Chapter 6: Statistics with R - 2nd Edition

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Student Exercises

1. Let x be the random variable described by the uniform probability distribution with its lower bound at $a = 120$, upper bound at $b = 140$.

   (a) What is the probability density function, $f(x)$?
   Answer: if $a = 120$ and $b = 140$, then

   $$f(x) = \frac{1}{b-a} = \frac{1}{140-120} = \frac{1}{20}$$

   (b) What is $E(x)$ and σ?
   Answer: the expected value and standard deviation are

   $$E(x) = \frac{a+b}{2} = \frac{140+120}{2} = \frac{260}{2} = 130$$

   $$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(140-120)^2}{12}} = \sqrt{33.33} = 5.77$$

2. What is $p(x = 130)$? Explain why $p(x = 130) \neq 1/20$.

   Answer: $p(x = 130) = 0$, just as it is for any value of x in the interval $120 \leq x \leq 140$. The probability density function $f(x)$ does not provide probabilities but only the height of the curve above the horizontal axis at a particular value of x. In this case, the height (but not the probability) is 1/20.

3. Referring to the previous exercise, find the following probabilities using $f(x)$ and R.

   (a) $p(125 \leq x \leq 135)$

   Answer: 0.5000

   $$p(125 \leq x \leq 135) = \frac{(135-125)}{(140-120)} = \frac{10}{20} = 0.5000$$

```r
# Subtract punif(125, 120, 140) from punif(135, 120, 140).

punif(135, 120, 140) - punif(125, 120, 140)

## [1] 0.5
```

(b) $p(125 \leq x \leq 131)$

Answer: 0.3000

$$p(125 \leq x \leq 131) = \frac{(131 - 125)}{(140 - 120)} = \frac{6}{20} = 0.3000$$

```r
# Subtract punif(125, 120, 140) from punif(131, 120, 140).

punif(131, 120, 140) - punif(125, 120, 140)

## [1] 0.3
```

(c) $p(129 \leq x \leq 131)$

Answer: 0.1000

$$p(129 \leq x \leq 131) = \frac{(131 - 129)}{(140 - 120)} = \frac{2}{20} = 0.1000$$

```r
# Subtract punif(129, 120, 140) from punif(131, 120, 140).

punif(131, 120, 140) - punif(129, 120, 140)

## [1] 0.1
```

(d) $p(120.50 \leq x \leq 139.50)$

Answer: 0.9500

$$p(120.50 \leq x \leq 139.50) = \frac{(139.50 - 120.50)}{(140 - 120)} = \frac{19}{20} = 0.9500$$

```r
# Subtract punif(120.50, 120, 140) from punif(139.50, 120, 140).

punif(139.50, 120, 140) - punif(120.50, 120, 140)

## [1] 0.95
```

4. Referring to the previous exercise, find the following probabilities using $f(x)$ and R.

(a) $p(x \geq 124)$

Answer: 0.8000

$$p(x \geq 124) = 1 - p(x<124) = 1 - \frac{(124 - 120)}{(140 - 120)} = 1 - (\frac{4}{20}) = 1 - 0.20 = 0.8000$$

```
# Use 1 minus punif()

1 - punif(124, 120, 140)

## [1] 0.8

# Use punif(124, 120, 140, lower.tail=FALSE).

punif(124, 120, 140, lower.tail = FALSE)

## [1] 0.8
```

(b) $p(x \geq 128)$

Answer: 0.6000

$$p(x \geq 128) = 1 - p(x<128) = 1 - \frac{(128 - 120)}{(140 - 120)} = 1 - (\frac{8}{20}) = 1 - 0.40 = 0.6000$$

```
# Use 1 minus punif().

1 - punif(128, 120, 140)

## [1] 0.6

# Use punif(128, 120, 140,lower.tail=FALSE).

punif(128, 120, 140, lower.tail = FALSE)

## [1] 0.6
```

(c) $p(x \geq 134)$

Answer: 0.3000

$$p(x \geq 134) = 1 - p(x<134) = 1 - \frac{(134 - 120)}{(140 - 120)} = 1 - (\frac{14}{20}) = 1 - 0.70 = 0.3000$$

```r
# Use 1 minus punif().

1 - punif(134, 120, 140)

## [1] 0.3

# Use punif(134, 120, 140,lower.tail=FALSE).

punif(134, 120, 140, lower.tail = FALSE)

## [1] 0.3
```

(d) $p(x \geq 139)$

Answer: 0.0500

$$p(x \geq 139) = 1 - p(x<139) = 1 - \frac{(139-120)}{(140-120)} = 1 - (\frac{19}{20}) = 1 - 0.95 = 0.0500$$

```r
# Use 1 minus punif().

1 - punif(139, 120, 140)

## [1] 0.05

# Use punif(139, 120, 140,lower.tail=FALSE).

punif(139, 120, 140, lower.tail = FALSE)

## [1] 0.05
```

5. A recent college graduate is moving to Houston, Texas to take a new job, and is looking to purchase a home. Since Greater Houston takes in a relatively large metropolitan area of nearly 8,000,000 people, there are many homes from which to choose. When consulting the real estate web sites, it is possible to select the price range of housing in which one is most interested. Suppose the potential buyer specifies a price range of \$200,000 to \$250,000, and the result of the search returns thousands of homes with prices distributed *uniformly* throughout that range. Please answer the following questions.

