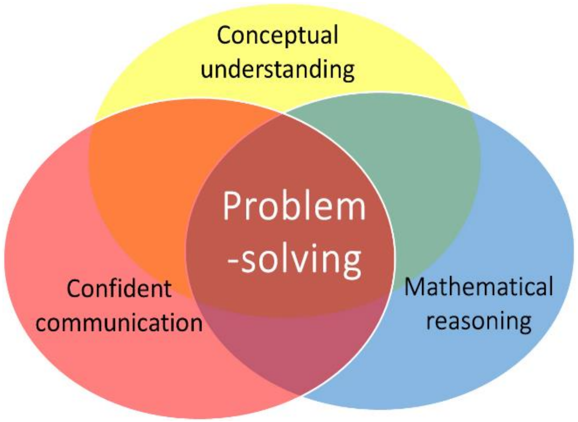
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

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Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

9. Written Methods for Addition and Subtraction **Add and subtract numbers mentally with increasingly large numbers, including decimals.** **Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.** **Solve addition and subtraction multi-step problems in context.** The aim here is to provide a real-life application of addition and subtraction calculations to money, in order to extend to more than two numbers when adding, and to cater for two decimal places (in the context of money notation only at this stage).	**The book token:** The teacher will need a number of books labelled with different prices between £5.00 and £20.00. For example: £5.57, £7.64, £8.31, £12.15, £13.86, £18.72, £21.42. Each child has a book token. They want to get as much value from their token as possible, because the shop does not give change or new tokens for unspent credit on tokens, and the children have no money of their own to add to the token. The aim is find the maximum amount they can spend (additions) and the unused value of their book token when they have done this (subtraction). The teacher should first demonstrate an example for a book token worth £20: i.e. they could buy the one book for £18.72, the two books at £5.57 + £13.86 = £19.43, the two books at £7.64 + £12.15 = £19.79. Are there any other possibilities? Which decision uses the maximum value from the book token? How much of the book token is not spent in each case? In pairs, the children investigate different ways of spending a £25 book token. For example: Meena selects two books costing £13.86 and £8.31, which she first estimates as £14 + £8 = £22, then calculates this to be £22.17. Charlie selects three books costing £12.15, £7.64 and £5.57, which he first estimates as £12 + £8 + £6 = £26, so he changes his mind. He selects books costing £8.31, £7.64 and £5.57. He estimates these as £8 + £8 + £6 = £22, and then calculates the actual total cost to be £21.52. Charlie and Meena swap results and check each other's calculation. They agree that Meena has got better value than Charlie from her book token. They then try again to see if they could get better value than £22.17. Investigate other values for the book token, e.g. £30, … £50. Alternatively simplify if necessary by using smaller amounts. For example, which items priced differently from 50p to 99p can I buy with a token for £5?	When using any formal or expanded written methods do children correctly align decimal points? If adjusting the calculation to whole numbers (pence) by scaling x100, do they re-scale the intermediate result correctly? Where partial calculations are able to be done mentally, this should be encouraged, so that children use the best/most appropriate strategies wherever possible. Can children make a reasonable estimate of the calculation before they attempt each addition or subtraction? Can children use the inverse operation to check their results?