PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN KS1

NATURE OF THE ACTIVITIES SUGGESTED HERE

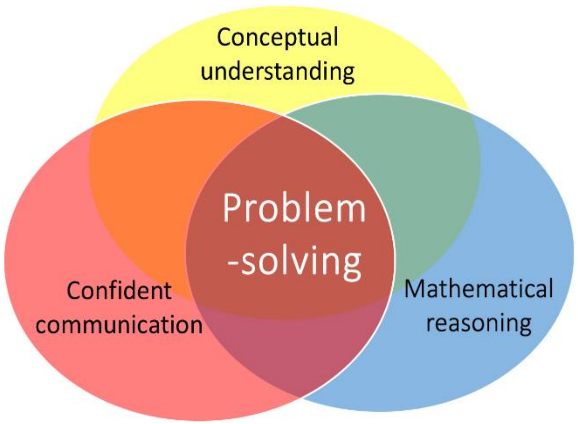
With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, which curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

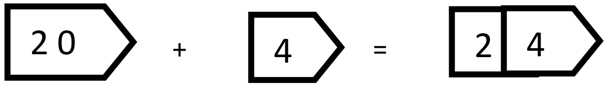
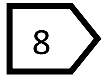
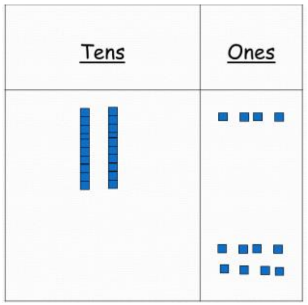
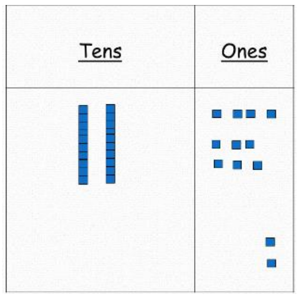
All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

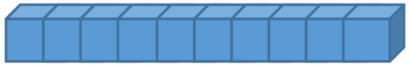
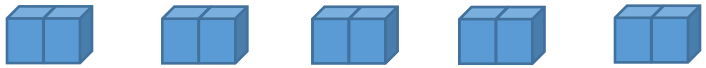
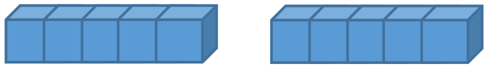
Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

9. Written Methods for Addition and Subtraction **Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.** Rather than introducing formal vertical written methods in KS1, we advise that in working towards these, activities for years 1–2 concentrate on practising skills and understanding in the partitioning of numbers which are crucial for developing the formal vertical written methods in KS2.	**Finding friendly pairs** The teacher should demonstrate with some examples, then in pairs, children separate place value cards into two shuffled piles: *tens* and *ones* placed face down on the table. Emily takes one place value card from the top of each pile to make a two-digit number, for example  Luke takes the next p.v. card from the top of the pile of ones cards, for example,  Both children write down the addition sentence for these numbers: ***24 + 8 =*** On a place-value mat, using base-10 apparatus Emily sets out 24 as ***20 + 4*** and Luke sets out 8, below left (see photocopiable resources):  They must look for a way of using all or part of the ***8*** to add easily to all or part of the 24. For example, Emily decides to add 6 to the 4 to make 10, So she partitions the 8 and rearranges the ones as on the right:  She can then rewrite the addition as: ***20 + 4 + 6 + 2*** and regroup it as ***20 + 10 + 2***	Do the children see how they can use knowledge of number bonds to 10 to see how to look for complements to make a 10? In time, children should make the connection to extend the 10-complement facts to make the next multiple of 10 for any addition. Children can explain that the actual value of, say, three *tens (longs)* is 30 Children should see that visualising the expression of their calculation as ***20 + 2 + 10*** is completely valid, and that we go on to group the *tens* and the *ones* appropriately. When confident, children can set out and add a pair of two-digit numbers for sums less than 100.