(a) What would be the probability density function that best describes this distribution of housing prices?

Answer:

$$f(x) = \frac{1}{(250000-200000)} = \frac{1}{50000}$$

(b) What are $E(x)$ and σ?

Answer:

$$E(x) = \frac{250000 + 200000}{2} = \frac{450000}{2} = 225000$$

$$\sigma = \sqrt{\frac{(250000 - 200000)^2}{12}} = 14434$$

(c) If the buyer ultimately selects a home randomly from this initial list of possibilities, what is the probability she will have to pay more than $235,000?

Answer: 0.3000

$$p(x{>}235000) = 1 - p(x \leq 235000) = 1 - \frac{(235000 - 200000)}{(250000 - 200000)} = 1 - 0.70 = 0.3000$$

```
# Use 1 minus punif().

1 - punif(235000, 200000, 250000)

## [1] 0.3

# Use punif( ,lower.tail=FALSE)

punif(235000, 200000, 250000, lower.tail = FALSE)

## [1] 0.3
```

6. Referring to the above question, what is the minimum price the buyer would pay if she refines her focus to the upper 25% of the price range from $200,000 to $250,000?

Answer: $237,500

```
# Use the qunif() function.

qunif(0.75, 200000, 250000)

## [1] 237500

# Use qunif(0.25, 200000, 250000, lower.tail = FALSE).

qunif(0.25, 200000, 250000, lower.tail = FALSE)

## [1] 237500
```

7. Use R to answer the following questions concerning z, the standard normal variable.

(a) $p(z \leq 2.33)$?

Answer: 0.9901. The probability that z is less than or equal to 2.33 is about 0.99.

```
# Use the pnorm() function.

pnorm(2.33)

## [1] 0.9900969
```

(b) $p(z \leq 2.05)$?

Answer: 0.9798. The probability that z is less than or equal to 2.05 is about 0.98.

```
# Use the pnorm() function.

pnorm(2.05)

## [1] 0.9798178
```

(c) $p(z \leq 1.96)$?

Answer: 0.9750. The probability that z is less than or equal to 1.96 is about 0.9750.

```
# Use the pnorm() function.

pnorm(1.96)

## [1] 0.9750021
```

(d) $p(z \leq 1.28)$?

Answer: 0.8997. The probability that z is less than or equal to 1.28 is about 0.90.

```
# Use the pnorm() function.

pnorm(1.28)

## [1] 0.8997274
```

8. Use R to answer the following questions about z.

(a) $p(z \leq 0.00)$?

Answer: 0.5000. The probability that z will be less than or equal to 0.00 is 0.5000.

```
# Use the pnorm() function.

pnorm(0.00)

## [1] 0.5
```

(b) $p(z \leq -1.28)$?

Answer: 0.1003. The probability that z will be less than or equal to -1.28 is about 0.10.

```
# Use the pnorm() function.

pnorm(-1.28)

## [1] 0.1002726
```

(c) $p(z \leq -1.96)$?

Answer: 0.025. The probability that z will be less than or equal to -1.96 is about 0.025.

```
# Use the pnorm() function.

pnorm(-1.96)

## [1] 0.0249979
```

(d) $p(z \leq -2.05)$?

Answer: 0.02018. The probability that z will be less than or equal to -2.05 is about 0.02.

```
# Use the pnorm() function.

pnorm(-2.05)

## [1] 0.02018222
```

(e) $p(z \leq -2.33)$?

Answer: 0.009903. The probability that z will be less than or equal to -2.33 is about 0.01.

```r
# Use the pnorm() function.

pnorm(-2.33)

## [1] 0.009903076
```

9. Use R to answer the following questions about z.

(a) $p(z \geq -2.33)$

Answer: 0.9901. The probability that z will be greater than or equal to -2.33 is about 0.99.

```r
# Use 1 minus the pnorm() function.

1 - pnorm(-2.33)

## [1] 0.9900969

# Use pnorm(-2.33, lower.tail = FALSE) to confirm.

pnorm(-2.33, lower.tail =  FALSE)

## [1] 0.9900969
```

