## NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by **Mastery** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning.**

Hence, the activities suggested here are designed to promote the following:
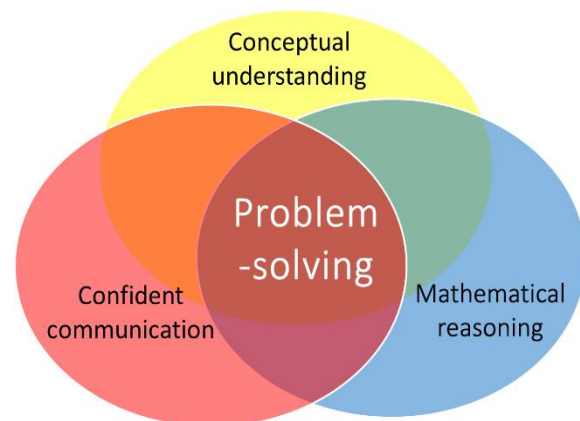
- practical activity manipulating concrete resources where possible;

- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;

- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;

- solving problems (or by playing games) with the potential for a useful or pleasing result;

- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

There is also a file of resource sheets used in some of the activities, which may be reproduced freely. However, please include any source information on each copy.

Related chapter, key learning & rationale	Plan for teaching and learning	Crucial points & barriers to understanding
6. Numbers and Place Value **Recognise the place value of each digit in a two-digit number (tens, ones).** This activity is intended to engage with the same number in four different representations: • symbolically and positionally on a number square; • using place value cards to partition into symbolic components in *tens* and *ones*; • using *base-10 apparatus* to partition it in a concrete/physical form; • speaking and hearing the English vocabulary language to describe the number and its place value components.	<u>Place race</u> The teacher first models the game and then the children play in pairs. They will need: • One set of place-value (p.v. or 'arrow') cards representing *tens* and *ones* for each pair of children; • *Base-10* or *Dienes' apparatus* (*tens* and *ones* only); • Place value (p.v.) mat (marked for just *tens* and *ones*, or fold back the *hundreds* on a three-digit p.v. mat); • 100-square (0–99 or 1–100); • Counters in two colours. Luke and Emily agree which colour of counters each will use. They shuffle the p.v. cards and set them out randomly, face down on the table. The base-10 apparatus is in a pile the other side of the p.v. cards. Luke chooses any uncovered two-digit number from the hundred square, pointing to and saying the number aloud. The pair then race each other: • Luke searches for the correct *tens* and *ones* p.v. cards needed to comprise the number and assembles these face up on the table; while … • Emily grabs the correct numbers of *tens* and *ones base-10* pieces and places these in the correct columns on the p.v. mat. The first child to complete their task calls out the number and claims the number by placing their coloured counter over it, then waits for the other to finish (and helps them if necessary). They both check each has the correct cards/numbers of *base-10* pieces. Then they return the *base-10* pieces to the pile and shuffle the p.v. cards face down on the table, to begin again. The next time Emily calls the number and searches for the p.v. cards, while Luke grabs the *base-10* pieces for the p.v. mat. The activity can be simplified by limiting the available range on the number square, or it can be extended to three-digit numbers to make it more challenging.	Do the children match the correct number of *base-10* pieces to each of the p.v. cards in the number? Do they have to recognise the equivalence of 'two tens' and 'twenty'? Do they speak and interpret correctly the irregular English vocabulary for the numbers 11 to 20? Do the children correct each other if they detect their partner is mistaken?

| 7. Addition and Subtraction Structures

Use the comparison structure to understand subtraction as the difference between two numbers.

The example activities here focus on subtraction, as these structures are more difficult for children to understand than addition. | <u>**What's the difference?**</u> The teacher first models this activity and then children work in pairs as partners. They will need:

- Multilink cubes in two colours (for example: 10 of each).

Luke and Emily agree which colour of multilink each will use. Without telling each other, each child takes a number of their cubes and makes a single column tower; for example, Luke makes a tower of 9 red cubes while Emily makes a tower of 3 green cubes.

They place their towers side by side to compare them. Both children write this comparison as a *subtraction* number sentence, starting with the longer tower:
9 – 3 =

They count the *extra* cubes found in the longer tower to find the *difference*. They agree what this is and then complete their number sentence with the answer:
9 – 3 = 6

They repeat this several times.

The activity can be simplified or extended by changing the number of cubes each child has. | Do the children ensure they align the base ends of the tower (for example, by placing both towers upright on the table)?

Do they understand the term **difference** as a form of subtraction? Children's experiences of subtraction often focus upon **take away** (partitioning).

Do they physically count on or count back correctly to identify the number of cubes which comprise the difference? |

| 8. Mental Strategies for Addition and Subtraction

Mentally add and subtract one-digit and two-digit numbers to 20 (and beyond!)

Represent and use number bonds and related subtraction facts within 20.

This activity is a *real life* use of money: that of selecting appropriate coins to combine to make a specified amount. | <u>**Loose change**</u> This activity requires the following resources for the children to use:
• Tray of coins of different denominations 1p, 2p, 5p, 10p;
• Number of classroom items labelled with different prices from 3p to 20p, for example: book, pencil, ruler, etc.
Ask children to choose an item to buy and then select the fewest coins needed to pay the exact price for each item in turn.

They write the number sentence which represents the sum of the coins. For example, for a pencil costing 9p, Luke writes:
    5p + 2p + 2p = 9p

When they have written their sums for each price the group explain their answers to each other in groups of 2–4 as a simple form of peer assessment.

A desktop number line can be used to support calculation (numbered 1–20 is sufficient)

As an extension, children can be asked to find alternative combinations of coins to make the same price (not just the fewest).

Challenge the children by reducing the available denominations, for example: just 1p, 2p and 5p.

The activity can be modified as needed by limiting or extending the range of prices. | Can the children make up an aggregate sum of money from the available coins to reach the required value?

Do the children recognise that there is a one to many relationship between different values of coin?

Can they make a correct substitution of other coins for one coin?

Do they try to find the highest available coin denomination to reduce the number of coins they need?

Do they add each additional amount correctly from the previously accumulated value?

When they have a partial value, can they subtract by counting on/back to find the remaining amount to make? |

9. Written Methods for Addition and Subtraction

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

Rather than introducing formal vertical written methods in KS1, we advise that in working towards these, activities for years 1–2 concentrate on practising skills and understanding in the partitioning of numbers which are crucial for developing the formal vertical written methods in KS2.

Finding friendly pairs The teacher should demonstrate with some examples, then in pairs, children separate place value cards into two shuffled piles: *tens* and *ones* placed face down on the table.

Emily takes one place value card from the top of each pile to make a two-digit number, for example

$$\boxed{2\,0} \quad + \quad \boxed{4} \quad = \quad \boxed{2\,4}$$

Luke takes the next p.v. card from the top of the pile of ones cards, for example, $\boxed{8}$

Both children write down the addition sentence for these numbers: **24 + 8 =**

On a place-value mat, using base-10 apparatus Emily sets out 24 as **20 + 4** and Luke sets out 8, below left (see photocopiable resources):

Tens	Ones

They must look for a way of using all or part of the **8** to add easily to all or part of the 24. For example, Emily decides to add 6 to the 4 to make 10, So she partitions the 8 and rearranges the ones as on the right:

She can then rewrite the addition as:
20 + 4 + 6 + 2 and regroup it as
20 + 10 + 2

Tens	Ones

Do the children see how they can use knowledge of number bonds to 10 to see how to look for complements to make a 10?

In time, children should make the connection to extend the 10-complement facts to make the next multiple of 10 for any addition.

