

KNOWLEDGE CHECK

18

MENTAL CALCULATIONS, CHANGING PROPORTIONS TO PERCENTAGES

Without using a calculator, rewrite these statements using percentages:

- a) A quarter of the pupils in my class have free school meals.
- b) Three-quarters of the pupils in my class do not have free school meals.
- c) Seven out of eight pupils in primary schools like their teacher.
- d) This year our school had to employ a supply teacher 17 days out of 20.
- e) Four-fifths of the lessons observed by Ofsted in our school were good or very good.
- f) A total of 273 pupils out of 300 achieved at least one GCSE at Grade 5 or above.

ANSWERS TO KNOWLEDGE CHECK 18

- a) 25% of the pupils... b) 75% of the pupils... c) 87.5% of primary pupils...
d) 85% of the days... e) 80% of the lessons... f) 91% of the pupils...

DISCUSSION AND EXPLANATION OF KNOWLEDGE CHECK 18

Per cent (%) means 'for each hundred'. For example, 27% (27 per cent) means '27 in each hundred', or '27 out of a hundred'. Percentages are useful because they give us a standard way of expressing proportions. This makes it easy to compare different proportions of pupils with free school meals in two schools if they are expressed as percentages (e.g. 37% and 35%). It is not so easy if all you have is the raw data (e.g. 170 out of 459 in one school and 238 out of 680 in the other).

Many simple proportions or fractions can be easily expressed as percentages, using mainly mental calculations. For example, the fraction one-half ($\frac{1}{2}$) might represent the proportion of a set of secondary pupils who walk to school. Without knowing how many pupils there are in the set, we can still express this proportion as an equivalent percentage. One-half as a proportion means 'one out of every two pupils walks to school. That's equivalent to 'fifty out of a hundred', or 50%. Knowing this we can easily deduce percentage equivalents for some other common fractions. Since a quarter ($\frac{1}{4}$) is 'half of a half', then it must be equivalent to half of 50%, that is 25%. And three-quarters ($\frac{3}{4}$) will be three times this, which is 75%.

Eighths are a bit trickier. One-eighth is 'half of a quarter', so expressed as a percentage it must be 'half of 25%', which gives 12.5%. Knowing this, you can then work out percentage equivalents for $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$. Actually, in example (c) I found it easier to think 'one-eighth of the pupils do not like their teacher', which is 12.5%, and then to subtract this from 100% to find the percentage who do like their teachers. This works because 100% represents the whole set of pupils, that is '100 out of 100'.

In example (d) the '17 out of 20' is easily converted to an equivalent proportion out of a hundred, just by multiplying by 5. This gives '85 out of 100', which is 85%.

In example (e) we can think of $\frac{4}{5}$ as '4 out of 5', which is equivalent to 80 out of 100, or 80%.

In example (f) we simply divide 273 by 3, to deduce that '273 out of 300' is equivalent to '91 out of 100', or 91%.

SUMMARY OF KEY IDEAS

- *Per cent* means 'for each hundred' (e.g. 35% means '35 out of 100')
- A proportion can be written as a percentage, by working out the equivalent number out of a hundred (e.g. '7 out of 20' is '35 out of 100', which is 35%; '240 out of 300' is '80 out of 100', which is 80%).
- The percentage equivalents of many simple proportions (such as $\frac{1}{2} = 50\%$, $\frac{3}{4} = 75\%$, $\frac{4}{5} = 80\%$) should be memorised.



FURTHER PRACTICE

Do these without using a calculator.

- 18.1 Work out the equivalent percentages for the following fractions and then commit them all to memory!

$$\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{20}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}$$

- 18.2 A pupil scores 21 marks out of 25 in one test and 17 out of 20 in another. Express these marks as equivalent percentages, to decide which mark is the higher proportion of the total marks available.
- 18.3 In a school of 600 pupils, there are 126 pupils with English as an additional language. In another school the number is 104 out of 400, Compare these proportions by expressing them as percentages.