(b) $p(z \geq -2.05)$

Answer: 0.9798. The probability that z will be greater than or equal to -2.05 is about 0.98.

```r
# Use 1 minus the pnorm() function.

1 - pnorm(-2.05)

## [1] 0.9798178

# Use pnorm(-2.05, lower.tail = FALSE) to confirm.

pnorm(-2.05, lower.tail =  FALSE)

## [1] 0.9798178
```

(c) $p(z \geq -1.96)$

Answer: 0.975. The probability that z will be greater than or equal to -1.96 is about 0.975.

```r
# Use 1 minus the pnorm() function.

1 - pnorm(-1.96)

## [1] 0.9750021

# Use pnorm(-1.96, lower.tail = FALSE) to confirm.

pnorm(-1.96, lower.tail =  FALSE)

## [1] 0.9750021
```

(d) $p(z \geq -1.28)$

Answer: 0.8997. The probability that z will be greater than or equal to -1.28 is about 0.90.

```r
# Use 1 minus the pnorm() function.

1 - pnorm(-1.28)

## [1] 0.8997274

#Comment2. Use pnorm(-1.28, lower.tail = FALSE) to confirm.

pnorm(-1.28, lower.tail =  FALSE)

## [1] 0.8997274
```

10. Use R to answer the following questions about z.

(a) $p(z \geq 1.28)$

Answer: 0.1003. The probability that z will be greater than or equal to 1.28 is about 0.10.

```r
# Use 1 minus the pnorm() function.

1 - pnorm(1.28)

## [1] 0.1002726

# Use pnorm(1.28, lower.tail = FALSE) to confirm.

pnorm(1.28, lower.tail =  FALSE)

## [1] 0.1002726
```

(b) $p(z \geq 1.96)$

Answer: 0.025. The probability that z will be greater than or equal to 1.96 is about 0.025.

```
# Use 1 minus the pnorm() function.

1 - pnorm(1.96)

## [1] 0.0249979

# Use pnorm(1.96, lower.tail = FALSE) to confirm.

pnorm(1.96, lower.tail =  FALSE)

## [1] 0.0249979
```

(c) $p(z \geq 2.05)$

Answer: 0.02018. The probability that z will be greater than or equal to 2.05 is about 0.02.

```
# Use 1 minus the pnorm() function.

1 - pnorm(2.05)

## [1] 0.02018222

# Use pnorm(2.05, lower.tail = FALSE) to confirm.

pnorm(2.05, lower.tail =  FALSE)

## [1] 0.02018222
```

(d) $p(z \geq 2.33)$

Answer: 0.009903. The probability that z will be greater than or equal to 2.33 is about 0.01.

```
# Use 1 minus the pnorm() function.

1 - pnorm(2.33)

## [1] 0.009903076

# Use pnorm(2.33, lower.tail = FALSE) to confirm.

pnorm(2.33, lower.tail =  FALSE)

## [1] 0.009903076
```

11. Use R to answer the following questions.

(a) If the area to the left of z is 0.99, what is z?

Answer: $z = 2.326$

```
# Use the qnorm() function.

qnorm(0.99)

## [1] 2.326348
```

(b) If the area to the left of z is 0.975, what is z?

Answer: $z = 1.96$

```
# Use the qnorm() function.

qnorm(0.975)

## [1] 1.959964
```

(c) If the area to the left of z is 0.95, what is z?

Answer: $z = 1.645$

```
# Use the qnorm() function.

qnorm(0.95)

## [1] 1.644854
```

(d) If the area to the left of z is 0.90, what is z?

Answer: $z = 1.282$

```
# Use the qnorm() function.

qnorm(0.90)

## [1] 1.281552
```

12. Use R to answer the following questions.

(a) If the area to the right of z is 0.10, what is z?

Answer: $z = 1.282$

```
# Use the qnorm() function.

qnorm(0.90)
```

```
## [1] 1.281552
# Use qnorm(0.10,lower.tail=FALSE) to confirm.
qnorm(0.10, lower.tail = FALSE)
## [1] 1.281552
```

(b) If the area to the right of z is 0.05, what is z?

Answer: $z = 1.645$

```
# Use the qnorm() function.
qnorm(0.95)
## [1] 1.644854
# Use qnorm(0.05, lower.tail = FALSE) to confirm.
qnorm(0.05, lower.tail = FALSE)
## [1] 1.644854
```

(c) If the area to the right of z is 0.025, what is z?

Answer: $z = 1.96$

```
# Use the qnorm() function.
qnorm(0.975)
## [1] 1.959964
# Use qnorm(0.025, lower.tail = FALSE) to confirm.
qnorm(0.025, lower.tail = FALSE)
## [1] 1.959964
```

(d) If the area to the right of z is 0.01, what is z?

Answer: $z = 2.326$

```
# Use the qnorm() function.
qnorm(0.99)
## [1] 2.326348
# Use qnorm(0.01, lower.tail = FALSE) to confirm.
```

```r
qnorm(0.01, lower.tail = FALSE)

## [1] 2.326348
```

13. Let x be a normally-distributed random variable with a mean $\mu = 25$ and standard deviation $\sigma = 5$.

    (a) What is the probability that x will be less than or equal to 35?