Children can explain that the actual value of, say, three *tens (longs)* is 30

Children should see that visualising the expression of their calculation as **20 + 2 + 10** is completely valid, and that we go on to group the *tens* and the *ones* appropriately.

When confident, children can set out and add a pair of two-digit numbers for sums less than 100.

Luke decides to use the 8 to make 10, so he needs 2 from the 4, below left:

He can then rewrite the addition as:
20 + 2 + 8 + 2 and regroup it as
20 + 10 + 2.

After exchanging 10 *ones* for 1 *ten*, they compare their calculations to see how both arrived at the result.

| 10. Multiplication and Division Structures

Use the inverse-of-multiplication (repeated subtraction) structure to understand division.

One problem in developing children's understanding of division is that concrete experiences in the lower primary years can overstate the equal-sharing structure. The equal-sharing structure is the basis of fractions, written methods of division, swap between equal-sharing and the inverse-of-multiplication (equal grouping or repeated subtraction). This activity is intended to develop children's understanding of the latter. | **How many groups?** Children take a given number of multilink and make a joined row of single cubes. They then see how many equal groups of the same length which they can break the row into without any left over. They write the division sentences for each one.
For example, starting with a row 10 cubes long:

Emily tells Luke they can make groups of 2, and demonstrates this:

They both write the number sentence: *10 ÷ 2 = 5*
Luke puts the row back together and attempts to break it into groups of 3, but finds he has 1 cube left over. Emily suggests groups of 4, but they find there are 2 cubes left. They find groups of 5:

And write the next number sentence: *10 ÷ 5 = 2*

They then do the same for other numbers, such as 9, 12, 16, 11 …

The activity can be simplified or extended by changing the numbers set for the rows the children are to make. | When dividing the tower, every group has to be the same size, with no cubes left over.

Emphasising the language '*equal groups of*' and that the children are *subtracting* the *same* group *many* times.

Children may still confuse which value represents the *group size or number in each group*, with the value which represents the *number of groups* when writing a number sentence.

Children should begin to see that some numbers (primes) can only be one single group or grouped in *ones.* |

11. Mental Strategies for Multiplication and Division
 	
Calculate mathematical statements for multiplications and divisions within the multiplication tables.
 	
Recall and use multiplication and division facts for the 2, 5 and 10-times multiplication tables.
 	
The aim here is to use very obvious concrete examples and visual images to illustrate mental strategies of multiplication and division as repeated additions and subtractions.
	<u>Groups of …</u> For each question children confer in pairs and show their answer on a mini-whiteboard, so that the teacher can assess everyone at the same time.
	
	'2 times …' questions - **Multiplication as repeated additions:**
	   a) How many feet does one child have?
	   b) How many feet do 2 children have?
	   c) How many feet do … children have? And so on.
	
	Division as inverse-of-multiplication/repeated subtractions:
	   a) There are 8 feet. How many children?
	   b) There are 10 feet altogether: how many children are there? And so on.
	
	'10 times …' questions: Do the same using the number of fingers and thumbs per *child*.
	
	'5 times …' questions: Do the same using the number of fingers and thumbs per *hand*.
	
	This visual model can be extended to '***4 times***', using the number of legs on a toy animal or the number of wheels on a car; to '***6 times***' using legs on an insect, and to '***8 times***' with legs on a spider.
		Inclusion of inverse questions to establish the inverse relationship between multiplication and division.
		
		The class has sufficient resources to model and physically check at least the 2-times, 10-times and 5-times questions.
		
		Do the children understand the divisor is the ***group size or number in each group***?

12. Written Methods for Multiplication and Division
 	
Recall and use multiplication and division facts for the 2, 5 and 10-times multiplication tables, including recognising odd and even numbers.
 	
This activity is a 'real-life' use of money, that of selecting and working out the value of a group of coins.
 	
While it is inappropriate to introduce formal written methods for multiplication and division to children in KS1, it is helpful for children to see that money can be counted using multiplication and practise strategies of multiplication and division as repeated additions and subtractions.
	Counting up the coins Demonstrate to the whole class, then children work in pairs. They will need:
	
	• Purse or wallet containing mixed coins of different denominations 1p, 2p, 5p, 10p;
	• Tray of additional coins;
	• Number of items labelled with different prices in multiples of 2p, 5p and 10p. For example: A pencil costing 16p, and eraser costing 35p and a book costing 60p, etc.
	
	First Emily and Luke must sort the coins in the purse into their different denominations. After this, they each write down the value of the coins of each type as multiplication. For example:
	• 6 × 1p = 6p; 9 × 2p = 18p; 5 × 5p = 25p; 7 × 10p = 70p.
	
	• Luke and Emily now swap their purse with another pair and find the values of the coins in a different purse.
	
	• After this they use the coin tray to help them count up and write down the calculation for each item they may buy, according to these rules:
	
	   o If the price is an exact multiple of 10p, write the number of 10p coins needed, for example: for the book, write 6 x 10p = 60p.
	
	   o If the price is an exact multiple of 5p, write the number of 5p coins, for example: the eraser is 7 x 5p = 35p and the book is 12 x 5p = 60p.
	
	   o If the price is not a multiple of 5p or 10p, but it is an even number, write the number of 2p coins, for example the pencil would be 8 x 2p = 16p.
	
	The activity can be simplified by reducing the numbers of coins or limiting the denominations.
	
	To extend the children, when they have counted the coins from different purses, challenge them to work out the total values of each purse, for example by using mental or informal strategies to calculate, say, 70p + 25p + 18p + 6p (as in the example above).
		Are the children secure in understanding that a coin has a **unitising** or **1:many relationship** with other coin denominations?
		
		Do they add these correctly according to their value, not simply counting the number of coins?
		
		Can children discuss which purse may contain the largest sum of money and why they think this is, say by comparing the number and relative values of the different coins?

13. Natural Numbers: Some Key Concepts **Investigate patterns of squares and begin to relate these to number.** **Recognise and understand some properties of common 2-D shapes (squares).** Although, children do not formally encounter square numbers in KS1, We can help children to develop their recognition of squares as shapes and their understanding of related patterns in number through the exploration of this puzzle.	**Seeing squares** In groups of 3 or 4, children explore individually and discuss their findings with one another. Each child will need: • Squared paper, cut into 4 × 4 squares – one for each child: There is a photocopiable resource for this activity, to ease preparation. Ask the class how many squares they can see on their piece of squared paper. Most children will simply be led by the squared paper and find 16. One or two may see that there also some larger squares present, if they look for arrangements of 2 × 2, or 3 × 3. If no child suggests this challenge them to look and see if they can see any more squares than just the 'little ones'. When they see that there are larger squares, ask them to describe the length of the sides ('2 little squares long' will be sufficient here). Once the children realise they can find more, set them off in groups of 3 or 4 and ask the group to tell you how many squares they can find of different sizes and how many altogether. By comparing and discussing what they see, Luke, Emily, Kasia and Nathan will have more support in developing their understanding of the properties of squares and latterly of square numbers. There are 16 × (1 × 1), 9 × (2 × 2), 4 × (3 × 3), 1 × (16 × 16) and 30 squares printed on the paper altogether.	Do the children see that there are several sizes of square? Do they have a way of explaining this? Do they realise that some squares overlap others? Ask the children how they can be sure they have found them all? Do they have a way of checking for the number of each size of square?

