

ACTIVITY IA8.5: Catch 22

Intended learning: To use written strategies for multi-digit addition and division, and to develop insight into place value.

Instructional mode: Longer, inquiry mode for individuals or groups.

Materials: Writing materials, a set of digit cards for each group.

Description: Each group plays the role of a number machine. Lead the groups through the five-step program for their machine.

1. Select three different digits (e.g. 1, 4, 7)
2. Calculate the sum of the three digits (e.g. $1 + 4 + 7 = 12$)
3. Write out the six different 2-digit numerals that can be made with the digits (e.g. 14, 17, 41, 47, 71, 74)
4. Calculate the sum of the six numbers (e.g. $14 + 17 + 41 + 47 + 71 + 74 = 264$)
5. Divide the big sum from step 4, by the little sum from step 2 (e.g. $264 \div 12 = 22$).

The machines report their output. They all report 22! Is there something wrong with the machines? Try running the program again.

The second phase of the activity is to investigate whether the program output is always 22, and why. Allow the students to develop their own investigative approaches. Writing the six numbers in column format and summing the tens and ones separately can help reveal the arithmetic pattern in the program. In the six numbers, each digit appears twice in both the ones and the tens. So the sum of the ones is 2 times the sum of the three digits, and the sum of the tens is 20 times the sum of the three digits. Therefore the sum of the six numbers is 22 times the sum of the three digits.

Responses, variations and extensions:

- An optional approach in the group machines is to make each group member responsible for a different step, like a production line.
- Running the machine program generates practice in multi-digit addition and division. After some turns being machines, students could pause to discuss their computation strategies, especially for division.
- While investigating the program, students might begin to organize their calculations into tens and ones.
- Instead of investigating whether the big sum $\div 22$ equals the little sum, students might switch to investigating the equivalent question of whether the little sum $\times 22$ equals the big sum, showing an insightful implicit understanding that multiplication is the inverse of division.