    Answer: 0.9772

$$p(x \leq 35) = p(\frac{x - \mu}{\sigma} \leq \frac{35 - 25}{5}) = p(z \leq 2) = 0.9772$$

```r
# Use the pnorm() function.

pnorm(35, 25, 5)

## [1] 0.9772499
```

    (b) What is the probability that x will be less than or equal to 32?

    Answer: 0.9192

$$p(x \leq 32) = p(\frac{x - \mu}{\sigma} \leq \frac{32 - 25}{5}) = p(z \leq 1.40) = 0.9192$$

```r
# Use the pnorm() function.

pnorm(32, 25, 5)

## [1] 0.9192433
```

    (c) What is the probability that x will be less than or equal to 30?

    Answer: 0.8413

$$p(x \leq 30) = p(\frac{x - \mu}{\sigma} \leq \frac{30 - 25}{5}) = p(z \leq 1) = 0.8413$$

```r
# Use the pnorm() function.

pnorm(30, 25, 5)
```

```
## [1] 0.8413447
```

(d) What is the probability that x will be less than or equal to 25?

Answer: 0.50

$$p(x \leq 25) = p(\frac{x-\mu}{\sigma} \leq \frac{25-25}{5}) = p(z \leq 0) = 0.50$$

```
# Use the pnorm() function.

pnorm(25, 25, 5)

## [1] 0.5
```

14. Assume x is a normally-distributed random variable with a mean $\mu = -63$ and standard deviation $\sigma = 4.5$.

(a) What is the probability that x will be less than or equal to -57?

Answer: 0.9088

$$p(x \leq -57) = p(\frac{x-\mu}{\sigma} \leq \frac{-57-(-63)}{4.5}) = p(z \leq 1.3333) = 0.9088$$

```
# Use the pnorm() function.

pnorm(-57, -63, 4.5)

## [1] 0.9087888
```

(b) What is the probability that x will be less than or equal to -60?

Answer: 0.7475

$$p(x \leq -60) = p(\frac{x-\mu}{\sigma} \leq \frac{-60-(-63)}{4.5}) = p(z \leq 0.6666) = 0.7475$$

```
# Use the pnorm() function.

pnorm(-60, -63, 4.5)

## [1] 0.7475075
```

(c) What is the probability that x will be less than or equal to -63?

Answer: 0.50

$$p(x \leq -63) = p(\frac{x-\mu}{\sigma} \leq \frac{-63-(-63)}{4.5}) = p(z \leq 0.00) = 0.50$$

```
# Use the pnorm() function.

pnorm(-63, -63, 4.5)

## [1] 0.5
```

(d) What is the probability that x will be less than or equal to -70?

Answer: 0.05991

$$p(x \leq -70) = p(\frac{x-\mu}{\sigma} \leq \frac{-70-(-63)}{4.5}) = p(z \leq -1.5556) = 0.05991$$

```
# Use the pnorm() function.

pnorm(-70, -63, 4.5)

## [1] 0.05990691
```

15. Let x be a normally-distributed random variable with a mean $\mu = 36$ and standard deviation $\sigma = 3$.

(a) What is the probability that x will be greater than 42?

Answer: 0.02275

$$p(x>42) = 1 - p(x \leq 42) = 1 - p(\frac{x-\mu}{\sigma} \leq \frac{42-36}{3}) = 1 - p(z \leq 2) = 0.02275$$

```
# Use 1 minus the pnorm() function.

1 - pnorm(42, 36, 3)

## [1] 0.02275013
```

```r
# Use pnorm(42,36,3,lower.tail=FALSE) to confirm.

pnorm(42, 36, 3, lower.tail = FALSE)

## [1] 0.02275013
```

(b) What is the probability that x will be greater than 39?

Answer: 0.1587

$$p(x>39) = 1 - p(x \le 39) = 1 - p(\frac{x-\mu}{\sigma} \le \frac{39-36}{3}) = 1 - p(z \le 1) = 0.1587$$

```r
# Use 1 minus the pnorm() function.

1 - pnorm(39, 36, 3)

## [1] 0.1586553

# Use pnorm(42,36,3,lower.tail=FALSE) to confirm.

pnorm(39, 36, 3, lower.tail = FALSE)

## [1] 0.1586553
```

(c) What is the probability that x will be greater than 33?

Answer: 0.8413

$$p(x>33) = 1 - p(x \le 33) = 1 - p(\frac{x-\mu}{\sigma} \le \frac{33-36}{3}) = 1 - p(z \le -1) = 0.8413$$

```r
# Use 1 minus the pnorm() function.

1 - pnorm(33, 36, 3)

## [1] 0.8413447

# Use pnorm(33,36,3,lower.tail=FALSE) to confirm.

pnorm(33, 36, 3, lower.tail = FALSE)

## [1] 0.8413447
```

(d) What is the probability that x will be greater than 27?