14. Integers: Positive and Negative **Understand ordinal use of number can be extended to negative numbers.** **Read and interpret temperature scales.** Temperature is a real-life context in which we can introduce KS1 children to positive and negative whole numbers in a meaningful way.	<u>**Temperature check**</u> An on-going activity, a regular observation to carry out with the whole class. You will need: • Large classroom number line: if possible, a vertical line with a range of at least –10 to (+)30; • Two large, easy-to-see, air-temperature thermometers. Rather than one in-depth specific lesson, it is helpful to introduce children as early as possible to the everyday use of a thermometer to measure temperature. In the UK, temperatures rarely exceed a range of between -10°C to (+)30°C, so it is a practical exercise to record the inside and outside temperatures each day as a real-life exercise in mathematical/scientific data-gathering. It helps to hang two separate thermometers that can be briefly compared side by side, but do not risk the outside thermometer changing its value if it is very sensitive, so it may be that you read it *in situ* with different volunteers each day. Use the inside thermometer to help the whole class learn to count along the unmarked divisions to the temperature indicated. Use the large classroom number line to mark the inside and outside temperatures, and then together find the difference (*comparison* structure of subtraction) between them. Record publicly the two temperatures and the difference between the temperature inside and outside. Make comparisons between different days of the week. Use the number line to illustrate differences in an informal, visual way. On very cold days of course, we can naturally introduce even young children to the idea of negative numbers, and they may hear this on TV and radio (although the word 'minus' is used). It is also a very natural way to help them see that you can find the difference between a positive and a negative number, by seeing the visual space between them on the number line. This is very helpful to see the difference between, say, +1°C and -1°C!	It does not matter at this stage that children may not know what is actually meant by degrees, just that they see that we measure temperature in degrees, just like we measure money in pence, and length in metres. Do the children realist that in this context 0°C does not represent the *absence* of temperature (or heat)? Do they see that 0 is simply a point in **ordering** the values of temperature, from which we count numbers positively in one direction (getting warmer) and negatively in the other (getting colder)? Can the children see the **difference** between two temperatures as the numerical 'space' between them on the number line, regardless of whether the temperatures are positive or negative values?

15. Fractions and Ratios

Recognise, find, name and write fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{4}$ of a set of objects or quantity.

To enable children to develop a solid understanding of fractions, it is important for them to have as much practical experience as possible in which they associate the language and notation used to express fractions with concrete examples. This activity concentrates on finding fractions of a number, as many of children's experiences of fractions are likely to be with shape.

**Sharing the sweets** Children work in pairs. Each pair will need:

- Counters (for *sweets*) in prepared paper bags of 4, 8, 12 and 16;
- Four 'soft toys', say *Teddy, Rover, Froggy and Rabbit* to be the recipients of sharing the sweets;
- *Post-it notes,* placed on a maths mat, one in front of each soft toy, used as 'plates' to collect the sweets.

Halves: Sharing equally between two toys:

- Starting with the bag of 4 sweets, show the children how to *share* the sweets *equally between 2* toys, *Teddy and Rover.*

  The children pour out the sweets on to their maths mat, and at the bottom of the mat they stick one *post-it* for *Teddy* and another *post-it* for *Rover*. They then start *sharing equally* the sweets between *Teddy* and *Rover* until there are none left.

  At the end emphasise that '*Teddy* has **half** of the 4 sweets, and *Rover* has **half** of the 4 sweets. **One half** of 4 is 2.' You may write this symbolically for the children to see '$\frac{1}{2}$ of 4 = 2'.
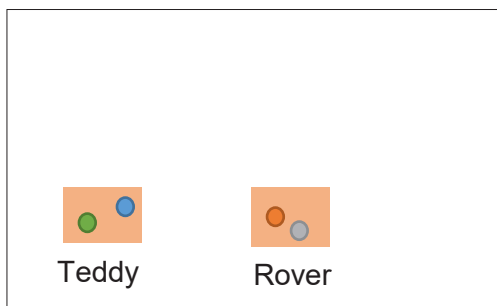
Teddy   Rover

Do the children connect the number who are sharing and the fraction this produces?

Do they see that every fraction must be an equal share of the number of sweets?

When finding quarters, do children realise how many quarters *Teddy, Rover* have together? (**two quarters**: if appropriate, you could write this as $\frac{2}{4}$),

Taking this further, do children see how many quarters *Teddy, Rover* and *Rabbit* have together?' (**three quarters**: you could write this as $\frac{3}{4}$).

Do the children recognise the equivalence between $\frac{2}{4}$ and $\frac{1}{2}$ of a number of sweets?

	• Repeat the demonstration with the bag of 8 sweets. This time **one half** is … Explain that between them *Teddy* and *Rover* actually have '**two halves**' and in each example ask the children 'How many sweets in **two halves**?' • Now Emily and Luke **share** the bag of 12 sweets **equally** between *Teddy* and *Rover*. They need to agree what is **one half** of 12. • Repeat this with the bag of 16 sweets. **Quarters: Sharing equally between four toys:** • Now explain that *Froggy* and *Rabbit* have also come to play with *Teddy* and *Rover.* How many will need to share the sweets altogether now? If necessary, demonstrate sharing equally between 4, to find **one quarter**, using $^1/_4$ of 4 and then $^1/_4$ of 8, before asking them to share the 12 and the 16.	

16. Decimal Numbers and Rounding	Pay the least amount	Do the children correctly recognise the values of different coins and add these values correctly?
The use of rounding in the context of money. **Combine amounts to make a particular value.** **Find different combinations of coins that equal the same amounts of money.** To enable children to develop a proper understanding of rounding, they will need to understand that the nearest number to round towards depends on the context of the problem. They can begin to see this in real contexts at an early age, for example when paying for things they wish to buy. In this problem, they have only a few coins from which to choose and they need to offer the smallest amount necessary to pay for the intended item.	<u>**Pay the least amount**</u> Children to work in groups of 4, possibly with some adult support. They will need: • A number of items labelled with different prices: 3p, 6p, 8p, 11p, 13p, 17p, 22p, 26p; • Three purses or bags each containing one each of the following coins only: 2p, 5p, 10p and 20p; • One small tray of mixed coins including 1p pieces, and/or a number line to assist in calculating the change; • Counters to award to the 'winners' of each round. First model how we may not have the exact money when we pay for something, and how to find the smallest value combination of coins, for example, to offer for an item costing 1p, 4p and 7p. In each case discuss different coins which could be used from the purse or bag and which would provide the smallest amount. Then set the children working in their groups, e.g.: • Emily is the shopkeeper with the items for sale. Starting with the cheapest of these, she places the 3p item in front of her. Luke, Kasia and Nathan select the smallest amount that they would need to offer to pay for this item. All should all choose 5p. Those who offer the smallest amount from their coins are winners and may take one counter each. • Emily works out the change that would be given for the smallest amount, in this case 2p. The other children discuss and help her if she has difficulty and there is no adult present. If the change is the correct amount, Emily takes one counter. • Note that the children do not hand over the coins, only show what they would offer. They keep the coins to use again for subsequent items. No change is actually given so that the coins in each purse or bag remain the same. • Luke passes his bag to Emily, and he now becomes the shopkeeper. The 6p item is next, so the children should offer 7p (5p + 2p) as the smallest amount, and Luke should find 1p in change. For higher attainers extend the amounts up to 50p. Vary the coins and prices to enable other combinations to be explored.	Do the children correctly recognise the values of different coins and add these values correctly? Do the children realise that it is always possible to offer an amount more than the price required, but we are trying to find the **nearest** possible amount to the cost of the item? Do children understand that we cannot round down to find the amount we need? We cannot offer an amount less than the price of the item (as we are not usually able to haggle with the shopkeeper!).