10. Multiplication and Division Structures **Use the inverse-of-multiplication (repeated subtraction) structure to understand division.** One problem in developing children's understanding of division is that concrete experiences in the lower primary years can overstate the equal-sharing structure. The equal-sharing structure is the basis of fractions, written methods of division, swap between equal-sharing and the inverse-of-multiplication (equal grouping or repeated subtraction). This activity is intended to develop children's understanding of the latter.	**How many groups?** Children take a given number of multilink and make a joined row of single cubes. They then see how many equal groups of the same length which they can break the row into without any left over. They write the division sentences for each one. For example, starting with a row 10 cubes long:  Emily tells Luke they can make groups of 2, and demonstrates this:  They both write the number sentence: *10 ÷ 2 = 5* Luke puts the row back together and attempts to break it into groups of 3, but finds he has 1 cube left over. Emily suggests groups of 4, but they find there are 2 cubes left. They find groups of 5:  And write the next number sentence: *10 ÷ 5 = 2* They then do the same for other numbers, such as 9, 12, 16, 11 ... The activity can be simplified or extended by changing the numbers set for the rows the children are to make.	When dividing the tower, every group has to be the same size, with no cubes left over. Emphasising the language '***equal groups of***' and that the children are ***subtracting*** the ***same*** group ***many*** times. Children may still confuse which value represents the ***group size or number in each group***, with the value which represents the ***number of groups*** when writing a number sentence. Children should begin to see that some numbers (primes) can only be one single group or grouped in *ones.*

14. Integers: Positive and Negative **Understand ordinal use of number can be extended to negative numbers.** **Read and interpret temperature scales.** Temperature is a real-life context in which we can introduce KS1 children to positive and negative whole numbers in a meaningful way.	**Temperature check** An on-going activity, a regular observation to carry out with the whole class. You will need: • Large classroom number line: if possible, a vertical line with a range of at least –10 to (+)30; • Two large, easy-to-see, air-temperature thermometers. Rather than one in-depth specific lesson, it is helpful to introduce children as early as possible to the everyday use of a thermometer to measure temperature. In the UK, temperatures rarely exceed a range of between -10^{o}C to (+)30^{o}C, so it is a practical exercise to record the inside and outside temperatures each day as a real-life exercise in mathematical/scientific data-gathering. It helps to hang two separate thermometers that can be briefly compared side by side, but do not risk the outside thermometer changing its value if it is very sensitive, so it may be that you read it *in situ* with different volunteers each day. Use the inside thermometer to help the whole class learn to count along the unmarked divisions to the temperature indicated. Use the large classroom number line to mark the inside and outside temperatures, and then together find the difference (*comparison* structure of subtraction) between them. Record publicly the two temperatures and the difference between the temperature inside and outside. Make comparisons between different days of the week. Use the number line to illustrate differences in an informal, visual way. On very cold days of course, we can naturally introduce even young children to the idea of negative numbers, and they may hear this on TV and radio (although the word 'minus' is used). It is also a very natural way to help them see that you can find the difference between a positive and a negative number, by seeing the visual space between them on the number line. This is very helpful to see the difference between, say, +1^{o}C and -1^{o}C!	It does not matter at this stage that children may not know what is actually meant by degrees, just that they see that we measure temperature in degrees, just like we measure money in pence, and length in metres. Do the children realist that in this context 0^{o}C does not represent the *absence* of temperature (or heat)? Do they see that 0 is simply a point in ***ordering*** the values of temperature, from which we count numbers positively in one direction (getting warmer) and negatively in the other (getting colder)? Can the children see the ***difference*** between two temperatures as the numerical 'space' between them on the number line, regardless of whether the temperatures are positive or negative values?