Answer: 0.9987

$$p(x>27) = 1 - p(x \leq 27) = 1 - p(\frac{x-\mu}{\sigma} \leq \frac{27-36}{3}) = 1 - p(z \leq -3) = 0.9987$$

```
# Use 1 minus the pnorm() function.
1 - pnorm(27, 36, 3)
## [1] 0.9986501
# Use pnorm(27,36,3,lower.tail=FALSE) to confirm.
pnorm(27, 36, 3, lower.tail = FALSE)
## [1] 0.9986501
```

16. Let x be a normally-distributed random variable with a mean of $\mu = 100$ and a standard deviation of $\sigma = 15$.

(a) If the area to the left of x is 0.99, what is x?

Answer: $x = 134.9$

Method 1. Recall

$$z = \frac{x-\mu}{\sigma}$$

and the area to the left of z is 0.99 when

$$z = 2.326$$

```
qnorm(0.99)
## [1] 2.326348
```

Substituting and solving for x

$$\frac{x-\mu}{\sigma} = 2.326$$

$$x = \mu + 2.326\sigma$$

Plugging in values for the mean and standard deviation $\mu = 100$ and $\sigma = 15$

$$x = \mu + 2.326\sigma = 100 + (2.326)(15) = 100 + 34.89 = 134.9$$

Method 2. Use R.

```
# Use the qnorm() function.

qnorm(.99, 100, 15)

## [1] 134.8952

# Use qnorm(0.01,100,15,lower.tail=FALSE) to confirm.

qnorm(0.01, 100, 15, lower.tail = FALSE)

## [1] 134.8952
```

(b) If the area to the left of x is 0.975, what is x?

Answer: $x = 129.4$

```
# Use the qnorm() function.

qnorm(.975, 100, 15)

## [1] 129.3995

# Use qnorm(0.025,100,15,lower.tail=FALSE) to confirm.

qnorm(0.025, 100, 15, lower.tail = FALSE)

## [1] 129.3995
```

(c) If the area to the left of x is 0.95, what is x?

Answer: $x = 124.7$

```
# Use the qnorm() function.

qnorm(.95, 100, 15)

## [1] 124.6728

# Use qnorm(0.05,100,15,lower.tail=FALSE) to confirm.

qnorm(0.05, 100, 15, lower.tail = FALSE)

## [1] 124.6728
```

(d) If the area to the left of x is 0.90, what is x?

Answer: $x = 119.20$

```r
# Use the qnorm() function.

qnorm(.90, 100, 15)

## [1] 119.2233

# Use qnorm(0.10,100,15,lower.tail=FALSE) to confirm.

qnorm(0.10, 100, 15, lower.tail = FALSE)

## [1] 119.2233
```

17. Let x be a normally-distributed random variable with a mean of $\mu = -280$ and a standard deviation of $\sigma = 35$.

    (a) If the area to the right of x is 0.10, what is x?

    Answer: $x = -235.1$

    Method 1. Recall

    $$z = \frac{x - \mu}{\sigma}$$

    and the area to the right of z is 0.10 when

    $$z = 1.28$$

```r
qnorm(0.10, lower.tail = FALSE)

## [1] 1.281552
```

    Substituting and solving for x

    $$\frac{x - \mu}{\sigma} = 1.28$$

    $$x = \mu + 1.28\sigma$$

    Plugging in the values for the mean and standard deviation $\mu = -280$ and $\sigma = 35$

    $$x = \mu + 1.28\sigma = -280 + (1.28)(35) = -280 + 44.8 = -235.1$$

    Method 2. Use R.

```r
# Use the qnorm() function.

qnorm(0.90, -280, 35)

## [1] -235.1457

# Use qnorm(0.10,-280,35,lower.tail=FALSE) to confirm.

qnorm(0.10, -280, 35, lower.tail = FALSE)

## [1] -235.1457
```

(b) If the area to the right of x is 0.05, what is x?

Answer: x=-222.4

```r
# Use the qnorm() function.

qnorm(0.95, -280, 35)

## [1] -222.4301

# Use qnorm(0.05,-280,35,lower.tail=FALSE) to confirm.

qnorm(0.05, -280, 35, lower.tail = FALSE)

## [1] -222.4301
```

(c) If the area to the right of x is 0.025, what is x?

Answer: $x = -211.4$

```r
# Use the qnorm() function.

qnorm(0.975, -280, 35)

## [1] -211.4013

# Use qnorm(0.025,-280,35,lower.tail=FALSE) to confirm.

qnorm(0.025, -280, 35, lower.tail = FALSE)

## [1] -211.4013
```

(d) If the area to the right of x is 0.01, what is x?