17. Calculations with Decimals Calculations with decimals would be inappropriate for this age group.	However, any KS1 activity which reinforces place value, i.e.*'10 of these is equal to 1 of those'* is foundational to the understanding of decimals and decimal fractions in future.	Do children recognise the principle of exchanging 10 *ones* for 1 *ten,* and that the maximum value of each column is the single digit 9?

18. Proportionality and Percentages

Recognise proportions within a set of objects or quantity.

To enable children to develop a solid understanding of ratio and proportion, it is important to gain practical experience in which they see how this is maintained with concrete examples. This activity can be done with children in KS1 as it demonstrates proportionality and equivalence without using symbolic notation. It is a prior activity to *Creative Cuboids* recommended for Y3 and Y4 in activities for Chapter 15.

Proportional patterns Working in pairs, children will need the following:

- Multilink cubes in several colours;
- Prepared sheets with tables to record results (see photocopiable resources):

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

First, model an example of a row of 3 multilink cubes, choosing from 2 colours:

Say that for this pattern there are '2 red cubes in 3 cubes' and '1 yellow cube in 3 cubes'.

Now explain that you want to keep using the same pattern, in another row. How many cubes would there be altogether? How many red? How many yellow? Demonstrate this:

Now use the language of proportion to describe the pattern: '2 red cubes *in every* 3, will be 4 red cubes *in every* 6. 1 yellow cube *in every* 3, will be 2 yellow cubes *in every* 6'.

Now ask the children to predict how many cubes of each colour would be needed altogether if there is another row.

Do the children see the idea of *ratio* in the continuity between '2 *in every* 3', and '4 *in every* 6', etc.?

Do the children to see that the step increase in each column is by the same number as in the first row every time?

Do they make connections with counting up groups of 2s, or 3s?

Can the children predict how many they will have of each colour in total, before they add the next row?

Demonstrate this, and express it: '6 red cubes *in every* 9. 3 yellow cubes *in every* 9.' Write this into a copy of the tables:

Colour: *red*		
2	in every	*3*
4	in every	*6*
6	in every	*9*

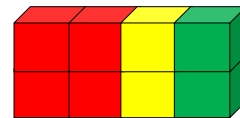| Colour: *yellow*		
1	in every	*3*
2	in every	*6*
3	in every	*9*

Colour:		
	in every	
	in every	
	in every	

Ask the children if they can see any patterns in the columns of numbers.

Now Luke and Emily each create their own patterns based on a row of 4 multilink cubes, choosing from 3 colours. For example, Luke creates:
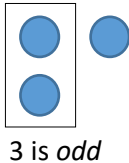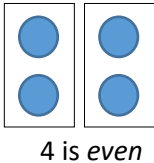
then:

… and so on.

The children each complete their own tables, then compare their patterns, to see what they notice about how the numbers increase in each column and any relationships between them.

After this, they can explore rows of 5 choosing from 3 colours, and perhaps rows of 6 choosing from 3 colours.

19. Algebraic Reasoning

Recognise odd and even numbers.

Understand the pattern for the outcomes of their additions.

It is very helpful in developing their understanding of algebra that they look for *generalisations*. Here, children explore the addition of numbers to determine the expectations of combining odd and/or even numbers.

Odds and evens In pairs. Children explore individually and compare their findings with one another. They will need:

- Number cards from 1 to 10;
- Counters, or *numicon* templates;
- Prepared tables to record whether odd/even (see photocopiable resources).

First establish/revise the odd/even property of numbers from 1 to 10. One basic visual representation to split the number of counters into two equal lines (halves) then if there is the same number in each line after this, the number of counters was *even*. If there is one more in one pile than the other, then the number of counters was odd.

Probably a more helpful way of testing whether a number is odd or even is to set out the counters in groups of 2. If there is a whole number of 2s without a counter 'left over', the number is even. If there is 1 counter left over, the number is odd. The counters can be arranged to show this.

This is naturally more powerfully demonstrated by taking a *numicon* template for each number and then attempting to place a series of templates for 2 along the top of the number. Any number which is completely covered by '2s' is *even*, while any which shows an uncovered '1' from the template below is *odd*. For example:

3 is *odd*　　4 is *even*

Do the children securely distinguish the property **odd** from **even**? For example, do they recognise that evens have the property of 'being shared equally between 2'?

Can children describe a rule in simple sentences? For example: 'When you add an *odd* number to another *odd* number, you will get an *even* number. When you add an *odd* number to an *even* number you always get an *odd* number.'

For each number Emily and Luke count out that number of counters, and see whether it is odd or even. They can then write the number under the appropriate heading in the table:

Odd	Even
1, 3, 5, 7, 9	2, 4, 6, 8, 10

Next ask Emily and Luke to see what happens when they combine different numbers by adding them together. Which additions give them a new *odd* number, which add together to make an *even* number? For example 3 + 3?
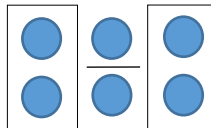
Again, this is something which is easy to demonstrate if the counters are grouped in '2s', or by using *numicon* templates, where the '1s' of two odd numbers combine by interlocking to make another '2'. Ask the children to write the additions under the appropriate table headings:

Odd	Even
1 + 2 = 3 2 + 3 = 5 3 + 4 = 7	1 + 1 = 2 2 + 2 = 4 1 + 3 = 4 3 + 3 = 6

Once the children have completed a number of additions can they see any rules for always getting an *odd* number, or always getting an *even* number?

To take it further, what happens when they combine three numbers? Can they begin to explain why?

20. Coordinates and Linear Relationships

Understand the use of simple row and column labelling to identify position in 2 dimensions.

In this activity, children explore position, identifying a row and column reference to locate a square on the paper. While this version does not use a frame of true co-ordinate references (which locate points, rather than whole squares), it is a helpful way for younger children to identify position in 2 dimensions. Row and column references are also used on some maps, for example, *A-Z street maps*.

Simple battleships Children play in pairs. Each pair will need:

- Sheets of squared paper, or prepared grids – one each (there is a photocopiable resource available);

- A paper screen to enable them to hide their grids from one another (see below how to make this).

The activity here is the traditional paper game of *battleships*.

Ships are set out as crosses in a line of whole squares, so there is no debate about whether a targeted square is part of a battleship, e.g.:

First of all, give out the prepared grids, or show the class how to mark out a large square grid on their squared paper, comprising 10 rows and 10 columns. Label the squares along the **horizontal** base with the letters A–J, and the squares up the left **vertical** edge with the numbers 1–10.

	A	B	C	D	E	F	G	H	I	J
10										
9						X				
8	X	X	X	X		X				
7					X				X	
6				X			X		X	
5							X		X	
4							X		X	
3		X	X	X	X		X			
2										
1						X	X	X	X	

Show how to locate any single square by identifying the combination of its row and column, for example, D6, F9, and practise this with the children. Emphasise that we give the horizontal location first when describing any row and column pair (some remember this as '*along* the hall **then** up the stairs'). This will be an important order to retain when making the transition to point co-ordinates later.

Now explain that this grid is an area of the ocean, where battleships are out 'on manoeuvres'. Show how children may place their battleships. Each battleship is represented by a single line of

Do the children understand the terms **horizontal** and **vertical?**

Do the children identify an intended square correctly?

Do the children recognise the relationship of the identification for each square with its neighbouring squares?

Do they associate the *rightward* direction in labelling *alphabetically* along the **horizontal** axis, and the *upward* direction for labelling *numerically* along the **vertical** axis?

four *crossed* squares, positioned horizontally, vertically or diagonally. Invite the children to fire a shot at one of your battleships, by naming a square it is covering. A 'hit' on any crossed square destroys the whole of that ship.