	• Repeat the demonstration with the bag of 8 sweets. This time ***one half*** is … Explain that between them *Teddy* and *Rover* actually have '***two halves***' and in each example ask the children 'How many sweets in ***two halves***?' • Now Emily and Luke ***share*** the bag of 12 sweets ***equally*** between *Teddy* and *Rover*. They need to agree what is ***one half*** of 12. • Repeat this with the bag of 16 sweets. **Quarters: Sharing equally between four toys:** • Now explain that *Froggy* and *Rabbit* have also come to play with *Teddy* and *Rover.* How many will need to share the sweets altogether now? If necessary, demonstrate sharing equally between 4, to find ***one quarter***, using $^1/_4$ of 4 and then $^1/_4$ of 8, before asking them to share the 12 and the 16.	

20. Coordinates and Linear Relationships

Understand the use of simple row and column labelling to identify position in 2 dimensions.

In this activity, children explore position, identifying a row and column reference to locate a square on the paper. While this version does not use a frame of true co-ordinate references (which locate points, rather than whole squares), it is a helpful way for younger children to identify position in 2 dimensions. Row and column references are also used on some maps, for example, *A-Z street maps*.

<u>Simple battleships</u> Children play in pairs. Each pair will need:

- Sheets of squared paper, or prepared grids – one each (there is a photocopiable resource available);
- A paper screen to enable them to hide their grids from one another (see below how to make this).

The activity here is the traditional paper game of *battleships*.

Ships are set out as crosses in a line of whole squares, so there is no debate about whether a targeted square is part of a battleship, e.g.:

10										
9							X			
8	X	X	X	X		X				
7					X				X	
6				X			X		X	
5							X		X	
4							X		X	
3		X	X	X	X		X			
2										
1						X	X	X	X	
	A	B	C	D	E	F	G	H	I	J

First of all, give out the prepared grids, or show the class how to mark out a large square grid on their squared paper, comprising 10 rows and 10 columns. Label the squares along the ***horizontal*** base with the letters A–J, and the squares up the left ***vertical*** edge with the numbers 1–10.

Show how to locate any single square by identifying the combination of its row and column, for example, D6, F9, and practise this with the children. Emphasise that we give the horizontal location first when describing any row and column pair (some remember this as '*along* the hall ***then*** *up* the stairs'). This will be an important order to retain when making the transition to point co-ordinates later.

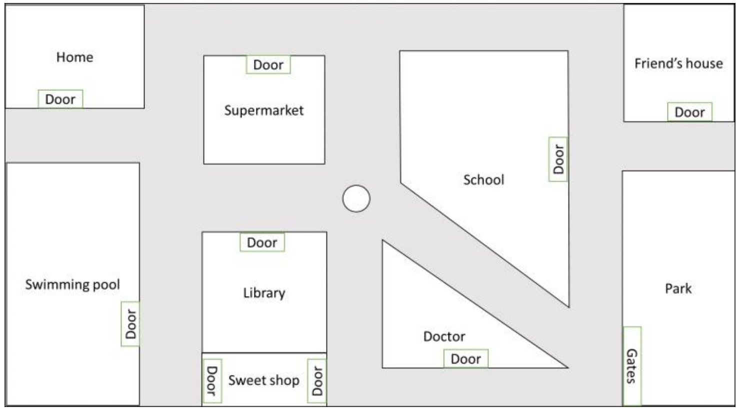
Now explain that this grid is an area of the ocean, where battleships are out 'on manoeuvres'. Show how children may place their battleships. Each battleship is represented by a single line of

Do the children understand the terms ***horizontal*** and ***vertical?***

Do the children identify an intended square correctly?

Do the children recognise the relationship of the identification for each square with its neighbouring squares?

Do they associate the *rightward* direction in labelling *alphabetically* along the ***horizontal*** axis, and the *upward* direction for labelling *numerically* along the ***vertical*** axis?

	One group can use the controllable toy to move it along the roads to different places on the village floor plan. When using the controllable toy, the simpler the device, the better. Many devices remember and accumulate a sequence of instructions, which can be confusing at this stage, so it is important to *clear all* previous instructions before each new instruction is given.  	

