KNOWLEDGE CHECK

4

THE ASSOCIATIVE LAWS

Each pupil in a class of A pupils requires B kilograms of flour for a series of food technology lessons. Flour costs £C per kilogram.

a) What will be the total cost if $A = 29$, $B = 2.5$ and $C = 0.46$?

b) Which of the following formulas gives the total cost in pounds: $A \times (B \times C)$ or $(A \times B) \times C$?

ANSWERS TO KNOWLEDGE CHECK 4

a) £33.35.

b) Either of the formulas can be used.

DISCUSSION AND EXPLANATION OF KNOWLEDGE CHECK 4

To calculate the total cost of the flour required here, you could first calculate the total cost per pupil (B × C) and multiply this by the number of pupils. This would be using the formula A × (B × C). The brackets indicate which bit of the calculation is to be done first. Alternatively, you could first calculate the total amount of flour required for the class (A × B) and then multiply this by the cost per kilogram. This would be using the formula (A × B) × C. Either way you get the same result.

This demonstrates what is called the *associative law* of multiplication: A × (B × C) = (A × B) × C, whatever numbers are chosen for A, B and C. This means that if you have three numbers to be multiplied together, the one in the middle can be 'associated' with either the first number or the last number and you get the same answer. This is very useful in mental calculations. For example, to calculate 4 × (5 × 13) it is much easier to think of it as (4 × 5) × 13, i.e. 20 × 13, which equals 260. So, if I had to calculate 12 × 35 mentally, I would think of the 35 as (5 × 7) and then associate the 5 with the 12 as follows: 12 × 35 = 12 × (5 × 7) = (12 × 5) × 7 = 60 × 7 = 420.

Because A × (B × C) and (A × B) × C are equal, we can just write 'A × B × C' without any brackets, recognising that it does not matter whether we start with the A ×B or with the B × C.

When we combine the associative property of multiplication with the com-mutative law, the upshot is that we can multiply three (or more) numbers together in any order you like! So, for example, if you had to calculate 25 × (86 × 4) you could rearrange this as 25 × (4 × 86), using the commutative law. Using the associative law, this then becomes (25 × 4) ×86, which is 100 × 86 = 8600.

All this is also true of addition. The associate law of addition is: A + (B + C) = (A + B) + C, allowing us to write just 'A + B + C' to mean either of these. Combined with commutativity, this property allows us to add three (or more) numbers in any order we like: which is probably what you would have done anyway, even if I had not given you this elaborate mathematical explanation!

You will probably have guessed by now that subtraction and division are not associative. These operations are explored in the further practice questions.

SUMMARY OF KEY IDEAS

- Addition is associative.
- In algebraic notation: $A + (B + C) = (A + B) + C$, for any numbers A, B and C.
- This allows us to write '$A + B + C$' to mean either of these.
- Multiplication is also associative.
- In algebraic notation: $A \times (B \times C) = (A \times B) \times C$, for any numbers A, B and C.
- This allows us to write $A \times B \times C$ to mean either of these.
- Subtraction and division are not associative.

FURTHER PRACTICE

4.1 Confirm that subtraction is not associative by evaluating '30 – (18 – 10)' and '(30 – 18) – 10'. Which of these corresponds to this situation: in a class of 30, the 18 pupils who still have to return their school–home con-tracts were asked to bring them in on Monday, but 10 of them forgot again; how many contracts were now returned?

4.2 Do two calculations involving the numbers 160, 8 and 4, to demonstrate that division is not associative.

4.3 Use the associative law to calculate mentally the cost of 28 books at £25 each, by thinking of the 28 as 7 × 4.