Luke and Emily hide their grids from one another behind the paper screen and mark out the same number of battleships. Then each takes it in turn to 'fire' a row and column location to the other:

- For example, if Luke goes first he may call 'C3'. Emily circles the location C3 on her grid and tells Luke whether it is a 'hit' or a 'miss';
- It is important that they answer truthfully whether a hit has been made upon one of their ships. It is useful that they each write a list of locations they have fired at, to check any disagreement at the end of the game;
- The winner is the first to score *at least* one hit on *every* one of their opponent's ships. When this happens, they take down the screen, and then check every shot they have recorded to confirm the result.

To make the game last longer, include ships of a shorter length. It is very hard to hit a ship whose length is just one square! Alternatively, use smaller grids to make the game shorter or easier.
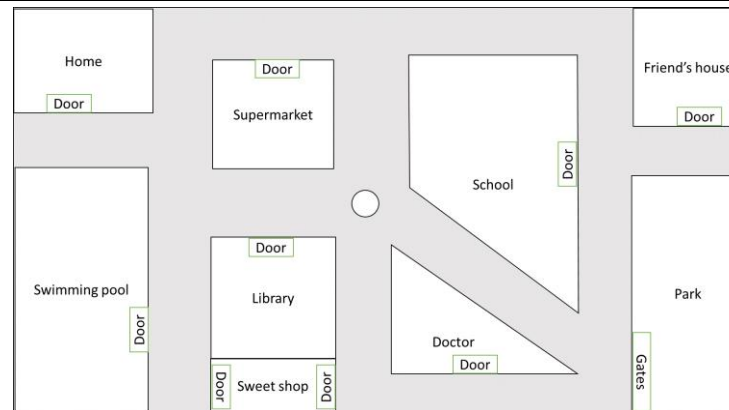
| 21. Concepts and Principles of Measurement

Developing understanding of length in different dimensions.

Use the comparative language of long, longer, longest, tall, taller, tallest, short, shorter, and shortest.

Children explore comparative language for length and height, but are also challenged to explore early ideas about ratio and proportion. | <u>**Comparisons**</u> Children working in groups of 4. They will need:

• Large roll of paper, of the size used to line a display board, or for covering tables.

Emily, Luke, Kasia and Nathan line up in order of their height. They record their names in order from shortest to the tallest.

Now they compare the lengths of their feet, the length of their middle fingers, the length of their right arm and the width of their hand spans. How does the order of each of these measures compare to the order of their heights?

If you can find a suitably long wall space, make a whole class tableau of the children. The group draws around the outline of each child as they take it in turns to lie on the large paper. They carefully cut around the outline. (If desired, these can be painted or covered with an appropriate collage by each child. The display space needed can be shortened by overlapping the children's images as in a 'crowd scene'. What is interesting here is that provided the top of each head is labelled, at another point much later in the year, the children compare themselves again and see whether there have been changes in the height order between them. If doing this with older children (Y2), they could actually compare heights in metres and centimetres, and make the other comparative measures in cm. | Do children use the vocabulary correctly?

Do they confuse 'near' terms – for example *'big'* with *'long'* and *'short'* with *'small'*?

Do they confuse *'tall'* with *'high'*, for example, if a child stands on a step?

Do the children recognise that proportions are general, and there some surprises in the expectations for who will have longer fingers, feet, hand spans, etc.? |

22. Perimeter, Area and Volume **Developing early understanding of perimeter as a measurement of length in non-linear situations.** **To measure length using a unit of 1 metre.** Children explore the notion of perimeter as the length of the complete boundary around a given space. Even young children can understand that fences which surround fields, or other spaces, have to be measured, so that we know how much we need.	<u>How much fencing?</u> Children work in groups of 2–4. Each groups will need access to: • 'Clicking' trundle wheel; • Metre stick. Mark out some prospective spaces with (P.E.) space markers or cones before the lesson. Explore the idea that there may be suggestions for the use of space outside the school buildings, which may need to be enclosed with fencing or for some other reason: for example, a new environmental area, a small garden or allotment, the school playing field, the playground itself, a children's patio, and so on. Show the children how a trundle wheel is used to measure 1 metre and demonstrate its equivalence to a measure of 1 metre using a metre stick. It is very helpful to have a trundle wheel which clicks every metre, as the children can count the number of clicks. Show the children how the trundle wheel could be used to measure a *perimeter.* It is ok to use this term – just help the children to understand that it is a measure of the length all the way along a boundary. Show how to measure the (approximate) perimeter of the classroom, say. Count the clicks and agree with the children the perimeter of the classroom to the nearest metre. Set out some P.E. space markers to mark an approximate rectangle or other large polygon on the floor. How long would a fence around the shape need to be? Help the children see how to traverse the boundary of this virtual shape with the trundle wheel to measure its perimeter. Emily, Luke, Kasia and Nathan, then go outside and measure the spaces to be enclosed for themselves. At this stage, do not worry about stopping at corners, just turn and continue when needed. All measurements can be approximate.	Do children see the equivalence between one turn of the trundle wheel and the straight length of the metre stick? Do the children understand that the perimeter is simply a measure of the distance around the boundary of a space? Do they traverse the boundary of the shape carefully? Do the children use the trundle wheel and count the number of metres correctly?

23. Angle

Developing the understanding of angle as a dynamic measure of a change in direction.

Use the vocabulary: *whole, half, quarter turn*, *clockwise* and *anticlockwise*.

The 'human robot' and controllable toys are very engaging, practical ways to explore position and direction. They are therefore a helpful way to explore the dynamic aspect of angle.

When discussing a change in direction, it is helpful to use the term ***angle***, even though we are describing this in terms of parts of a ***turn***. It helps children to begin to understand the concept of angle as a dynamic measure of a change in direction.

<u>Robots</u> In groups of 2–4 children. Children will need:

- (Freestanding) controllable moving toy such as a 'beebot';
- Large sheet of sugar paper with a simple village road plan drawn upon it, suitable to be traversed by the 'beebot' (or whatever is used). See example in photocopiable resources, which could be enlarged if you have a suitable copier.

Begin with the whole class standing practising turning in response to a demonstration and then by instruction alone, to teach/reinforce the terms for changing direction: ***whole, half, quarter turn, clockwise*** and ***anti-clockwise.***

Then appoint a child to become a 'human robot', and explain that the robot only understands certain commands – the instructions for changing direction, and the commands forward and back a given <number> of steps. Humorously test some trial instructions and non-instructions suggested by individual children in the class, reminding the 'robot' that unless the instruction is given in the correct form, they will have to ignore it. As a class provide instructions to direct the 'robot' to another part of the classroom.

The children then carry this out in their groups. Emily, Luke, Kasia and Nathan each take an opportunity to become the 'robot', and the others give instructions to direct the robot to a place in the classroom (avoiding other 'robots' en route!).

Do children understand that ***angle*** refers to a concept of dynamic change in the direction of a line of travel,

Do they understand angle can be measured simply to begin with in terms of ***whole, half*** and ***quarter turns***, together with the orientation of the turn – ***clockwise*** or ***anti-clockwise***?

Can the children visualise the forward view from the position of the robot, whether human or toy? This is necessary in order to give the direction commands accurately to the robot.

One group can use the controllable toy to move it along the roads to different places on the village floor plan.

When using the controllable toy, the simpler the device, the better. Many devices remember and accumulate a sequence of instructions, which can be confusing at this stage, so it is important to *clear all* previous instructions before each new instruction is given.