26. Handling Data **To use a *tally* chart to organise data received in an unsorted order.** The purpose of a tally chart is to sort data that is being received in an unpredicted order, so that it can be easily organised during the collecting phase. To demonstrate the tally chart's effectiveness to children, the data they collect needs to be arriving in this way too. Incorrect use of a tally chart is a common problem observed among trainee teachers, and sadly this is sometimes due to bad guidance in published schemes of work. I have seen children being introduced to tallies by counting the number of their peers, say, who walk to school, by a show of hands, which can be totalled at the same time, rendering the need for a tally chart redundant.	**Traffic survey** In small groups with an adult or as a whole class with additional adults to supervise as necessary. Each group will need: • Prepared charts for recording the tallies (see photocopiable resources); • Clipboards and spare pencils. An interesting example of the need for a ***tally chart*** is to carry out a traffic survey. Of course, this needs to be properly supervised if the children need to leave the school premises to carry it out; it may be most effective for a teaching assistant to take a small group at a time, with comparisons made of traffic at different times during the day; alternatively, the teacher could take the whole class with additional adult helpers to supervise groups at a safe place from which to observe passing traffic. In many schools an actively used road can be observed from within the school grounds. Set up the initial premise that the class has been asked to survey traffic to gather information for changes to routing or other forms of traffic management, such as where traffic lights and crossings need to be placed. Ask the children to discuss between them and identify the different categories of vehicles it may be helpful to count: *car, lorry, bus/coach, motorcycle, bicycle, van, tractor and 'other'* (just in case). Children write these categories in their chart. From a safe vantage point, children watch and tally observations into the different categories, for a fixed period, for example, 10 minutes.	Do the children realise they must make one stroke on the *correct* row of the tally chart for every vehicle they see? Do the children know how to make the fifth stroke on each tally as the 'bar' across the four previous strokes? Do they see how they can completed tallies help us to count up each category more easily in groups of 10s and 5s? Do the children realise there is no point in creating a tally chart if we can easily find out the total of each category without one?

Therefore, to show the use of a tally for ‘journeys to school’, ensure that the children ask their peers individually one at a time, so that the data arrives in an unpredictable order.

Upon their return, they count up their tallies and record the totals for each category, e.g.

Vehicle	Tally	Total
car	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	33
lorry	𝍸 𝍸 ||	12
bus/coach	|||	3
van	𝍸 ||||	9
tractor	|	1
motorcycle	𝍸 |	6
bicycle	||||	4
other	|	1

There needs to be a meaningful discussion of their findings, as if preparing for a council transport meeting. Use the opportunity to teach/reinforce traffic awareness and road safety. Discuss the different vehicles:

- How many two-wheeled vehicles? (motorcycles + bicycles)
- How many were large vehicles? (lorries + buses)
- How many were small vehicles? (cars + vans)
- Why might there be more of some types than others?
- Were there any they did not see? Why?

28. Probability

To experience the equal probability of events involving symmetrical outcomes.

Although *Probability* does not presently appear in the English Primary curriculum, it is a worthwhile and interesting area of mathematics for children, and it is included here as it is still featured in other international curricula.

Probability is one of the most important areas for using and applying mathematics in real life. It is a key part of our mathematical skills to respond to life's chance events with intelligent strategies based on our understanding of probability.

(To use a *tally* chart to organise data received in an unsorted order – c.f. activity for Chapter 26.)

Snakes and Ladders Children work in small groups of 3 or 4. They will need:

- A single prepared chart for recording the tallies (See photocopiable resources);
- *Snakes and ladders* game board, a single die and a different coloured counter for each player.

Simple *chance* games with playing cards such as *Snap* and *Beat your neighbour out of doors* and simple dice games such as *Snakes and Ladders* help children begin to experience chance from an early age, though unfortunately, often acquiring some wrongly established intuitive ideas along the way! Often children's *subjective* experience of throwing a die – when they require a *six* in order to start – is that it is harder to get a *six* than any other number! The problem is, of course, the heightened emotional attachment the child has to the outcome, hence previous unpleasant experience is more likely to be recalled. This activity is to help to dispel such false notions.