Answer: $x = -198.6$

```r
# Use the qnorm() function.

qnorm(0.99, -280, 35)
```

```
## [1] -198.5778

# Use qnorm(0.01,-280,35,lower.tail=FALSE) to confirm.

qnorm(0.01, -280, 35, lower.tail = FALSE)

## [1] -198.5778
```

18. According to British weather forecasters, the average monthly rainfall in London during the month of June is $\mu = 2.09$ inches. Assume the monthly precipitation is a normally-distributed random variable with a standard deviation of $\sigma = 0.48$ inches.

    (a) What is the probability that London will have between 1.5 and 2.5 inches of precipitation next June?

    Answer: 0.694

    $$p(1.5 \leq x \leq 2.5) = p(-1.23 \leq z \leq 0.85) = 0.694$$

    ```
    # From pnorm(2.5,2.09,0.48) subtract pnorm(1.5,2.09,0.48)

    pnorm(2.5, 2.09, 0.48) - pnorm(1.5, 2.09, 0.48)

    ## [1] 0.693989
    ```

    (b) What is the probability that London will have 1 inch or less of precipitation?

    Answer: 0.01158

    $$p(x \leq 1) = p(z \leq -2.27) = 0.01158$$

    ```
    # Use the pnorm() function.

    pnorm(1, 2.09, 0.48)

    ## [1] 0.01157853
    ```

    (c) If London authorities prepare for flood conditions when the monthly precipitation falls in the upper 5% of the normal June amounts, how much rain would have to fall to cause local authorities to begin flood preparations?

    Answer: $x = 2.88$

Since $z = 1.645$ cuts off the upper 5% of the standard normal probability distribution, we solve for the corresponding value of x

$$\frac{x-\mu}{\sigma} = 1.645$$

$$x = \mu + (1.645)\sigma = 2.09 + (1.645)(0.48) = 2.09 + 0.79 = 2.88$$

```
# Use the qnorm() function.

qnorm(.95, 2.09, 0.48)

## [1] 2.87953

# Use qnorm(0.05,2.09,0.48,lower.tail=FALSE) to confirm.

qnorm(0.05, 2.09, 0.48, lower.tail = FALSE)

## [1] 2.87953
```

19. The time required by students to complete an organic chemistry examination is normally-distributed with a mean of $\mu = 200$ minutes and a standard deviation of $\sigma = 20$ minutes.

    (a) What is the probability a student will complete the examination in 180 minutes or less?

    Answer: 0.1587

$$p(x \leq 180) = p(z \leq -1) = 0.1587$$

```
# Use the pnorm() function.

pnorm(180, 200, 20)

## [1] 0.1586553
```

    (b) What is the probability a student will take between 180 and 220 minutes to complete the examination?

    Answer: 0.6827

$$p(180 \leq x \leq 220) = p(-1 \leq z \leq 1) = 0.6827$$

```r
# From pnorm(220,200,20) subtract pnorm(180,200,20).

pnorm(220, 200, 20) - pnorm(180, 200, 20)

## [1] 0.6826895
```

(c) Since this particular class is a large lecture section of 300 students, and the final examination period lasts 240 minutes, how many students would we expect to submit the completed exam on time?

Answer: Around 293 to 294 students should finish on time.

$$p(x \leq 240) = p(z \leq 2) = 0.9772$$

$$(0.9772)(300) = 293.16 \text{ or } 293 - 294 \text{ students}$$

```r
pnorm(240, 200, 20) * 300

## [1] 293.175
```

20. A large agricultural producer in Spain produces melons with diameters that are normally-distributed with a mean of $\mu = 15$ centimeters (cm) and a standard deviation of $\sigma = 2$ cm.

(a) What is the probability that a melon will have a diameter of at least 12 cm?

Answer: 0.9332

$$p(x \geq 12) = p(z \geq \frac{12 - 15}{2}) = p(z \geq -1.5) = 0.9332$$

```r
# Use 1 minus the pnorm() function.

1 - pnorm(12, 15, 2)

## [1] 0.9331928

# Use pnorm(12,15,2,lower.tail=FALSE) to confirm.

pnorm(12, 15, 2, lower.tail = FALSE)

## [1] 0.9331928
```

(b) What is the probability that a randomly selected melon will have a diameter of no less than 12 cm but no more than 16 cm?

Answer: 0.6247

$$p(12 \leq x \leq 16) = p(-1.5 \leq z \leq 0.50) = 0.6247$$

```r
# Subtract pnorm(12,15,2) from pnorm(16,15,2).

pnorm(16, 15, 2) - pnorm(12, 15, 2)

## [1] 0.6246553
```

(c) The producer has an arrangement with several gourmet shops by which it will receive a slightly higher price for melons with a diameter that falls in the top 10%. What is the minimum diameter a melon must have in order to qualify for the higher price?