24. Transformations and Symmetry **Developing a practical understanding of reflection as an inverse of a shape or movement.** **Use the vocabulary:** *reflect*, *reflection*, *mirror*. When we look in a mirror the degree of symmetry in our bodies often fools us into thinking that this is the view the world has of us, when of course it is a reflection of that. Someone with a visible asymmetrical feature, such as a birthmark, will unconsciously believe that it appears to everyone on the same side of the face as it appears to them in the mirror. This activity is to help children get used to the idea of reflection being an inverse. It can be carried out as a dance activity during a P.E. session.	**Dancing reflections** Children work in pairs. The teacher may optionally introduce a piece of music or a poem as a stimulus for movement. Begin with the whole class standing facing you. Tell the children that they are to pretend to be your reflection in a mirror. As you (slowly) move a part of your body, even simply to wink an eye, ask the children to make the move that you would see in the mirror. Throw in some occasional quicker moves and see if the children remember to reflect with correct body part. Then ask the children to work in pairs. Emily leads a series of movements, and Luke endeavours to reflect each movement that Emily makes at the same time. Then they swap over. With the teacher playing the music (or reciting the poem) as a stimulus, if possible, Luke and Emily make up a sequence of moves (four is sufficient), in which they alternate the lead and the reflection between them with each move. Ask them to practise this so that they make their moves as perfectly reflected as possible. The pairs perform their dances to the rest of the class. To speed this up, the pair can perform in sets, and ask different parts of the class to watch specific pairs' performances. Discuss the performances, who led and who reflected? Could the observers see or was the performance too good to identify that? How could the reflections be improved? During the paired work, it becomes very interesting if the children turn so that they cannot see their partner! Suggest that they include only moves that enable them to see each other. Challenge higher attaining children to make some of their moves very subtle, just a little turn of the head, or even extending a finger.	When looking at half of a shape reflected in a mirror, many children will say that the two halves they see *'are the same'*, rather than recognise that every feature of one half has an *inverse orientation and position* in the reflection. Do the children understand that a *reflection* is an inverse orientation of a shape or a movement? Does the leader's *left* hand appear 'in the mirror' as the reflector's *right* hand, and so on?

25. Classifying Shapes

Recognise and name common 2-dimensional shapes.

Use the vocabulary to identify properties: *number of sides*, *number of vertices*.

A common misconception in children's recognition of shapes is that they often see very few *irregular* shapes*. Most manufactured shapes, whether in the built environment or simply plastic shapes provided for maths resources, are usually *prototypical*; that is, the shapes are either *regular* (equal sides and equal angles) or they are presented more frequently in one irregular form than any other (e.g. quadrilaterals as rectangles). If you do not have any *atypically irregular* shapes in your resources, make some by cutting from card or thin plastic, to show variation from the prototypical shapes available.
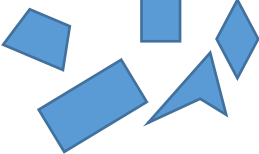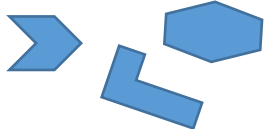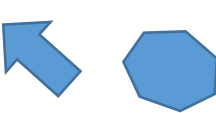
Shape sorter Children to work in groups of 3 or 4. They will need:

- Set of many prepared plastic or card shapes (N.B. see rationale);
- Large sheet of sugar paper divided into rectangles for classifying the shapes (see photocopiable resource to enlarge).

Tell the class you have had all your shapes muddled up and they need to be sorted so that you know how many of each type there are in the sets. Ask the children to think of ways in which they could sort them. Help the children to consider categories such as a particular number of (straight) sides: 3, 4, 5, 6 and so on. Help them to consider any shapes which have curves separately.

Display and say the names of specific polygons and the number of sides and vertices each has. Ask the children to write these in the 'boxes' on their sheet of paper. Ensure they also have a box for 'curves' and 'don't know'.

Emily, Luke, Nathan and Kasia must now work through their set of shapes, discuss and agree which box each shape should be placed in, and explain their reasons when asked.

Do children begin to sort shapes by properties such as number of sides and number of vertices?

Do they recognise that some prototypical forms are just special types of certain shapes? For example, a *rectangle* is a special type of quadrilateral, and that a *square* is a special type of rectangle.

When they see shapes which have curves – are any of these *circles*? We might say that circles are 'as tall as they are wide'. Others will be *ovals*. Some may have a combination of sides and curves. What would we call these? (For example, there may be a *semi-circle*.)

*Hershkowitz, R. (1990) 'Psychological aspects of learning geometry', in Nesher, P. and Kilpatrick, J. (eds), *Mathematics and Cognition.* Cambridge: Cambridge University Press, pp 70–95.	The children's discussion should be able to describe the properties they have used to classify the shapes. As a class, you highlight some shapes which may be of specific interest and the 'don't knows'. Children enjoy exploring mathematical words, so don't worry about using 'big words' and providing some polygons of 8 or even more sides.	

26. Handling Data	**Traffic survey** In small groups with an adult or as a whole class with additional adults to supervise as necessary. Each group will need:	Do the children realise they must make one stroke on the *correct* row of the tally chart for every vehicle they see?
To use a *tally* chart to organise data received in an unsorted order. The purpose of a tally chart is to sort data that is being received in an unpredicted order, so that it can be easily organised during the collecting phase. To demonstrate the tally chart's effectiveness to children, the data they collect needs to be arriving in this way too. Incorrect use of a tally chart is a common problem observed among trainee teachers, and sadly this is sometimes due to bad guidance in published schemes of work. I have seen children being introduced to tallies by counting the number of their peers, say, who walk to school, by a show of hands, which can be totalled at the same time, rendering the need for a tally chart redundant.	• Prepared charts for recording the tallies (see photocopiable resources); • Clipboards and spare pencils. An interesting example of the need for a **tally chart** is to carry out a traffic survey. Of course, this needs to be properly supervised if the children need to leave the school premises to carry it out; it may be most effective for a teaching assistant to take a small group at a time, with comparisons made of traffic at different times during the day; alternatively, the teacher could take the whole class with additional adult helpers to supervise groups at a safe place from which to observe passing traffic. In many schools an actively used road can be observed from within the school grounds. Set up the initial premise that the class has been asked to survey traffic to gather information for changes to routing or other forms of traffic management, such as where traffic lights and crossings need to be placed. Ask the children to discuss between them and identify the different categories of vehicles it may be helpful to count: *car, lorry, bus/coach, motorcycle, bicycle, van, tractor and 'other'* (just in case). Children write these categories in their chart. From a safe vantage point, children watch and tally observations into the different categories, for a fixed period, for example, 10 minutes.	Do the children know how to make the fifth stroke on each tally as the 'bar' across the four previous strokes? Do they see how they can completed tallies help us to count up each category more easily in groups of 10s and 5s? Do the children realise there is no point in creating a tally chart if we can easily find out the total of each category without one?

Therefore, to show the use of a tally for 'journeys to school', ensure that the children ask their peers individually one at a time, so that the data arrives in an unpredictable order.	Upon their return, they count up their tallies and record the totals for each category, e.g.	

