

KNOWLEDGE CHECK

19

MENTAL CALCULATIONS, CHANGING MORE PROPORTIONS TO PERCENTAGES

Do not use a calculator.

- a) School X has 248 pupils out of 400 living within a mile of the school; School Y has 135 out of 250. Which has the larger proportion living within a mile of the school? To answer this, express the proportions as percentages.
- b) A pupil scores 22 marks out of 40 for part A of a mathematics test and 39 out of 60 for part B. In terms of percentages, in which part did the pupil score higher marks?
- c) Three schools expressed the proportion of pupils on free school meals in different ways.

School A: $\frac{1}{5}$ of the pupils have free school meals.

School B: 17% of pupils have free school meals.

School C: 77 pupils out of 350 have free school meals.

Which school has the lowest and which school has the highest proportion of pupils on free school meals?

ANSWERS TO KNOWLEDGE CHECK 19

- a) School X: 62%. School Y: 54%. School X has the larger proportion.
- b) Part A: 55%. Part B: 65%. Part B was the higher percentage mark.
- c) The lowest is School B with 17%, the highest is School C with 22%; School A has 20 %.

DISCUSSION AND EXPLANATION OF KNOWLEDGE CHECK 19

Proportions such as these can be changed into equivalent proportions out of 100, by simple multiplications and divisions, especially doubling and halving where possible. They can then be expressed as percentages. My strategy is to look at the total number involved and ask how I can relate it to 100. So, for example, given a proportion of 400, I will halve and halve again to get an equivalent proportion of 100. Given a proportion of 250, I will double and double again to get to 1000 and then divide by 10 to get to 100. Below I show how I reasoned for the proportions in these Knowledge check questions. You may well have done these differently, which is fine, of course. I have used an arrow (\rightarrow) to mean 'is equivalent to'.

248 out of 400 \rightarrow 124 out of 200 [halving] \rightarrow 62 out of 100 [halving] = 62%

135 out of 250 \rightarrow 270 out of 500 [doubling] \rightarrow 540 out of 1000 [doubling] \rightarrow 54 out of 100 [dividing by 10] = 54%.

22 out of 40 \rightarrow 11 out of 20 [dividing by 2] \rightarrow 55 out of 100 [multiplying by 5] = 55%.

39 out of 60 \rightarrow 13 out of 20 [dividing by 3] \rightarrow 65 out of 100 [multiplying by 5] = 65%

$\frac{1}{5}$ \rightarrow 1 in 5 \rightarrow 2 in 10 [doubling] \rightarrow 20 in 100 [multiplying by 10] = 20%.

77 in 350 \rightarrow 11 in 50 [dividing by 7] \rightarrow 22 in 100 [doubling] = 22%.

Note that in all these questions the relationships between the numbers are such that it is not difficult to manipulate them mentally to obtain proportions out of 100. In the last example it was easy to do this once I spotted that 7 divided exactly in 77 and 350. If, however, the proportion had been, say, 79 out of 350 or 77 out of 352, this would have been far more difficult to handle by purely mental methods and it would make more sense to use a calculator.

SUMMARY OF KEY IDEAS

- Many proportions can be expressed as percentages by changing them to equivalent proportions out of 100, by simple multiplications and divisions.
- Look for ways of using doubling, halving, multiplying by 5 or 10, or using simple divisions to relate the total number involved to 100.
- If there is no simple way of changing a proportion to an equivalent proportion out of 100, then use a calculator to do this.



FURTHER PRACTICE

Do these without using a calculator.

- 19.1 A pupil scores 19 out of 25 in a spelling test and 42 out of 70 marks in a mathematics test. Express these marks as equivalent percentages.
- 19.2 In Ofsted inspections of a sample of 250 schools, 35 schools were graded as unsatisfactory, and 130 were graded as good, for leadership and management. Express these proportions as percentages. What percentage of schools were graded satisfactory (the only other grade awarded)?
- 19.3 Three primary schools reported the proportions of their year 6 pupils achieving a score of 100 or above for mathematics as follows:

School P $\frac{3}{5}$ of the pupils achieved a score of 100 or above

School Q 74% of the pupils achieved a score of 100 or above

School R 36 out of 45 pupils achieved a score of 100 or above

Put these proportions in order from the highest to the lowest.