Answer: $x = 17.56$ cm

Method 1. Since $z = 1.28$ cuts off the upper 10% of the standard normal probability distribution, we solve for the corresponding value of x

```r
qnorm(0.10, lower.tail = FALSE)

## [1] 1.281552
```

$$\frac{x - \mu}{\sigma} = 1.28$$

$$x = \mu + (1.28)\sigma = 15 + (1.28)(2) = 15 + 2.56 = 17.56$$

Method 2. Use R to confirm that a melon with a diameter of 17.56 cm sets the cut-off point for the top 10%.

```r
# Use the qnorm() function.

qnorm(.90, 15, 2)

## [1] 17.5631

# Use qnorm(0.10,15,2,lower.tail=FALSE) to confirm.

qnorm(0.10, 15, 2, lower.tail = FALSE)

## [1] 17.5631
```

21. A variable is exponentially-distributed with a mean of $\mu = 5$.

(a) What is the form of the probability density function?

$$f(x) = \frac{1}{5}e^{-\frac{x}{5}}$$

(b) What is the cumulative probability function?

$$p(x \leq x_0) = 1 - e^{-\frac{x_0}{5}}$$

(c) What is $p(x \leq 2)$?

Answer: 0.3297

$$p(x \leq 2) = 1 - e^{-\frac{2}{5}} = 1 - 0.6703 = 0.3297$$

```
# Use the pexp() function.
pexp(2, 1/5)
## [1] 0.32968
```

(d) What is $p(x \leq 4)$?

Answer: 0.5507

$$p(x \leq 4) = 1 - e^{-\frac{4}{5}} = 1 - 0.4493 = 0.5507$$

```
# Use the pexp() function.
pexp(4, 1/5)
## [1] 0.550671
```

(e) What is $p(x \leq 5)$?

Answer: 0.6321

$$p(x \leq 5) = 1 - e^{-\frac{5}{5}} = 1 - 0.3679 = 0.6321$$

```
# Use the pexp() function.
pexp(5, 1/5)
## [1] 0.6321206
```

(f) What is $p(x \leq 8)$?

Answer: 0.7981

$$p(x \leq 8) = 1 - e^{-\frac{8}{5}} = 1 - 0.2019 = 0.7981$$

```
# Use the pexp() function.
pexp(8, 1/5)
## [1] 0.7981035
```

(g) What is $p(x \leq 15)$?

Answer: 0.9502

$$p(x \leq 15) = 1 - e^{-\frac{15}{5}} = 1 - 0.0498 = 0.9502$$

```
# Use the pexp() function.
pexp(15, 1/5)
## [1] 0.9502129
```

22. Referring to the preceding exercise, answer the following questions.

(a) What is $p(x \geq 2)$?

Answer: 0.6703

$$p(x \geq 2) = 1 - p(x<2) = 1 - (1 - e^{-\frac{2}{5}}) = e^{-\frac{2}{5}} = 0.6703$$

```
# Use 1 minus the pexp() function.
1 - pexp(2, 1/5)
## [1] 0.67032
# Use pexp(2,1/5,lower.tail=FALSE) to confirm.
pexp(2, 1/5, lower.tail = FALSE)
## [1] 0.67032
```

(b) What is $p(x \geq 4)$?

Answer: 0.4493

$$p(x \geq 4) = 1 - p(x<4) = 1 - (1 - e^{-\frac{4}{5}}) = e^{-\frac{4}{5}} = 0.4493$$

```
# Use 1 minus the pexp() function.
1 - pexp(4, 1/5)
## [1] 0.449329
# Use pexp(4,1/5,lower.tail=FALSE) to confirm.
pexp(4, 1/5, lower.tail = FALSE)
## [1] 0.449329
```

(c) What is $p(x \geq 5)$?

Answer: 0.3679

$$p(x \geq 5) = 1 - p(x<5) = 1 - (1 - e^{-\frac{5}{5}}) = e^{-\frac{5}{5}} = 0.3679$$

```r
# Use 1 minus the pexp() function.
1 - pexp(5, 1/5)
## [1] 0.3678794
# Use pexp(5,1/5,lower.tail=FALSE) to confirm.
pexp(5, 1/5, lower.tail = FALSE)
## [1] 0.3678794
```

(d) What is $p(x \geq 8)$?

Answer: 0.2019

$$p(x \geq 8) = 1 - p(x{<}8) = 1 - (1 - e^{-\frac{8}{5}}) = e^{-\frac{8}{5}} = 0.2019$$

```r
# Use 1 minus the pexp() function.
1 - pexp(8, 1/5)
## [1] 0.2018965
# Use pexp(8,1/5,lower.tail=FALSE) to confirm.
pexp(8, 1/5, lower.tail = FALSE)
## [1] 0.2018965
```

(e) What is $p(x \geq 15)$?

Answer: 0.04979

$$p(x \geq 15) = 1 - p(x{<}15) = 1 - (1 - e^{-\frac{15}{5}}) = e^{-\frac{15}{5}} = 0.04979$$

```r
# Use 1 minus the pexp() function.
1 - pexp(15, 1/5)
## [1] 0.04978707
```

```r
# Use pexp(15,1/5,lower.tail=FALSE) to confirm.
pexp(15, 1/5, lower.tail = FALSE)
## [1] 0.04978707
```

23. Referring to the previous exercise, answer the following questions.

    (a) What is $p(2 \leq x \leq 4)$?

    Answer: 0.221

    $$p(2 \leq x \leq 4) = p(x \leq 4) - p(x \leq 2) = 0.5507 - 0.3297 = 0.221$$

    ```r
    # Subtract pexp(2,1/5) from pexp(4,1/5).
    pexp(4, 1/5) - pexp(2, 1/5)
    ## [1] 0.2209911
    ```

    (b) What is $p(2 \leq x \leq 5)$?

    Answer: 0.3024

    $$p(2 \leq x \leq 5) = p(x \leq 5) - p(x \leq 2) = 0.6321 - 0.3297 = 0.3024$$

    ```r
    # Subtract pexp(2,1/5) from pexp(5,1/5).
    pexp(5, 1/5) - pexp(2, 1/5)
    ## [1] 0.3024406
    ```

    (c) What is $p(2 \leq x \leq 8)$?

    Answer: 0.4684

    $$p(2 \leq x \leq 8) = p(x \leq 8) - p(x \leq 2) = 0.7981 - 0.3297 = 0.4684$$

    ```r
    # Subtract pexp(2,1/5) from pexp(8,1/5).
    pexp(8, 1/5) - pexp(2, 1/5)
    ```

```
## [1] 0.4684235
```

(d) What is $p(2 \leq x \leq 15)$?

Answer: 0.6205

$$p(2 \leq x \leq 15) = p(x \leq 15) - p(x \leq 2) = 0.9502 - 0.3297 = 0.6205$$

```
# Subtract pexp(2,1/5) from pexp(15,1/5).

pexp(15, 1/5)-pexp(2, 1/5)

## [1] 0.620533
```

24. The number of visits to the Book4Less.com discount travel website is a Poisson-distributed random variable with a mean arrival rate of 10 visits per minute.

(a) If the Poisson arrival rate is 10 visitors per minute, what is the mean of the associated exponential probability density function?

Answer: If the mean Poisson arrival rate is 10 visitors per minute (every 60 seconds), then the time between passenger visitors is exponentially-distributed with a mean of 6 seconds.

(b) What is the exponential probability density function?

Answer:

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$f(x) = \frac{1}{6} e^{-\frac{x}{6}}$$

(c) What is the cumulative probability function?

$$p(x \leq x_0) = 1 - e^{-\frac{x_0}{\mu}}$$

$$p(x \leq x_0) = 1 - e^{-\frac{x_0}{6}}$$

(d) What is the standard deviation of the distribution?

$$\sigma = \mu = 6$$

25. Referring to the preceding exercise, answer the following questions.

(a) If the internet server experiences a brief power failure of 18 seconds duration during which time people would be denied access to the website, what is the probability that no one attempted to visit the Book4Less.com website anyway and thus no business was lost during the down time? Use the exponential framework to answer this question.

Answer: 0.04979

$$p(x \geq 18) = 1 - p(x \leq 18) = 1 - (1 - e^{-\frac{18}{6}}) = e^{-\frac{18}{6}} = 0.04979$$

```
# Use 1 minus the pexp() function.

1 - pexp(18, 1/6)

## [1] 0.04978707
```

(b) Answer question (a) using the Poisson probability approach. Confirm that the answer is equal to that in (a).

Answer: 0.04979

Since we are told that there is an average of 10 visitors per minute, or 10 visitors per 60 seconds, we need to convert this parameter: μ=10 visitors/60 seconds=3 visitors/18 seconds.

Recall that the Poisson probability function is

$$p(x|\mu) = \frac{\mu^x e^{-\mu}}{x!}$$

$$p(x = 0|\mu = 3) = \frac{3^0 e^{-3}}{0!} = e^{-3} = 0.04979$$

```r
# Use the dpois(0,3) function to find probability of 0 visitors
# during a period (18 seconds) when the mean is 3 visitors.

dpois(0, 3)

## [1] 0.04978707
```

(c) Comment on the fact that the answers to parts (a) and (b) are exactly the same regardless of the approach (Poisson or exponential) employed.

Answer: If the interval of time (or distance) between occurrences is distributed exponentially, then the number of occurrences in that interval must be Poisson-distributed. The two distributions are related.