

Chapter 9: Statistics with R - 2nd Edition

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Students Exercises

The following exercises test your understanding of hypothesis testing in the context of the triangle-taste test described in Chapter 9. In this case, the test consists of 14 identical trials on which the subject attempts to identify the odd sample on each trial. Assume that there are 2 possible rejection regions: (1) $RR_8 = \{8, 9, 10, 11, 12, 13, 14\}$ and $RR_{10} = \{10, 11, 12, 13, 14\}$.

1. With a Rejection Region of $RR_8 = \{8, 9, 10, 11, 12, 13, 14\}$, what is the probability of a Type I error? Recall that since a Type I error occurs when the subject has no taste-discrimination ability, $p = 1/3$.

Answer: 0.0576. This is equal to the probability of 8 or more correct identifications if the subject has no taste discrimination ability (and $p = 1/3$)

```
1 - pbinom(7, 14, 1/3)

## [1] 0.0576163

#or

sum(dbinom(8 : 14, 14, 1/3))

## [1] 0.0576163
```

2. With a Rejection Region of $RR_8 = \{8, 9, 10, 11, 12, 13, 14\}$, what is the probability of a Type II error, if the subject has a probability of $p = 0.80$ of identifying the odd sample?

Answer: 0.0116. That is, the probability of 7 or fewer correct identifications if the subject is able to identify the odd sample with 0.80 probability.

```
pbinom(7, 14, 0.80)

## [1] 0.01160991

#or

sum(dbinom(0 : 7, 14, 0.80))
```

```
## [1] 0.01160991
```

3. With a Rejection Region of $RR_{10} = \{10, 11, 12, 13, 14\}$, what is the probability of a Type I error?

Answer: 0.0040. This is equal to the probability of 10 or more correct identifications if the subject has no taste discrimination ability (and $p = 1/3$)

```
1 - pbinom(9, 14, 1/3)

## [1] 0.004039541

#or

sum(dbinom(10 : 14, 14, 1/3))

## [1] 0.004039541
```

4. With a Rejection Region of $RR_{10} = \{10, 11, 12, 13, 14\}$, what is the probability of a Type II error, if the subject has a probability of $p = 0.80$ of identifying the odd sample?

Answer: 0.1298. This is equal to the probability of 9 or fewer correct identifications if the subject is able to identify the odd sample with 0.80 probability.

```
pbinom(9, 14, 0.80)

## [1] 0.1298396

#or

sum(dbinom(0 : 9, 14, 0.80))

## [1] 0.1298396
```

5. Please answer the following questions about this triangle-test taste.

- (a) Which of the Rejection Regions should we prefer? RR_8 or RR_{10} ? Why?

Answer: we would prefer RR_{10} since the probability of a Type I error is considerably lower.

- (b) In general, a hypothesis test can result in 2 different types of errors. Describe those 2 errors in this case where we are attempting to identify someone with a high degree of taste-discrimination ability. Which is more serious?

Answer: in the case of the triangle-taste test, a Type I error occurs when we reject the null hypothesis when it is true; that is, when we conclude that someone with no taste-discrimination ability actually has it. A Type II error occurs when we do not reject the null hypothesis when it is false; in other words, when we conclude that someone who has ability does not actually have it after all. The consequences of committing a Type I error are more serious: the person hired as taster is less likely to discern problems with the product. The consequences of committing a Type II error are less serious since they mean that we fail to identify a subject who has taste-discrimination ability and we must continue interviewing and testing until the next qualified person applies for the position. Whereas the brewery can withstand a Type II error, it might not survive a Type I error.

- (c) We saw in Chapter 9 that α and β are usually in a trade-off relationship. That is, if we select one of two possible Rejection Regions—such as either RR_8 or RR_{10} —we can reduce α only if we are willing to have a higher β . Can you think of anything we might do to reduce both α and β simultaneously? What would that be?

Answer: it is possible to reduce both α and β simultaneously by increasing the sample size n . In this case, this would involve increasing the number of trials.