Emily, Luke, Kasia and Nathan play a game of *Snakes and ladders* with a difference. At every throw they record in the *same* tally chart the number shown on the die, thus using their game to collect data from their experiences of throwing the die. For example:

Number	Tally	Total
1	卌 卌 卌 ||||	19
2	卌 卌 卌 |	16
3	卌 卌 ||||	14
4	卌 ||||	9
5	卌 卌 |||	13
6	卌 卌 卌 |	16

After different groups of children have played and collected a reasonable number of throws, collect and display the accumulated totals from all the groups for each number, and discuss the data with the children. It should become clear that in practice the number of throws will not be the same for each number, but that *six* is not a particularly disadvantaged number!

Do the children see that the frequency of each number being rolled is reasonably similar across the numbers? For any particular number that may have occurred many fewer times, ensure that the children see data from another group where this was not the case.

See also the potential crucial points and barriers to understanding for tallying (Y1–2 activity for Chapter 26).

Common equipment recommended

Notes about some of the common equipment recommended for use with the activities.

100 square

It is helpful to have available class sets of paper 100 squares of size about the width of a piece of A5. Two sets are useful in different arrangements: 1–100 and 0–99.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

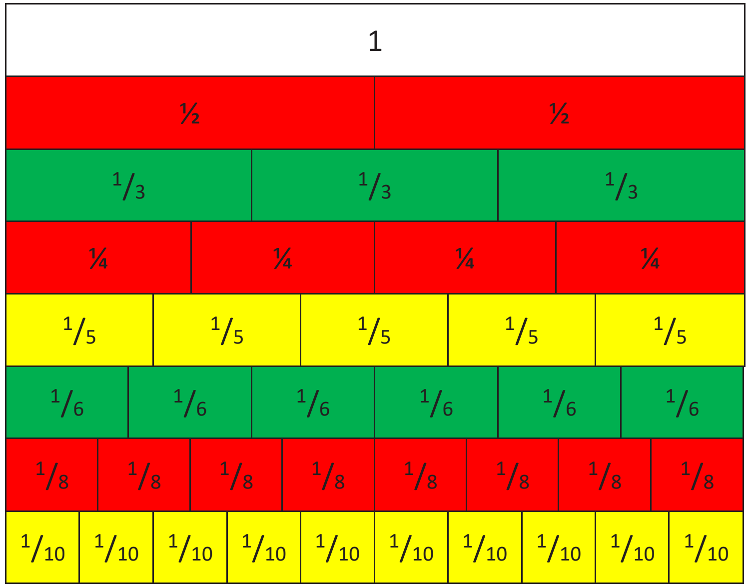
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counters

A large box of 500–1000 counters in many colours, of about 1.2cm diameter.

Cuisenaire Rods

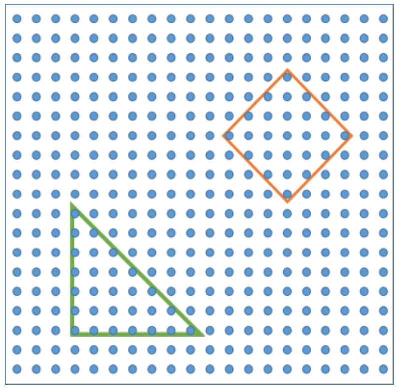
These coloured rods of different lengths are great for helping children to develop a practical sense of ratio and of algebra from an early age. The rods were invented by a Belgian primary school teacher, Georges Cuisenaire. The right to use the name ‘Cuisenaire’ is now owned by a specific company, but other manufacturers supply this resource by different descriptions.



Fraction Wall

A rectangular tray containing plastic or wooden strips, each strip sub-divided into equal fractions of a different denominator, enabling the pieces to be moved and recombined in different ways to make 1. It is a physical model of the fraction chart pictured here.

Children can be encouraged to create their own Fraction Wall with identical paper strips which they fold into $^1/_2$, $^1/_3$, $^1/_4$, $^1/_5$, $^1/_6$, and $^1/_8$ and paste to a backing sheets.



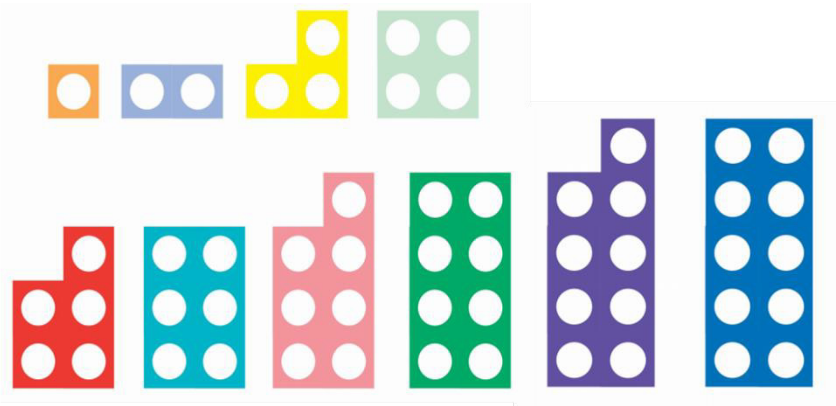
Geoboards

Typically plastic or wooden boards with raised pins or points in a square matrix formation around which elastic bands can be looped to create 2-dimensional shapes of various shapes and sizes.

Maths mat

Simply ***a sheet of A4 or A3 paper***, on which children arrange counters or other concrete apparatus they actually use in a calculation. This helps children to identify the counters which form the calculation, separately from the 'spare'/unused counters on the table.

Numicon templates ©



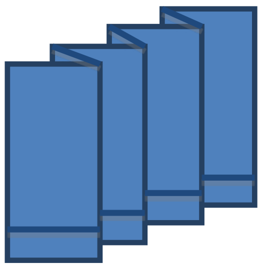
These are another commercially produced resource for exploring number and calculation. We have not seen an alternative provider of a similar resource of the same quality. It is especially helpful in reinforcing odd/even numbers, and for overlaying numbers upon others, for example when exploring multiplication and division as repeated groups.

Paper/plastic cups

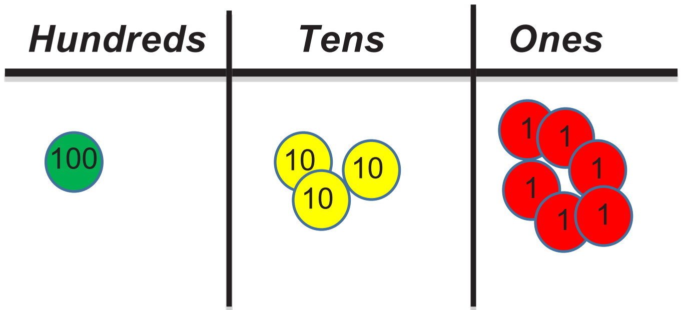
When demonstrating/modelling the use of counters in calculations to the whole class, it is helpful for the class teacher to use a larger and more visible resource to represent the counters, such as paper cups.

Paper screens

This is a simple and useful device for enabling a pair of children to hide something from one another for the purposes of a game. It can be made simply from a sheet of A3 (or larger) sugar paper with a series of vertical 'concertina' folds and one small 'turn-up' folded horizontally along the bottom. For example:



Place value (p.v.) counters



These are an abstraction from base-10 apparatus, so that equal-sized labelled counters are used to represent place value on a place value mat, rather than having items which are scaled in size. The counters usually represent each place value with a different colour, as for the number 136 in the example here. The significant learning developments are that the value of an item is not proportional to its size compared with other items and that one counter ***unitises*** or has a ***1:many*** relationship with other counters which may be of equivalent size or even larger. A similar experience which children will encounter before using this resource is when exchanging coins of different values.

Playing cards

Playing cards are a very cheap, reusable and shared resource, so it is not expensive to buy sufficient packs to equip a class. Even quite young children enjoy learning to shuffle cards properly, but as long as they can make sure the cards are 'mixed up' to some extent, that is sufficient!