the children have the pre-requisite skills to use the appropriate written addition and subtraction methods where needed to calculate partial and combined journeys? They will need to make more than one addition to compare different routes

work' when the children arrive and during registration, and are later explored with the whole class. Other than that it is helpful to present a purpose for calculations, so the activity requires the children to practise their written methods in order to do something purposeful and meaningful. These activities could also be used to explore problem-solving whilst practising more informal or expanded methods, if desirable.	The teacher and children explore where cities are located on a map to show the relevance of this activity to travel. The teacher teaches the children *how to interpret* the distance chart, recognising that the table is reduced because we only need to find the distance in one direction ***or*** the other. Starting on the vertical axis, find the distance between London and Carlisle (308 miles), then between London and Liverpool (212 miles). Then how do we find the distance between London and Newcastle (283 miles)? In pairs, the children to look up other distances for routes directly between pairs of cities. **Partial journeys:** In pairs the children, work out the remaining distances to travel if the coach stops at a service station: e.g. 1. From Carlisle to Manchester, stopping after 65 miles? 2. From Bristol to Leeds, stopping after 129 miles? 3. From Swansea to Liverpool, stopping after 191 miles? **Combining journeys:** How could we travel between two cities when there is no direct route? For example, between London and Portsmouth, via Southampton (80 + 22 = 102 miles). Is this the ***shortest*** route? Then ask the children in pairs to work out the lengths of journeys between, e.g: 1. From Manchester to Exeter (248, via Bristol); 2. From Southampton to Liverpool (292 miles via London, but 235 miles via Birmingham); 3. From London to Aberdeen (560 miles via Liverpool and Edinburgh, but 530 miles via Newcastle and Edinburgh).	for combined journeys to find the shortest. Can children make a reasonable estimate of the calculation before they attempt each addition or subtraction? Can children use the inverse operation to check their results? Can the children demonstrate the steps in their calculation to one another informally or using base-10 place value resources? Where some partial calculations are able to be done mentally, this should be encouraged, so that children use the best/most appropriate strategies wherever possible.