

ACTIVITY IA8.4: Palindromization

Intended learning: To add multi-digit numbers using semi-formal or formal written strategies.

Instructional mode: Longer, inquiry mode for individuals or groups.

Materials: Writing materials, a hundreds chart per group.

Description: Palindromes are words or sentences that read the same when the order of the letters is reversed, for example: level; Hannah; do geese see God? Examples of palindromic numbers include 66, 121 and 2002. Begin the activity by introducing palindromes and palindromic numbers, writing a few on the board. Then introduce the palindromization technique. Write 36 on the board. *Thirty-six isn't a palindrome. But there is a technique that can turn a number into a palindrome: reverse the digits, and add the original number.* Write 63 beneath 36, and complete writing the sum in vertical format – 36 plus 63 is 99 – we got a palindromic number. Let's try 37: 37 requires two steps to palindromize (see Figure 8.8). Set some for the class to try: 25, 52, 84.

$ \begin{array}{r} 36 \\ 63+ \\ \hline 90 \\ 9 \\ \hline 99 \text{ in one step} \end{array} $	$ \begin{array}{r} 37 \\ 73+ \\ \hline 100 \\ 10 \\ \hline 110 \text{ not yet} \\ 011+ \\ \hline 121 \text{ in two steps} \end{array} $
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Figure 8.8 Palindromizing 36 and 37

The activity can finish here, with students practising the palindromization technique, and hence practising column addition. Alternatively, begin a class investigation of the following question: *Does palindromization work for any number up to 100?* Support a discussion of how to organize the investigation. Students can use a hundreds chart to record which numbers need testing. Students can realize that about half the numbers don't need testing, because they are already palindromes (e.g. 22, 33) or palindromes after one easy step (e.g. 21, 36). Some students may also recognize that half the remaining numbers don't need testing because they are reversals: if 37 works, then so will 73. That accounts for all but 20 of the numbers. The results for the 20 numbers are: 1 step (29, 38, 47, 56); 2 steps (19, 28, 37, 39, 46, 48, 49, 57, 58, 67); 3 steps (59, 68); 4 steps (69, 78); 6 steps (79); 24 steps (89).

Responses, variations and extensions:

- The activity generates considerable multi-digit addition practice.
- The interest in generating a palindrome brings attention to the digits in the numerals while students are calculating. Some students who use a semi-formal written strategy, such as

partial sums, may begin to use more abbreviated forms, focusing on the digits in each column.

- Watch for 79 and 97, which require a 4-digit addition with regrouping.
- Watch especially for 89 and 98. Requiring 24 steps to palindromize into the 13-digit number 8813200023188, this case requires a calculator, determination and patience. Still, a good approach is not to reveal an answer that students are willing to find for themselves.
- In pursuing the investigation, allow the students to develop their own investigative approaches. They are learning how to test a conjecture, as well as how to calculate an addition.
- Palindromization can be extended to 3-digit numbers, and beyond. The conjecture that palindromization works for *all* numbers remains an open question: mathematicians have not been able to prove it true or false. Some numbers appear to be unpalindromizable: 196 remains unpalindromized after millions of iterations (e.g. <http://mathworld.wolfram.com/196-Algorithm.html>).