Vehicle	Tally	Total
car	ĦĦ ĦĦ ĦĦ ĦĦ ĦĦ ĦĦ |||	33
lorry	ĦĦ ĦĦ ||	12
bus/coach	|||	3
van	ĦĦ ||||	9
tractor			1
motorcycle	ĦĦ		6
bicycle	||||	4
other			1

There needs to be a meaningful discussion of their findings, as if preparing for a council transport meeting. Use the opportunity to teach/reinforce traffic awareness and road safety. Discuss the different vehicles:

- How many two-wheeled vehicles? (motorcycles + bicycles)
- How many were large vehicles? (lorries + buses)
- How many were small vehicles? (cars + vans)
- Why might there be more of some types than others?
- Were there any they did not see? Why?

| 27. Comparing Sets of Data | This chapter is mostly focused on the professional needs of teachers, rather than the learning needs of children. However, it is important that by the end of Key Stage 2 children understand and use the terms *range*, *minimum*, *maximum* and the *mean* as a measures of average. So an activity is suggested here on these ideas, but only for children in upper Key Stage 2. | |

28. Probability

To experience the equal probability of events involving symmetrical outcomes.

Although *Probability* does not presently appear in the English Primary curriculum, it is a worthwhile and interesting area of mathematics for children, and it is included here as it is still featured in other international curricula.

Probability is one of the most important areas for using and applying mathematics in real life. It is a key part of our mathematical skills to respond to life's chance events with intelligent strategies based on our understanding of probability.

(To use a *tally* chart to organise data received in an unsorted order – c.f. activity for Chapter 26.)

Snakes and Ladders Children work in small groups of 3 or 4. They will need:

- A single prepared chart for recording the tallies (See photocopiable resources);
- *Snakes and ladders* game board, a single die and a different coloured counter for each player.

Simple *chance* games with playing cards such as *Snap* and *Beat your neighbour out of doors* and simple dice games such as *Snakes and Ladders* help children begin to experience chance from an early age, though unfortunately, often acquiring some wrongly established intuitive ideas along the way! Often children's *subjective* experience of throwing a die – when they require a *six* in order to start – is that it is harder to get a *six* than any other number! The problem is, of course, the heightened emotional attachment the child has to the outcome, hence previous unpleasant experience is more likely to be recalled. This activity is to help to dispel such false notions.

Emily, Luke, Kasia and Nathan play a game of *Snakes and ladders* with a difference. At every throw they record in the *same* tally chart the number shown on the die, thus using their game to collect data from their experiences of throwing the die. For example:

Number	Tally	Total
1	IIII IIII IIII IIII	19
2	IIII IIII IIII I	16
3	IIII IIII IIII	14
4	IIII IIII	9
5	IIII IIII III	13
6	IIII IIII IIII I	16

After different groups of children have played and collected a reasonable number of throws, collect and display the accumulated totals from all the groups for each number, and discuss the data with the children. It should become clear that in practice the number of throws will not be the same for each number, but that *six* is not a particularly disadvantaged number!

Do the children see that the frequency of each number being rolled is reasonably similar across the numbers? For any particular number that may have occurred many fewer times, ensure that the children see data from another group where this was not the case.

See also the potential crucial points and barriers to understanding for tallying (Y1–2 activity for Chapter 26).

Common equipment recommended

Notes about some of the common equipment recommended for use with the activities.

100 square

It is helpful to have available class sets of paper 100 squares of size about the width of a piece of A5.
Two sets are useful in different arrangements: 1–100 and 0–99.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counters

A large box of 500–1000 counters in many colours, of about 1.2cm diameter.
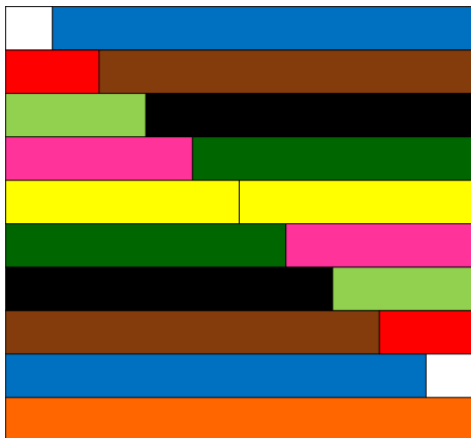
Cuisenaire Rods

These coloured rods of different lengths are great for helping children to develop a practical sense of ratio and of algebra from an early age. The rods were invented by a Belgian primary school teacher, Georges Cuisenaire. The right to use the name 'Cuisenaire' is now owned by a specific company, but other manufacturers supply this resource by different descriptions.

1					
½			½		
⅓		⅓		⅓	
¼	¼		¼		¼
⅕	⅕	⅕	⅕		⅕
⅙	⅙	⅙	⅙	⅙	⅙
⅛	⅛	⅛	⅛	⅛	⅛ ⅛ ⅛
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$

Fraction Wall

A rectangular tray containing plastic or wooden strips, each strip sub-divided into equal fractions of a different denominator, enabling the pieces to be moved and recombined in different ways to make 1. It is a physical model of the fraction chart pictured here.

Children can be encouraged to create their own Fraction Wall with identical paper strips which they fold into $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{8}$ and paste to a backing sheets.
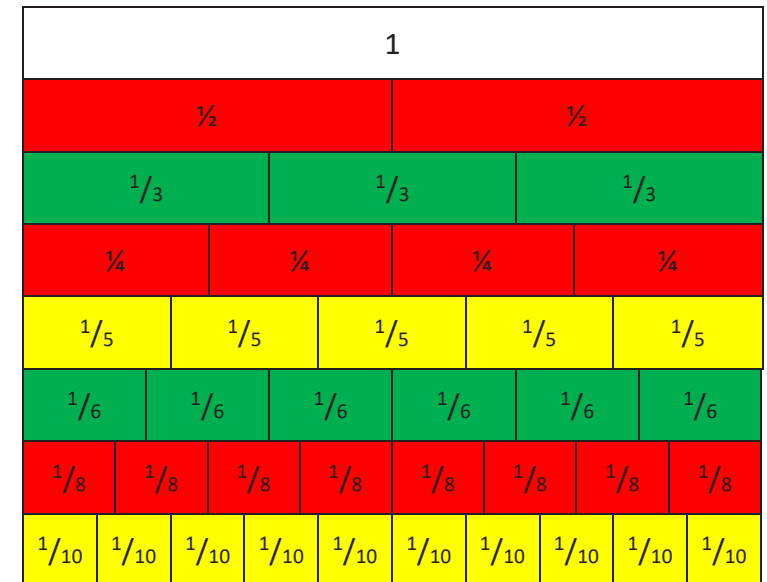
Geoboards
Typically plastic or wooden boards with raised pins or points in a square matrix formation around which elastic bands can be looped to create 2-dimensional shapes of various shapes and sizes.

Maths mat
Simply **a sheet of A4 or A3 paper**, on which children arrange counters or other concrete apparatus they actually use in a calculation. This helps children to identify the counters which form the calculation, separately from the 'spare'/unused counters on the table.
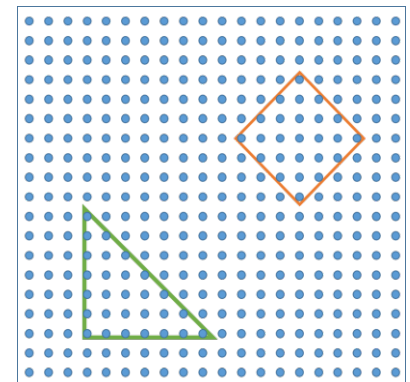
Money

Plenty of coins of all denominations. It is more realistic and better for learning about money, if these can be real coins. However, plastic money will suffice if it is a reasonable representation of actual coins in size and appearance. It is also surprising how easy it is to come across out-of-date coins in sets when the Royal Mint has changed them.
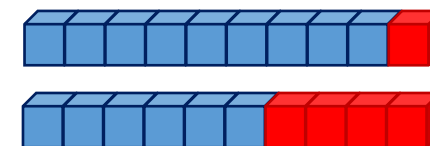
Geoboard

Multilink ©

Plastic, interlocking cubes (edges about 1.8 cm in length) of different colours, which can be connected to one another in different ways to construct an endless variety of solid 3D creations. The name 'multilink' is the trademark of a particular manufacturer, and other variants of interconnecting cubes are available, though we have found *multilink* to be the most reliable.

Number lines

In the early years and Y1 it is helpful to have a number line which is scaled to fit the multilink pieces, so children learn to match the edge of a piece of multilink with a complete whole number:

0 1 2 3 4 5 6 7 8 9 10 11 12 …
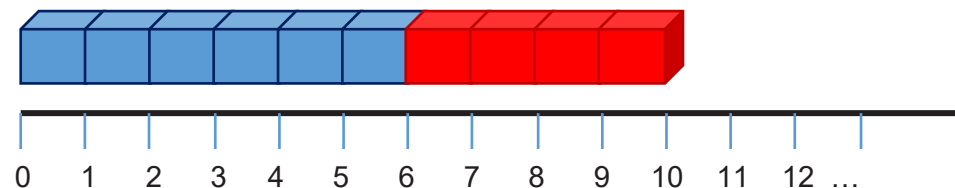
After that in KS1 it is helpful to have wipe-clean laminated number lines up to 20 or 30 which children can use individually at their desks, and a longer number line from 0 to 100 along a wall at a comfortable height for a child to point and touch individual numbers. It is important also to display vertically arranged number lines, and certainly from KS2 if not earlier, to show negative numbers to -10 on number lines.

Numicon templates ©
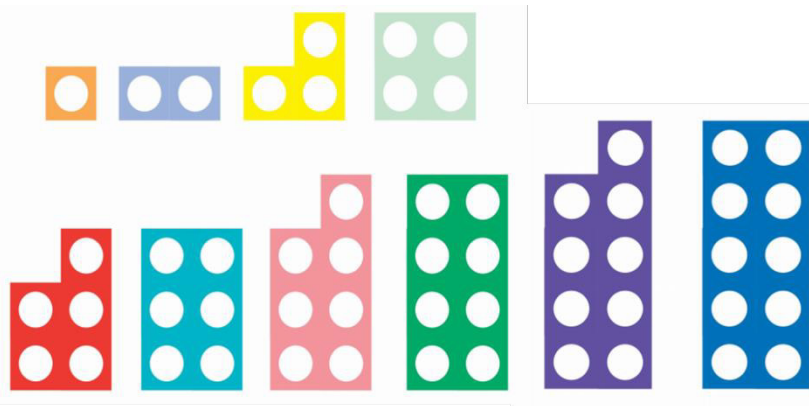
These are another commercially produced resource for exploring number and calculation. We have not seen an alternative provider of a similar resource of the same quality. It is especially helpful in reinforcing odd/even numbers, and for overlaying numbers upon others, for example when exploring multiplication and division as repeated groups.

Paper/plastic cups

When demonstrating/modelling the use of counters in calculations to the whole class, it is helpful for the class teacher to use a larger and more visible resource to represent the counters, such as paper cups.

Paper screens

This is a simple and useful device for enabling a pair of children to hide something from one another for the purposes of a game. It can be made simply from a sheet of A3 (or larger) sugar paper with a series of vertical 'concertina' folds and one small 'turn-up' folded horizontally along the bottom. For example:
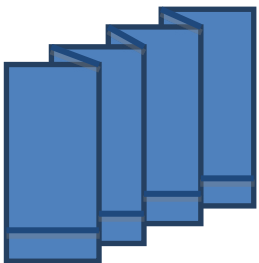
Place value base-10 blocks or Dienes' apparatus

Hundreds	Tens	Ones
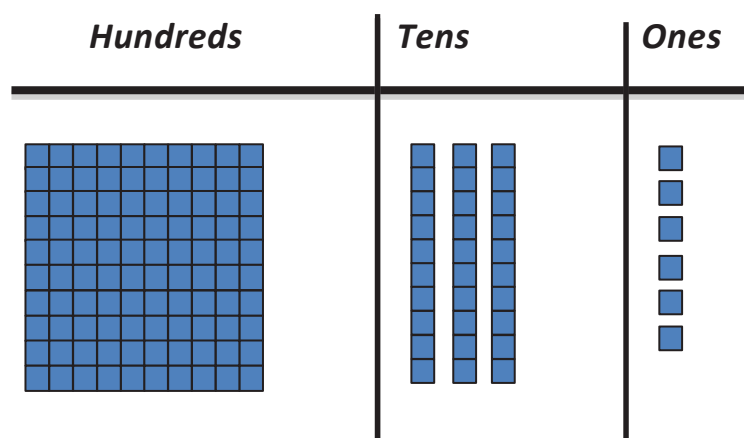
These are described in Chapter 6, 'Numbers and Place Value'. Invented by Zoltan Dienes, base-10 apparatus is a set of scaled manipulative blocks, in 1s, 10s, 100s and 1000s which are used to make scaled concrete representations of place value in number, e.g. the number 136 is represented concretely by the arrangement pictured left.
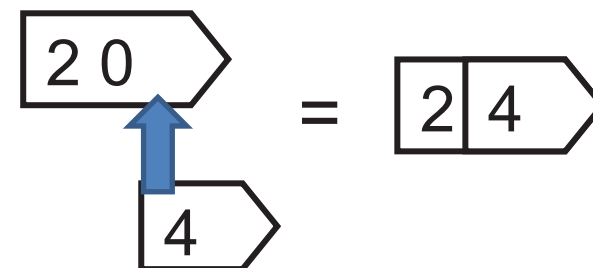
Place value (p.v.) mat

When using base-10 apparatus, it is helpful for the child to have a simple A4 or A3 mat upon which Hundred, Tens and Ones columns have been created in which to place the different denominations of the base-10 apparatus.

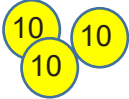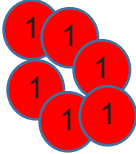A free photocopiable resource for this is supplied.

Place value (p.v. or 'arrow') cards

These are described in Chapter 6. Place value cards are often called 'arrow' cards because of their shape, and are number cards which can be assembled to represent a number so that the number can also be partitioned into hundreds, tens and ones, to show the actual place value of each digit e.g. for 20 + 4 = 24

There are many variations of this resource freely available on the internet, and sets which can be purchased ready-made for schools to provide for pupils.

$$20 \quad 4 \quad = \quad 2\,4$$

Place value (p.v.) counters

Hundreds	Tens	Ones

These are an abstraction from base-10 apparatus, so that equal-sized labelled counters are used to represent place value on a place value mat, rather than having items which are scaled in size. The counters usually represent each place value with a different colour, as for the number 136 in the example here. The significant learning developments are that the value of an item is not proportional to its size compared with other items and that one counter *unitises* or has a *1:many* relationship with other counters which may be of equivalent size or even larger. A similar experience which children will encounter before using this resource is when exchanging coins of different values.

Playing cards
Playing cards are a very cheap, reusable and shared resource, so it is not expensive to buy sufficient packs to equip a class. Even quite young children enjoy learning to shuffle cards properly, but as long as they can make sure the cards are 'mixed up' to some extent, that is sufficient!

Polydron frameworks ©

We are not paid to advertise *Polydron,* and you may find a cheaper alternative, but this really is the most effective resource we have come across for helping children to easily construct their own 3-dimensional shapes from pre-formed 2-dimensional shapes that click together. The skeletal faces in the *Frameworks* also have the benefit of being able to open into one flat open net, allowing children to draw around the inside of each face to create an approximate representation of the whole net. For example, a net for a triangular prism could look like any of the 'opened' shapes below: