PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UKS2 5-6

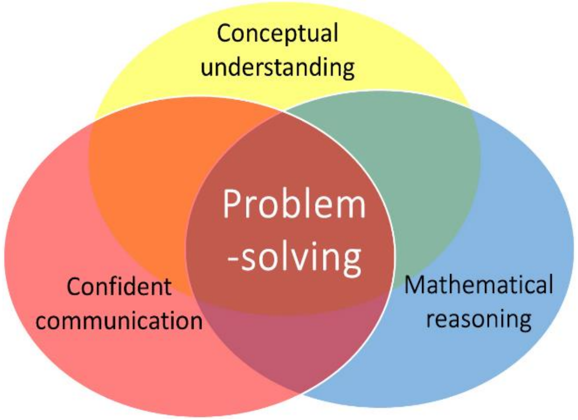
NATURE OF THE ACTIVITIES SUGGESTED HERE

With the surge of interest and sometimes confused interpretations of what is meant by ***Mastery*** in mathematics, a number of different claims have been made about what it means and what is required. The efficacy of different approaches to implementing a Mastery approach to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in the Far East, as measured by PISA* and TMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, that curriculum developers and educationalists in the Far East have used, have been known for many years and including Bloom's* theories of *Mastery,* the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*, and the seminal *Cockcroft Report*,* particularly, its emphasis on the importance of *practical experiences* and *problem-solving*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of Mastery in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. These aims are also highlighted in the 2014 National Curriculum Aims*. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***



Hence, the activities suggested here are designed to promote the following:

- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

NATURE OF THE ACTIVITIES SUGGESTED HERE

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

In some of the activities, it could be argued that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Cockcroft, W. H. (1982) *Mathematics Counts,* London: HMSO.

DfE (2013) 'Mathematics', in *National Curriculum in England: Primary Curriculum*, DFE-00178-2013, London: DfE.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

NATURE OF THE ACTIVITIES SUGGESTED HERE

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

There is also a file of resource sheets used in some of the activities, which may be reproduced freely. However, please include any source information on each copy.

Related chapter, key learning & rationale	Plan for teaching and learning	Crucial points & barriers to understanding
6. Numbers and Place Value **Read, write order and compare numbers up to 10,000,000 and determine the value of each digit.** **Identify the value of each digit in numbers given to three decimal places.** In the game a player must subtract the exact place value of a single digit within the number, thus reinforcing their knowledge of how to express the actual value of a specific digit from its position in a number.	<u>**Place invaders**</u> Demonstrate using an on-screen calculator, then children play against one another in pairs. They will need: • Simple (non-scientific) calculator for each pair; • Counters for scoring. *Space Invaders* was an arcade video game from the 1970s. The aim was to shoot down as many alien spacecraft as possible. Versions of this game are available on the internet today. *Place invaders* is based on this idea, to 'shoot' specified digits in a number displayed on a calculator, by making them zero (or blank for the most and least significant digits in the number) whilst leaving the other digits unchanged. Here is an example: • Meena clears the calculator and enters a number, say, with up to 6 integer digits and 2 decimal digits, for example 763981.72; • Charlie selects a digit for Meena to shoot, e.g.: 'Take out the 3, Meena'; • Meena has to enter the correct subtraction into the calculator to do this. She enters: '–' '3000' '='; • They both check that the calculator display has been correctly changed to 760981.72, and Meena gets 1 point; • Meena selects a target digit: 'Shoot the 7 on the right, Charlie' • Charlie has to enter the correct subtraction. He enters: '–' '0.7' '=' • The calculator now shows 760981.02 and Charlie gets 1 point; • They continue alternating turns until the display is 0. If the display is incorrect after a subtraction, the player does not get a point, but they can continue to work with the changed number. If needed, simplify the game by reducing the number of digits, and/or removing decimal places, or extend it further with more digits in the integer and/or decimal parts.	Does the child correctly recognise the actually place value of a specific digit from its position within the number? It is important that the children have already had sufficient teaching and experience in how to manipulate and interpret the calculator. Can the child use the calculator correctly to enter the number and press the keys in the correct order to carry out the subtraction? Can the children be further challenged, by giving an additional point to the target-setter if they can describe the target in terms of its actual place value? For example: 'Take out the three thousand, Meena', or 'Shoot the seven tenths, Charlie.'

7. Addition and Subtraction Structures **Recognition of a variety of addition and subtraction structures, using decimal numbers, by connecting language and symbols.** At younger ages children are asked to make up stories to fit a given calculation. This extends that idea but requires more thought to make sense of the numbers being used.	**<u>Tricky story-writing</u>** The teacher gives the children an example of a 'story' for '17 + 16': for example, 'There were 17 boys on the bus and 16 girls; how many children were there?' The children are then challenged to make up stories for, say, both '7.28 + 2.75' and '104.75 - 98.50' using: a) shopping as a context; b) measuring length as a context; c) the word 'increased'; d) the word 'decreased'; e) the word 'more'; f) the word 'less'; g) the word 'added'; h) the word 'difference'. Use the responses to (a) and (b) first to discuss with the class a variety of stories and language that use different structures.	Can the children use the words 'increased', 'more' and 'added' in a subtraction story? It is a challenge! For example: 'The shop increased the digital radio from £98.50 to £104.75. By how much was it increased?' corresponds to the subtraction '104.75 - 98.50'. Likewise can children use the words 'decreased', 'less' and 'difference' in an addition story? For example: 'Meena's garden is 7.28 metres long. Charlie's is longer. The difference between the lengths of their gardens is 2.75 metres. How long is Charlie's garden?' corresponds to the addition '7.28 + 2.75'.

8. Mental Strategies for Addition and Subtraction

Add and subtract numbers mentally with increasingly large numbers.

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Solve addition and subtraction multi-step problems in context.

The aim here is to get the children practise mental and informal strategies for addition and subtraction.

In effect, the children will have to do several subtractions to solve the square, then check every total by additions of every row, column and diagonal.

<u>**Magic squares**</u> The children will need a magic square with some numbers missing.
A magic square is one where the sums of the numbers in every row, column and diagonal are all the same. The easiest way to construct one is start with a very simple square, and then multiply the number in each cell by the same number; or add the same number to each square. For example, starting with a simple magic square, where each row, column and diagonal sum to 15:

2	7	6
9	5	1
4	3	8

But a more complicated magic square is produced by multiplying each number by 33:

66	231	198
297	165	33
132	99	264

Now the sum of each row, column and diagonal is 33 × 15 = 495.
Provide the children with a square where some cells have been removed and tell them children that each row, column and diagonal adds up to 495. Tell them we need all the numbers to complete the missing cells.
You can dress this as a story that the completed square provides the combination of numbers for the lock to a pirate's treasure chest, the vault of the Bank of England, or the staff room biscuit tin and you need their help to unlock it!

Do the children realise that they can calculate the value of any cell in a row, column or diagonal for which they already know two other cells?

Do they recognise they need to add two numbers, then carry out a subtraction, to find a missing cell?

Do children add and subtract values correctly?

Do they check their calculations by adding up in other directions to confirm the total is the same for every row, column and diagonal?

<table>
<tr>
<td>It could also be used for practising expanded written or formal methods too, if desired.</td>
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<table>
<tr><td>66</td><td></td><td>198</td></tr>
<tr><td></td><td>165</td><td></td></tr>
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</table>
Children compete in pairs and compare solutions. Encourage children to compare and check each other’s mental and informal strategies.
The activity can be simplified by using a smaller multiplier, or including more numbers in the cells.
Extend the challenge by using a 4 × 4 square, for example:
<table>
<tr><td>11</td><td>14</td><td>2</td><td>7</td></tr>
<tr><td>4</td><td>5</td><td>9</td><td>16</td></tr>
<tr><td>13</td><td>12</td><td>8</td><td>1</td></tr>
<tr><td>6</td><td>3</td><td>15</td><td>10</td></tr>
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<td></td>
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9. Written Methods for Addition and Subtraction **Add and subtract numbers mentally with increasingly large numbers, including decimals.** **Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.** **Solve addition and subtraction multi-step problems in context.** The aim here is to provide a real-life application of addition and subtraction calculations to money, in order to extend to more than two numbers when adding, and to cater for two decimal places (in the context of money notation only at this stage).	**<u>The book token</u>:** The teacher will need a number of books labelled with different prices between £5.00 and £20.00. For example: £5.57, £7.64, £8.31, £12.15, £13.86, £18.72, £21.42. Each child has a book token. They want to get as much value from their token as possible, because the shop does not give change or new tokens for unspent credit on tokens, and the children have no money of their own to add to the token. The aim is find the maximum amount they can spend (additions) and the unused value of their book token when they have done this (subtraction). The teacher should first demonstrate an example for a book token worth £20: i.e. they could buy the one book for £18.72, the two books at £5.57 + £13.86 = £19.43, the two books at £7.64 + £12.15 = £19.79. Are there any other possibilities? Which decision uses the maximum value from the book token? How much of the book token is not spent in each case? In pairs, the children investigate different ways of spending a £25 book token. For example: Meena selects two books costing £13.86 and £8.31, which she first estimates as £14 + £8 = £22, then calculates this to be £22.17. Charlie selects three books costing £12.15, £7.64 and £5.57, which he first estimates as £12 + £8 + £6 = £26, so he changes his mind. He selects books costing £8.31, £7.64 and £5.57. He estimates these as £8 + £8 + £6 = £22, and then calculates the actual total cost to be £21.52. Charlie and Meena swap results and check each other's calculation. They agree that Meena has got better value than Charlie from her book token. They then try again to see if they could get better value than £22.17. Investigate other values for the book token, e.g. £30, … £50. Alternatively simplify if necessary by using smaller amounts. For example, which items priced differently from 50p to 99p can I buy with a token for £5?	When using any formal or expanded written methods do children correctly align decimal points? If adjusting the calculation to whole numbers (pence) by scaling x100, do they re-scale the intermediate result correctly? Where partial calculations are able to be done mentally, this should be encouraged, so that children use the best/most appropriate strategies wherever possible. Can children make a reasonable estimate of the calculation before they attempt each addition or subtraction? Can children use the inverse operation to check their results?

10. Multiplication and Division Structures

Use the inverse-of-multiplication structure with ad hoc strategies to solve a division.

The aim here is to reinforce the children's understanding of the relationship between multiplication and division, showing how they can use multiplication to carry out a division. They also develop strategies to find unknown multiplication facts from those facts that they have already established.

Division by multiplying! The children will need to have had experience of finding some multiplications they don't know using number facts they do know in ad hoc ways.
For example, from 6 × 3 = 18, they can work out:
6 × 6 = 36 (by doubling 6 × 3);
6 × 30 = 180 (by multiplying 6 × 3 by 10);
6 × 60 = 360 (by multiplying 6 × 6 by 10), and so on.

Now for the problem: the children have been asked by the car manufacturer 'Nissota' to work out the sizes of petrol tanks needed for different models of its new cars. Nissota wants each car to be able to drive at least 500 miles between visits to filling stations. The company needs to know the minimum size each car's fuel tank must be in litres. Here is a table of the number of miles per litre of petrol each new model can drive:

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Miles/litre	6	8	11	12	14

(See photocopiable resource.)

The teacher models an example with the children: Rival car manufacturer 'Kionda' has a car which goes 7 miles on 1 litre of petrol. To find how much petrol is needed to drive at least 500 miles, we will build up gradually to that figure. This can be done in a number of ways, depending on the child's confidence with mental multiplication by 7. For example:
10 litres will cover 7 × 10 = 70 miles;
so 20 litres will cover 7 × 20 = 140 miles (doubling the result for 10 litres);
and 40 litres will cover 7 × 40 = 280 miles (doubling the result for 20 litres);
but 80 litres would cover 7 x 80 = 560 miles (doubling again, but this is too large);
so 60 litres will cover 140 + 280 = 420 miles (adding results for 20 litres and 40 litres);
so 70 litres will cover 420 + 70 = 490 miles (adding results for 60 litres and 10 litres);
and 2 litres will cover 7 × 2 = 14 miles.

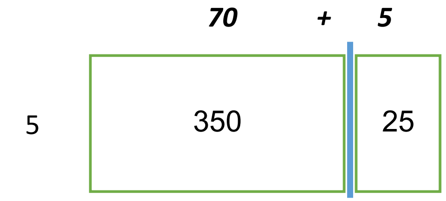
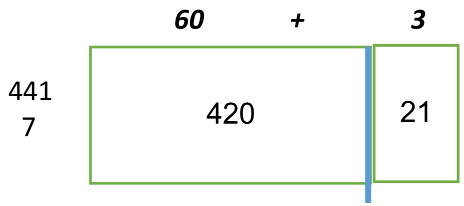
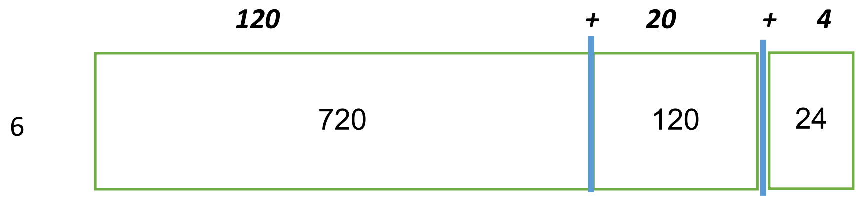
Are the children confident in adapting and developing new calculations from those they already know?

Can they do this methodically to develop a progressive sequence of facts?

With successive calculations, do the children use what they already know to find the largest multiples as soon as possible wherever they can? This will help when developing vertical written methods of division too.

Children need to be confident in scaling multiplication facts by 10 and 100.

	So if we add the results for 70 litres and 2 litres we pass the 500 miles target: 390 + 14 = 504 miles. So to travel at least 500 miles the Kionda car's petrol tank needs to hold at least 70 + 2 = 72 litres. Meena and Charlie now use the same approach each work out the minimum size of each Nissota model's fuel tank in litres. They compare their strategies and check each other's calculation. To simplify this use only single-digit mileage/litre figures or reduce the minimum driving distance to, say, 200 miles. To make it more of a challenge, add some more economical models, for example with fuel consumption at 19 or 23 miles/litre or extend the driving range to, say, 550 miles or 720 miles.	

11. Mental Strategies for Multiplication and Division **Divide numbers of up to 4 digits by 1 digit, using mental strategies.** The ability to partition numbers is key to the distributive law, which is the underlying strategy for the written methods of division (long and short). This activity is for children to practise solving divisions by 1-digit numbers using mental strategies based upon ad hoc partitioning of the dividend. This reinforces the inverse relationship with multiplication	<u>**Division by partitioning**</u> The teacher models some examples, asking how a number, the dividend could be partitioned (broken up) to make it easier to divide? For example:  375 ÷ 5 = (350 + 25) ÷ 5 (chosen because we know 35 ÷ 5 = 7) = (350 ÷ 5) + (25 ÷ 5) = 70 + 5 = 75  ÷ 7 = (420 + 21) ÷ 7 (chosen because we know 42 ÷ 7 = 6) = (420 ÷ 7) + (21 ÷ 7) = 60 + 3 = 63  864 ÷ 6 = (720 + 120 + 24) ÷ 6 (chosen because we know 72 ÷ 6 = 12) = (720 ÷ 6) + (120 ÷ 6) + (24÷ 6) = 120 + 20 + 4 = 144	Can children adapt the strategy of using the distributive law to division, and see how each part expresses a subset of repeated subtractions of the divisor? Can children use sketches to illustrate the partitions? Children need to be confident in scaling multiplication facts by 10 and 100.

	Charlie and Meena each try the following then compare how they each chose to partition the numbers and check each other's calculation: 552 ÷ 4; 276 ÷ 6; 660 ÷ 8; 747 ÷ 9; 1750 ÷ 7; 2568 ÷ 4; Encourage children to see if there are other ways to partition the same numbers: e.g. 441 ÷ 7 = (350 + 91) ÷ 7. Ask them to explain which partitions they found to be the most helpful or the most efficient and why. Simplify or challenge with simpler/larger numbers for the dividends. Challenge higher attainers with some straightforward 2-digit divisors, for example: 1001 ÷ 11; 3756 ÷ 12; 2775 ÷ 25; or some divisions that yield remainders.	

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<td>12. Written Methods for Multiplication and Division

Divide numbers up to 4 digits by a two-digit number (leading to understanding and using the formal methods of long and short division).

Solve problems involving multiplication and division using their knowledge of factors and multiples.

Both long and short division formal methods rely on the structure of repeated subtractions (inverse of multiplication) and both these methods partition numbers into their powers of ten: H, T, U and so on. In order to work, at every step it is important to find the largest subtraction (chunk) possible within each power of ten (or order of magnitude), and</td>
<td><u>The Meatiest Chunks!</u> Individually, then peer-assessment in pairs, and exchange solutions with other groups.

The teacher tells the children that a friend has been doing division by subtracting chunks, but finds that each has taken a long time and used up a lot of paper. Show one of your friend’s divisions. Could they write a shorter solution? Can the children see where the friend could have found bigger chunks to subtract, and have less to write out?

For example, to divide 544 by 17, the friend has subtracted ten 17s (to leave 374), another ten 17s (to leave 204), another ten 17s (to leave 34) and then two 17s to complete the division, giving the answer as 10 + 10 + 10 + 2 = 32. This is set out as shown below. Explain how this could be done quicker, for example, by starting with a chunk of twenty 17s (340), or even a chunk of thirty 17s (510), as shown below. It can also be written more simply, as shown on the right.

32
17) 544
10 x 17 | 170
374
10 x 17 | 170
204
10 x 17 | 170
34
2 x 17 | 34
32 x 17 | 0

Could be simplified to:
32
17) 544
30 x 17 | 510
34
2 x 17 | 34
32 x 17 | 0

or written more simply as:
32
17) 544
30 | 510
34
2 | 34
32 | 0

Provide a number of such examples of division set out using ad hoc subtraction and challenge the children to find bigger chunks to subtract.</td>
<td>Does the child recognise the distributive law being applied to division?

Do they see that in effect the number is being partitioned into subgroups which are helpful multiples of the divisor?

When there is more than one subtraction for any power of ten, does the child see a quicker way of estimating the highest multiple?

Can the children quickly work out some helpful multiplication ‘facts’ for a divisor, to find the largest chunk to subtract from the dividend?</td>
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then repeating this with the remainder of the number.

By the time children are using large two digit divisors it is inefficient and error-prone to model this concretely with p.v. counters, so it is not surprising to see children writing out several multiplications to find the largest chunk!

Ofsted (2011) identified that schools had difficulty in teaching the formal methods of division and that those who taught written division by chunking as a step towards this often did not successfully help children learn to look for the bigger, more efficient chunks. This activity gives children practice in doing this.

Meena and Charlie each try to simplify the division calculations. They aim to use the shortest number of steps, each time trying to find the biggest *chunk* that can be subtracted from the *thousands, hundreds, tens* and *ones* in turn. They assess each other's calculations, to check that they are correct and to see who has the most efficient solution.
For example:

	117
	8)936
50 x 8	400
	536
50 x 8	400
	136
10 x 8	80
	56
5 x 8	40
	16
2 x 8	16
117 x 8	0

	73
	13)949
20 x 13	260
	689
20 x 13	260
	429
20 x 13	260
	169
10 x 13	130
	39
2 x 13	26
	13
1 x 13	13
73 x 13	0

	48
	27)1296
20	540
	756
20	540
	216
5	135
	81
2	54
	27
1	27
48	0

	32
	34)1598
20	680
	918
20	680
	238
5	170
	68
2	68
47	0

Then they compare their solutions with other children's.
To simplify, provide calculations with single-digit divisors. Extend by introducing divisions which produce quotients with fractional parts to be simplified. E.g. $935 \div 12 = 77\ ^{11}/_{12}$
N.B. It is more appropriate to develop the most compact standard form of long division before applying to numbers which produce decimal places.

E.g. when dividing by 17:
2 x 17 = 34 *double 17*
4 x 17 = 68 *double 34*
10 x 17 = 170
5 x 17 = 85 *halve 170*

From these, other multiples of 17 could be found easily from adding or multiplying combinations of the above.

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13. Natural Numbers: Some Key Concepts

Explain, extend and infill other number sequences which follow a progressive sequence.

To explore and recognise the sequence of triangular numbers.

This is a practical exercise for exploring number patterns to find the sequence of triangular numbers.

N.B. It is not necessary for the children to arrive at a formula for the sequence, which is difficult to see unless they can double and factorise the pattern for the sequence.

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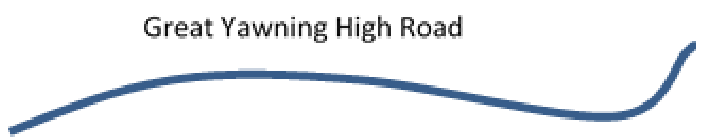
<u>**Road planning**</u> After demonstration, children explore the activity in groups of 3 or 4, to discuss and share ideas. Resources needed for each group:

- Sheet of A3 paper;
- Different colour felt pens;
- Counters to model the increasing sequence of intersections;
- A construction manager's hard hat for the teacher (optional!).

The children are town planners for *Great Yawning Development Corporation*. *Great Yawning* is going to be a new town, and the children's project is to design the road network. Wearing your 'hard hat', introduce the project to the class. *Great Yawning* is going to be built on land through which an existing main road already runs. This will be renamed *Great Yawning High Road.* Show them a picture of this road:



Explain that new roads need to be added to the plan to create the new town. The children can design any routes and shapes for the roads that they wish, but each time a new road is added, it must have ***one and only one intersection with every other*** road. It is also important for safety, and to reduce traffic congestion, that only two roads cross at any point. There will need to be sets of traffic lights at each intersection. How many sets of traffic lights are needed will depend on the number of intersections in the whole road network.

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Do they ensure that each new road crosses every existing road once and once only? The may need to extend the length of some of the existing roads to do this.

After they have added several roads, can they identify any patterns here?

What is the difference in the number of intersections when each is added? (They should see that the difference increases by 1 with each new road they add.)

Could they predict how many intersections there will be for any given number of roads in *Great Yawning* without drawing them?

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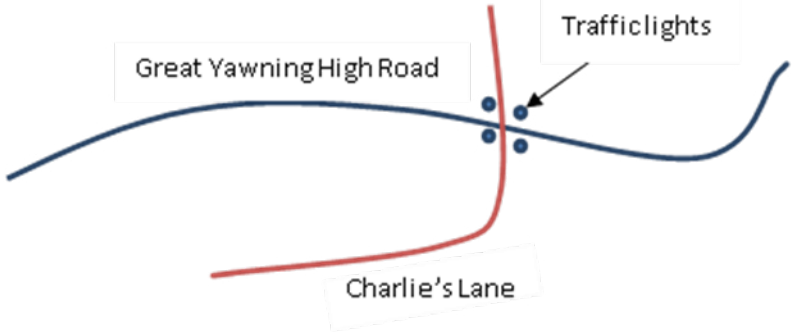
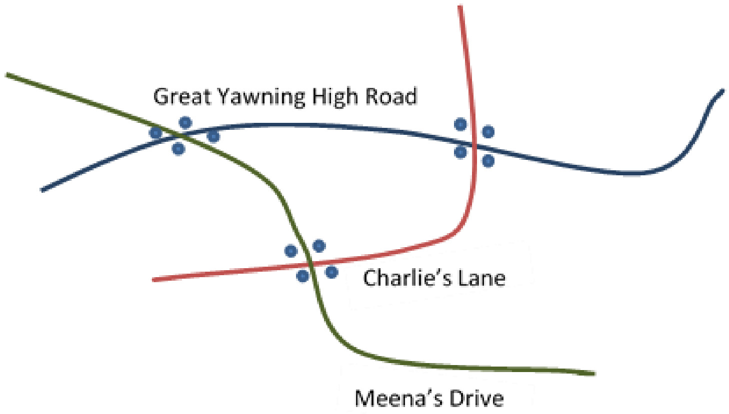
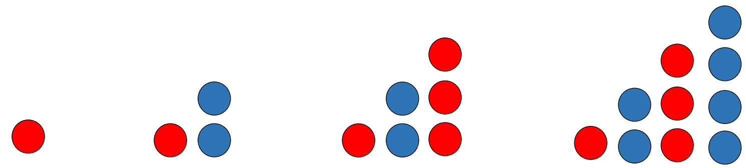
	Model a design for the network with two roads:  How many intersections with just two roads? (1) Now add a third road: There are three roads, requiring 3 intersections. Now hand the project over to the road designers. Ask them to write how many intersections there are for the total number of roads every time they add another road.  1 road requires 0 intersections, 2 roads require 1 intersection, 3 roads " 3 intersections, 4 roads " 6 intersections, … and so on.	When arranging the sequence with counters, do they recognise the *triangular* shape of each number in the sequence?

Explore further this pattern of the increasing number of intersections. Ask the children to see if they can use counters to arrange a sequence of the number of intersections every time a new road is added:



No. of roads:	1	2	3	4	5
No. of intersections:	0	1	3	6	10

It is called a series of *triangular* numbers. Why might that be?

14. Integers: Positive and Negative

Developing understanding and application of negative numbers in data.

Solve comparison problems using information presented in tables.

League tables After demonstration, children working together in pairs. They will need:

- Paper copy of the Premier league football table, sliced into horizontal strips for each team (but without their ranking), for example:

Norwich City	27	7	7	13	20	39	-19	28

Arsenal	27	18	5	4	52	27	25	59

Etc.

- Large sheet of plain paper with the premier league headings at the top:

#	Team	GP	W	D	L	GF	GA	GD	PTS

- Glue-stick

Display the top portion of the Premier League table, for example:

#	Team	GP	W	D	L	GF	GA	GD	PTS
1	Chelsea	27	18	6	3	49	21	28	60
2	Arsenal	27	18	5	4	52	27	25	59
3	Man City	26	18	3	5	69	27	42	57

Do children understand the ordinal use of negative numbers? For example: -19 is less than -10, or ***-19 < -10***

Can the children give answers to interesting questions such as whether a club with a negative goal difference could be in a higher position than a club with a positive goal difference?

Do the children justify and convince their partners and agree on their ranking before sticking them down?

Ask for some volunteers (there is bound to be at least one!) who can help you to explain what the headings mean and how to interpret the table (Rank, Team Name, Games Played, Games Won, Games Drawn, Games Lost, Goals For, Goals Against, Goal Difference, Points).

Ask the children to check whether the goal differences (GD) have been calculated correctly (GF – GA). Display some teams further down the table. What does it mean when some goal differences are negative? What happens to the order of the teams, when they have the same number of points? For example:

#	Team	GP	W	D	L	GF	GA	GD	PTS
13	Aston Villa	27	7	7	13	27	37	-10	28
14	Norwich City	27	7	7	13	20	39	-19	28

Now explain that you have all the team data in but it has been muddled. Please can the children create the league table of the teams in time for the sports reporter to announce the order? One of the children can do this, in role, at the end of the activity.

Meena and Charlie then take the strips of team data and have to order them by using the information in the points and goal difference columns. Note that if points and goal difference are identical for two or more teams, the rank is then decided by the higher number of goals scored (Goals For).

15. Fractions and Ratios

Identify, name and write equivalent fractions of a given fraction.

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

This activity is to help children understand how equivalent fractions can be substituted during addition and subtraction of fractions.

<u>Goofy ways to make 1</u>! Children work in pairs. Each pair will need:

- A blank table for completing equivalent fractions (see photocopiable resources):
- Fraction wall (to explore combinations visually and manipulatively, if helpful).

$^{1}/_{2}$				
$^{1}/_{3}$				
$^{1}/_{4}$				
$^{1}/_{5}$				

First Meena and Charlie complete a blank table of the 4 equivalent fractions for each of the first 10 unit fractions: $^{1}/_{2}$, $^{1}/_{3}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{1}/_{6}$, $^{1}/_{7}$, $^{1}/_{8}$, $^{1}/_{9}$ and $^{1}/_{10}$:

$^{1}/_{2}$	$^{2}/_{4}$	$^{3}/_{6}$	$^{4}/_{8}$	$^{5}/_{10}$
$^{1}/_{3}$	$^{2}/_{6}$	$^{3}/_{9}$	$^{4}/_{12}$	$^{5}/_{15}$
$^{1}/_{4}$	$^{2}/_{8}$	$^{3}/_{12}$	$^{4}/_{16}$	$^{5}/_{20}$
$^{1}/_{5}$	$^{2}/_{10}$	$^{3}/_{15}$	$^{4}/_{20}$	$^{5}/_{25}$
$^{1}/_{6}$	$^{2}/_{12}$	$^{3}/_{18}$	$^{4}/_{24}$	$^{5}/_{30}$
$^{1}/_{7}$	$^{2}/_{14}$	$^{3}/_{21}$	$^{4}/_{28}$	$^{5}/_{35}$
$^{1}/_{8}$	$^{2}/_{16}$	$^{3}/_{24}$	$^{4}/_{32}$	$^{5}/_{40}$
$^{1}/_{9}$	$^{2}/_{18}$	$^{3}/_{27}$	$^{4}/_{36}$	$^{5}/_{45}$
$^{1}/_{10}$	$^{2}/_{20}$	$^{3}/_{30}$	$^{4}/_{40}$	$^{5}/_{50}$

Next model, with the children, how the number 1 can be the sum of two or more fractions and then how any of these fractions could be replaced by an equivalent fraction of a different denomination. For example:

$1 = {}^{1}/_{2} + {}^{1}/_{2} = {}^{1}/_{2} + {}^{2}/_{4} = {}^{1}/_{2} + {}^{3}/_{6} = {}^{1}/_{2} + {}^{2}/_{6} + {}^{1}/_{6} = {}^{1}/_{2} + {}^{2}/_{6} + {}^{2}/_{12}$

Now challenge Meena and Charlie to find as many different ways to make 1 as they can – the more creative, the better! Tell them to check their solutions by writing down the simplest equivalent fraction under each fraction in the sum. For example:

$1 = {}^{3}/_{15} + {}^{5}/_{25} + {}^{1}/_{10} + {}^{3}/_{18} + {}^{4}/_{12}$ and $1 = {}^{1}/_{5} + {}^{1}/_{5} + {}^{1}/_{10} + {}^{1}/_{6} + {}^{1}/_{3}$ … and so on.

Simplify or challenge some by reducing or increasing the denominators involved.

Do the children have a sufficient understanding of ratio yet, that it is possible to exchange equivalent fractions as these maintain the same ratio between numerator and denominator?

Do the children have enough experience of substituting equivalent fractions?

A concrete fraction wall will help here if they have not. It will enable them to swap and substitute difference pieces physically.

16. Decimal Numbers and Rounding **Identify the value of each digit in numbers given to three decimal places.** **Round decimal numbers to one decimal place.** Quite often, we expect children to round numbers up or down to the nearest tenth. This activity helps the children to see this through an inverse approach. Being given the rounded figure, how imaginative can they be in finding different numbers which will round to this amount?	<u>**Round about**</u> Children work in pairs. Display the numbers 3.172, 3.2, 3.177, 3.17, 3.271, 3.1 and, through discussion with the children, put these into ascending order (3.1, 3.17, 3.172, 3.177, 3.2, 3.271). Now ask the children to round all the given numbers to the ***nearest tenth*** (one decimal place), to give: 3.1, 3.2, 3.2, 3.2, 3.2, 3.3. Note that four of them round to 3.2 to the nearest tenth. Ask the children if they can find *4 more* different numbers with two or three decimal places that round to 3.2 (to the nearest tenth, for example 3.215, 3.19, 3.23, 3.192). Charlie and Meena now work as a pair to write, in order, eight different numbers with two or three decimal places in each case that all round to the same number to the nearest tenth, for example: to 4.5, 6.9, 12.1, 271.4. Extend this by challenging children to find numbers with more decimal places which round to one decimal place, and then to two decimal places.	When comparing two numbers, do the children realise that the one with more decimal places is not necessarily the larger number? Do they realise that they must compare each of the values in the *same decimal place*, working from the left-most digit, when placing them in order? Do children see that only the value the *first* decimal place to the right of the rounded number of places has any impact on whether it is rounded up or down? E.g. 5.192 rounds to 5.2 whereas 5.149 rounds to 5.1, both owing to the value of the *100ths* decimal place. Similarly, to two decimal places, 5.4948 rounds to 5.49 and 5.4973 rounds to 5.5 owing to the values of the *1000ths* place.

17. Calculations with Decimals **Multiply one digit numbers with up to two decimal places by whole numbers.** **Solve problems which require answers to rounded to specific degrees of accuracy.** This activity requires some additions and multiplications with decimal numbers, in a typically real context.	**<u>The new carpet</u>** Working in pairs, children will need the following: • Metre stick or long tape measure. This activity is to calculate the (wall-to-all) area of (non-patterned) carpet needed for the classroom. It is reasonable to make some adjustment for the actual shape of the room, so that an approximate rectangle can be calculated. This is natural, as the carpet will be cut as rectangular pieces from a roll. However, it is unlikely that the room will be small enough to carpet the floor from just one complete piece. It is likely to require two or more pieces joined together side by side. First model how the carpet would be laid. Use a roll of kitchen towel to represent the carpet and use the top of a table to represent the floor. Show how the 'carpet' cannot cover the 'floor' unless more than one piece is cut from the roll and laid down side by side. To find out how much carpet is needed add up the combined length ***L*** of all the pieces. ***L*** is the total length of carpet to cut from the carpet roll to cover the floor. The area of carpet to buy will be this length ***L*** multiplied by the width ***W*** of the carpet roll. Note that the actual dimensions of the room will mean that some carpet is wasted, as it is unlikely to fit an exact number of carpet widths from wall to wall. The price of the carpet is calculated per square metre, so what is the area of carpet that has to be bought? Meena and Charlie use the tape measure or metre stick to measure the length (say, 10.5 m) and width (say, 7 m) of the room. Given a *standard* carpet width of 3.7 m, they need to decide how many carpet widths are needed to cover the width of the room, hence the number of pieces (2) and the total length ***L*** of carpet needed (21 m). They calculate the area as 3.7 × 21 = 77.7 square metres.	Do children align, add and multiply decimal numbers correctly when calculating? Do they estimate an answer to any calculation using an appropriate rounding of the numbers involved? Do they use an appropriate calculation method correctly? Do they realise that multiplying by a number which is less than one will make a product which is smaller than the original multiplicand? Do they understand that area is the product of length and width?

	If different carpet widths are available, for example 3.9 m or 4.6 m, what area of carpet would be needed? Would the joins need to be aligned with the room length or room width to minimise the waste? For lower attainers, this activity could be carried out with rounded measurements or simplified decimals, for example, to the nearest 0.5 m. Extend the activity by giving a price per square metre, which may vary with the different carpet widths and the children calculate the cost of the different options.	

18. Proportionality and Percentages

Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'.

Write percentages as a fraction with denominator 100.

Solve problems which require knowing percentage and equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those fractions with a denominator of a multiple of 10 or 25.

A very common real need to use percentages is to compare prices before and after discounts, price reductions or price rises. In this activity the Meena and Charlie work for *Barry's Bikes* and their job is to update the prices on the

<u>Catalogue changes</u> Children in pairs, working individually but with peer-assessment. They will need:

- An interesting page from a *real* catalogue is best, but for the purposes of this description here is a fictitious example (see photocopiable resources):

Barry's Bikes Catalogue		Accessories page	
A. LED light set	£ 20.59	F. Bike helmet	£ 14.57
B. Twin mudguards	£ 16.21	G. 'D' lock	£ 16.99
C. Tyre pump	£ 8.34	H. Cycle computer	£ 9.35
D. Rack	£ 23.98	I. Basket	£ 11.93
E. Gel cycle seat	£ 25.89	J. Puncture repair kit	£ 2.49

Barry has reviewed his sales and has decided that some items are not selling well, so in the next catalogue, these will be reduced. Other items are very popular or have been replaced by better quality products and Barry wants to increase their prices.

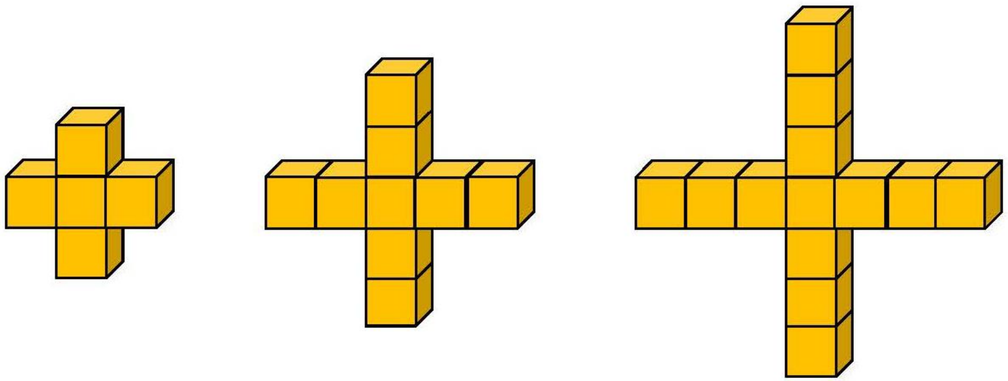
First, model/revise the calculation of an example price reduction and/or increase. For example, if a puncture repair kit increases in price by 20%, that would be 50p increase (typically rounded up – see below*), so the new price would be £2.99.

Do the children know that, if the whole value of something is 100%, then to find:

- 10% is $^{1}/_{10}$, so divide by 10?
- 1% is $^{1}/_{100}$, so divide by 100 (or find $^{1}/_{10}$, and then $^{1}/_{10}$ of that)?
- 5% is $^{1}/_{20}$, so halve 10%?
- 20% is $^{1}/_{5}$, so divide by 5 or double 10%?
- 25% is $^{1}/_{4}$, so divide by 4 or halve and halve again?

Do they see that it is often easier to calculate the new price by using the percentage to add to or subtract from the original amount, rather than multiply or divide by a factor?

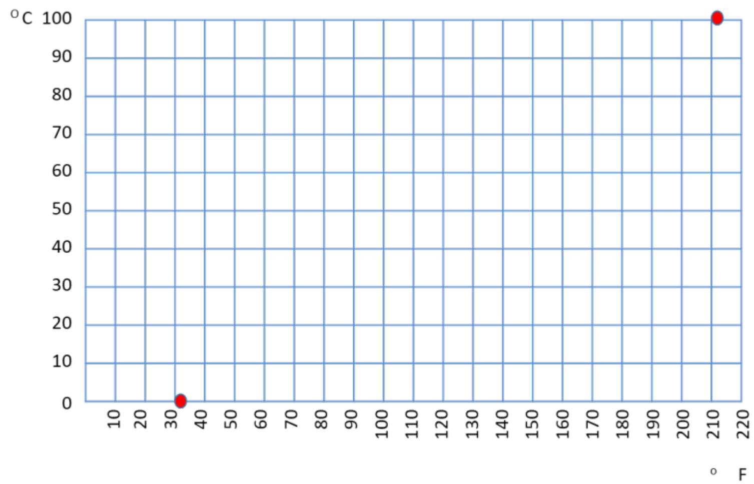
accessories page of the shop's catalogue.	Meena and Charlie have the following changes to make for the catalogue and need to work out the new prices, for example: • Items C and J increased by 10% (C = £9.17; J = £2.74); • Items H and I reduced by 5% (H = £8.88; I = £11.33); • Items B and D increased by 25% (B = £20.26; J = £29.98); • Items A and E reduced by 20% (A = £16.47; E = £20.71); • Items F and I increased by 40% (F = £20.40; I = £16.70). The children estimate, calculate and compare their results to check and agree the figures they will present to Barry. *In Chapter 18, ad hoc methods are used to build up 'awkward' percentages such as 17% from its component parts: 10% + 5% + 1% + 1%. This is fine if the price is a whole number of pounds, or for estimating the result. However, for *realistic* prices in a catalogue including odd numbers of pence, this approach will yield some inaccuracy if fractions of pennies are rounded at each intermediate step, so some tolerance may be needed if this is done. Therefore when setting problems for the children to do on *realistic* prices without a calculator, it is wise to limit these to one or two step calculations on a single number to limit the number of intermediate steps at which amounts may be rounded. For higher attainers, children can work with further decimal places (fractions of pence) until the calculation is complete before rounding just the result to nearest penny.	

19. Algebraic Reasoning **To develop a simple formula to generalise a linear number sequence.** This is a practical exercise for exploring number patterns to find a simple formula for generalising a linear number sequence.	<u>**Generalising sequences**</u> Work in groups of 3 or 4, to discuss and share ideas. They will need access to: • A class set of *multilink* cubes, or some counters. Start by asking Charlie, Meena, Alexi and Woljca to make the following sequence of shapes:  1^{st} 2^{nd} 3^{rd} What do they think the 4^{th} and 5^{th} shapes would look like? Ask them to make them? How many cubes are there in each shape? (5, 9, 13, 17, 21) Could they work out how many cubes they would need to make the 6^{th} shape, without making it? (add 4 to the 5^{th} shape) How many would they need to make 10^{th} shape? (add another 4 × 4)	Do children see that each shape is established by adding 4 to the previous shape? Can they write a similar calculation for each shape in terms of the number of times they need to add 4? If they cannot derive the formula, can they describe verbally the increment to make each successive shape, and work out the number of cubes needed for a specific shape, e.g. the 10^{th}?

	Suppose we wanted to make the ***n******th*** shape? If ***n*** could be any whole number, could they describe a *formula:* a rule to work out how many cubes they would need? **<u>1</u>**st shape needs 5 cubes = 1 + (4 **×** **<u>1</u>**) **<u>2</u>**nd " " 9 cubes = 1 + (4 **×** **<u>2</u>**) **<u>3</u>**rd " " 13 cubes = 1 + (4 **×** **<u>3</u>**) **<u>4</u>**th " " 17 cubes = 1 + (4 **×** **<u>4</u>**) Highlight the part of the calculation which changes with each shape and ask if they can see any connection with the changing part and the number of the shape (called the *term* of the sequence). Suppose this was any number in the sequence, ***n***? ***n***th shape needs 1 + (4 **×** ***n***) or 1 + 4*n* cubes (Explain that we can write the multiplication **4 ×** ***n*** simply as **4*n***.) To extend the challenge, can they create their own simple sequence of a pattern of shapes that increases by the same number each time, and challenge others in their group to work out their formula?	

20. Coordinates and Linear Relationships **To understand a real-life linear relationship in a practical example.** This is a practical exercise for exploring the linear relationship between the Celsius and Fahrenheit scales of temperature. Although most countries now use the Celsius scale, the Fahrenheit scale used to be used in the UK, and is still more familiar to some older citizens. The Fahrenheit scale continues to be used in a few countries, most notably the United States of America.	**Some like it hot!** Children work in pairs, to discuss and share ideas. They will need: • A4 Squared paper (1 cm^2); • Classroom thermometer in both °C and °F (optional). Start by explaining that in the United States they measure temperature using a different scale to us, and find out what the children may already know. Establish that we are going to make a 'ready reckoner' to convert temperatures easily between the two countries' scales. Display a squared grid with the longer axis horizontal, and label this axis Fahrenheit. Label the vertical axis Celsius. Now mark the grid lines on each scale as 10° intervals. Do any children know the equivalent values in each scale for the freezing point and boiling point of water? (At standard atmospheric pressure, of course!) Establish that these are respectively 0°C: 32°F and 100°C: 212°F, and explain that these values are co-ordinates to mark the points where they are represent the same temperature on each scale. Mark these (approximately) on the display: Explain that both temperature scales measure in regular intervals so that a difference of 1 degree on one scale always represents same interval in temperature on that scale. Could they use this to work out the approximate equivalent values between the scales for other temperatures? How would they do this?	Do the children see that the graph shows the relationship between two variables? One variable is the temperature measure in degrees Celsius, and the other variable is the temperature measured in degrees Fahrenheit. Do they see that a straight line shows there is a ***linear relationship*** between two variables? Do they understand that this means the values of the Celsius and Fahrenheit scales vary proportionally to one another?

The children can discuss this between them. They may conclude from other experiences that joining the two known points with a straight line will show the intersection of the two scales for any other temperature. If not, make this point and draw the line on your display.



Charlie and Meena now create their own conversion graphs. Insist that they position the axes so that they use the **uppermost 10** grid squares for the Celsius temperatures and the **rightmost 22** grid squares for Fahrenheit. Tell them that this will be useful for some later questions. Then ask them to plot the freezing and boiling points. They can use mm measures on their rulers to make a more accurate point for each one on the Fahrenheit axis. They connect the two points with a straight line and they are ready to find at least approximate answers to the following questions:

	• What is 20°C in Fahrenheit? • What is 180°F in Celsius? • If an American tells you it is 110°, what temperature would a European think this is? • … and so on. Challenge Charlie and Meena to think about how to use the conversion graph for other values: • What is -10°C in Fahrenheit? • What is 0°F in Celsius? • What is -20°F in Celsius? To answer these questions, the children will need to extend the graph into the 3rd and 4th quadrants. There should be room on an A4 sheet of $1cm^2$ paper to extend the axes below zero on both scales and to extend the line of the relationship. A further challenge could be to calculate the conversion between Fahrenheit and Celsius. *(Hint: Start by equating the difference between boiling point and freezing point in both scales: 100°C = 180°F.)* A few children may even be able to work out that, if *n* is the temperature in Celsius and *m* is the equivalent in Fahrenheit, then $n = (m - 32) \times {}^5/_9$	

21. Concepts and Principles of Measurement **To use and calculate with appropriate measures in real situations.** Athletics is an aspect of P.E. where children can gain rich, practical, real-life experience of using measures, particularly in length and time. In these lessons, Year 5–6 children can take, and actually enjoy, the responsibility for measuring and recording their own and others' performance data and making comparisons between their previous and current attainments. It is also a cross-curricular application of mathematics in another part of the curriculum.	<u>**Sports maths**</u> In groups of 4–6, to measure and record measures. Each group will need access to: • Recording chart (see photocopiable resources); • Stop-watches, metre stick/trundle wheel, long tape measure, as appropriate for the event. In a typical primary athletics lesson, it is possible to include a carousel of events, such as sprints and other runs, long jump, high jump (standing or running jump depending on the school's facilities), skipping, distance throwing events (ball, bean bag, foam javelin, foam discus, etc.) In all of these there is the provision for recording times in seconds and possibly to the nearest tenth of a second, as well as distances in metres to two decimal places. For a particular event they can: • find the difference between their latest and previous distances, or times; • calculate the difference as a percentage change in the previous performance; • calculate their mean speed in metres/second over an event. Typically these calculations may need the assistance of a calculator or spreadsheet, or times and distances may be suitably rounded to enable informal or written methods to be practised. Always encourage the children to focus on *ipsative* assessment: how they are improving their *present* performance compared to *their own* past performance, rather than comparing themselves against that of other children in their group or class. Top athletes focus primarily on how they can improve their *personal best*, rather than the difference between their performance and other people's.	Do the children use the different items of measuring equipment accurately, and interpret the scales and divisions correctly? Do they record the units they used to measure, not just the numbers?

22. Perimeter, Area and Volume **To use and calculate areas for real purposes.** When painting a room, the liquid volume of paint to buy depends on the area to be painted, the number of coats to fully colour the walls, and the coverage property of the chosen paint, usually given on the tin. The coverage is usually described in terms of m^2/litre of paint.	**Painting wall areas** In groups of 4, to measure, calculate and check their findings. They will need: • Metre sticks or tape measures; • Recording chart (see photocopiable resources). Explain that you think it is time the classroom walls were repainted (or some other part of the school). The children can help by working out how much paint the school needs to buy. Tell them we need to know the size of the area we wish to paint. Ask the children to remind you how to calculate the area of a rectangle and revise this if needed. Explain this is fine if we only need to paint one rectangular wall, but what should we do if we want to paint all the visible parts of the walls–ignoring cupboards, display boards, doors and other covered parts which do not need to be painted the same colour as the plastered walls? Help the children to see that the total area to be painted comprises a number of smaller areas and that these can be divided into rectangles. Charlie, Meena, Alexi and Woljca set off to measure the length and height of different parts of the walls to be painted. As they do so they complete the group's recording chart:	Do the children understand the idea of 'coverage' in terms of area per liquid volume of paint? Do the children divide the areas to be painted into a reasonable distribution of rectangles? Do they measure and calculate the areas of each of these rectangles correctly? Do they add all the areas together to find the total area needed to be covered?

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>Section of wall</td><td>Length (m)</td><td>Height (m)</td><td>Area of section (m^2)</td></tr>
<tr><td>Back of classroom</td><td>6 m</td><td>2.5 m</td><td>15 m^2</td></tr>
<tr><td>Above the big display board</td><td>8 m</td><td>1 m</td><td>8 m^2</td></tr>
<tr><td>Below the big display board</td><td></td><td></td><td></td></tr>
<tr><td>Right of big display board</td><td></td><td></td><td></td></tr>
<tr><td>Above the fixed cupboards</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Total area to be painted:</td><td></td><td></td><td></td></tr>
</table>
Provide the children with the height of the walls, so that they can work out the actual height of each area to paint by measuring and subtracting the heights of cupboards, boards etc.,
</td>
<td>Do they take account of the number of coats needed and multiply the area accordingly?</td>
</tr>
</table>

	which are not to be painted. This will save the children needing to climb up to reach unsafe parts of the walls. Decide the precision necessary for the measurements and whether calculators may be used or children are to practise informal or formal written methods of multiplication. In real life, measurements of length could typically be rounded up/down to the nearest 1 m or 0.5 m. When they have measured all the smaller areas they can then calculate the paint to buy. For example: • What is the total area that needs to be painted? • If three coats of paint are needed to fully colour the walls, how much is the area that has to be covered if we count the area being painted three times? • If the coverage of a typical emulsion paint is 16 m^2/litre and the paint is sold in 5-litre, 2.5-litre and 1-litre tins, how many of each size should we buy? (It is more economical to buy paint in larger volumes). • How much would the paint cost, if the prices of a these tins are: 5 litres for £29.50, 2.5 litres for £17 and 1 litre for £11?	

23. Angle **To use a protractor to measure angles accurately for a purpose.** This is a practical investigation into the sum of the internal angles of different irregular polygons. It provides opportunities for measuring many different angles, as well as multiple additions.	**How many degrees?** Children work in pairs, to discuss and compare findings. They will need: • Protractors (try to ensure these are 360º, for measuring angles greater than 180º); • Recording chart (see photocopiable resources); • Scissors. With the whole class ask them if they know the sum of all the internal angles of a triangle. (They should: 180º.) Ask them *how* they know? Have they checked? Some children may know the activity for tearing off the vertices and placing them adjacently to make a straight line. Ask them if this works to find the sum of the internal angles for a quadrilateral? Get the children to draw an irregular quadrilateral and cut it out, then tear off the angles to try it. They should find that they create a complete circle (360º). Now ask them to see if they can work out the sum of the internal angles for a pentagon? Again, get the children to try this, drawing and cutting out an irregular pentagon. They should find that the angles overlap one another. Suggest that we test these by measuring the angles and adding them up. Check that the children can use the protractor correctly.	Do the children align the protractors accurately upon the angles and read and interpret the scale correctly? Do they check they have measured every internal angle? How can they do this systematically? Do they re-measure and revise their results if they find differences between polygons with the same number of angles? Do they realise that all measurements are approximate and that even though we try to be as accurate as possible a small imprecision in measuring is unavoidable.

Charlie and Meena draw different (irregular) pentagons, measure the angles and add these up. As they do so they complete a recording chart:

Shape	Angles	Sum of angles
Pentagon(1)	35°, 72°, 69°, 123°, 243°	542°
Pentagon(2)	161°, 88°, 117°, 146°, 31°	543°
Pentagon(3)	97°, 182°, 152°, 46°, 30°	538°

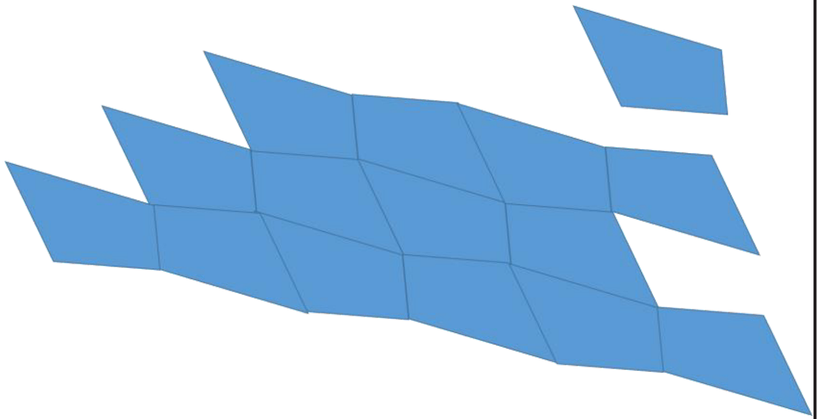
After several examples, ask the class whether it is about the same every time, and if there may be a reason for values to differ? (Some inaccuracies in measuring the individual angles.)

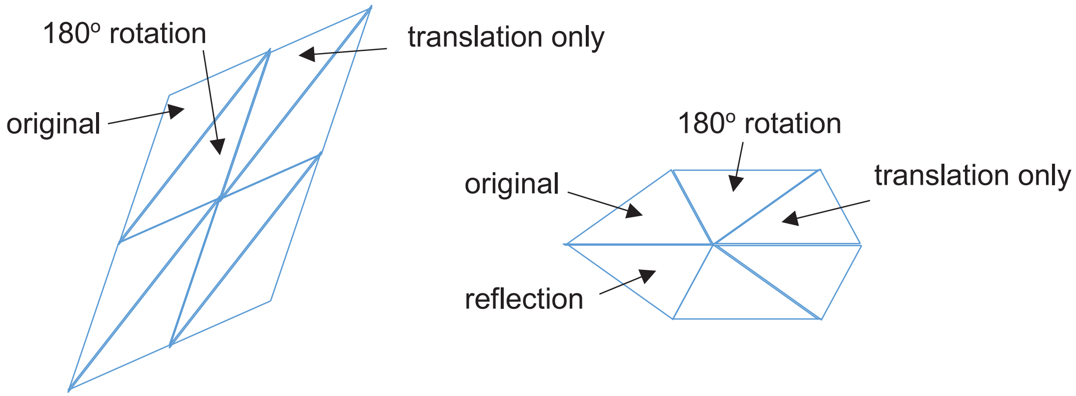
Ask the children to suggest what they think is ***sum of the angles*** for a ***pentagon*** (540°).

This exercise can be extended to ***hexagons*** (720°), ***heptagons*** (900°), ***octagons*** (1080°), and so on.

Ask the children if they can see any pattern in the relationship between the number of sides and the sum of the angles? *(Hint: how many 180s are there in each total?)*

Children do not need to know the formula, but this is further opportunity to develop their algebraic reasoning.

24. Transformations and Symmetry **To understand how transforming some 2-dimensional shapes enables them to tessellate.** **To use the vocabulary: *orientation*, *rotate*, *translate*, *reflect*, and *tessellate*.** A practical activity for children to explore how irregular shapes can be transformed (but remain congruent) in order to tessellate. It is a particularly helpful use of technology with which children can rapidly replicate and manipulate shapes.	**Irregular tessellations** Children will work in pairs. Each pair will need • PC/tablet with a simple drawing program/app. Introduce/revise the drawing program. Show how an *irregular* polygon can be drawn, how it can be *moved in the same orientation* ***(translated)***, ***rotated*** *about the centre of the shape*, and *flipped over* ***(reflected).*** Show that it is still the same shape – the same number and lengths of sides and the same angles. Through copying and pasting the created shape, show the children how to make several duplicate *tiles* of the same shape. Show how you can transform (***reflect, rotate*** and/or ***translate***) the tiles so that some of the sides of separate tiles can be made to touch one another. Remind children how some shapes can ***tessellate***, so that they can be arranged to touch one another without leaving any gaps. Demonstrate tessellation with a rectangle, then with an irregular quadrilateral. For example: 	Do the children understand the difference between each of the different transformations? Can they identify where two or more transformations have been made and what these are? Can the children see that tessellating polygons also create larger, intrinsically tessellating polygons with more sides? For tessellating quadrilaterals create tessellating octagons, and tessellating triangles create tessellating quadrilaterals and hexagons?

	Using their PC/tablet, Meena and Charlie create an *irregular* triangle. Ask them to create duplicate tiles. Challenge the children to see if they can transform and arrange the tiles so that the tiles will ***tessellate***. • Will this work for an irregular triangle? (Yes) • In what ways have they transformed the original triangle? As shown in the examples below, the tessellation may involve just ***rotations*** through 180° and ***translations***; or it might involve ***reflections*** as well. For example: 	

	Meena and Charlie go on to investigate the question 'Will any quadrilateral shape tessellate?' (Yes) In what ways do they have to transform the original shape to do this? Discuss the children's findings as a class. Challenge higher attainers to find some pentagons and hexagons that will tessellate. The house-shaped' pentagon and 'arrow-shaped' hexagon shown below are examples that do tessellate, but most irregular and regular pentagons and most irregular hexagons don't.  Children can also explore various websites which enable them to create very complex shapes which will tessellate. Where shapes do tessellate, can the children describe any conditions which enable this? E.g. sides of same length and orientation.	

25. Classifying Shapes **To discover that there is a special number (π) which is the defining property of a circle.** **To use the vocabulary: *diameter*, *radius*, *circumference*, *ratio*, *π (pi).*** A practical exploration and investigation into the property of a circle that we know as 'π'. For this type of investigation, involving divisions which may produce several decimal places, it is a good idea for the children to use a calculator, to help focus on what is common each calculation.	<u>**The life of Pi**</u> Children explore in pairs. They will need access to: • An *ad hoc* class collection of objects with circular faces: cups, bowls, plates, cylindrical tins, jars, a wall clock, and so on; • Ruler, tape measure and calculator; • Recording sheet (see photocopiable resources). This is best presented to the whole class as a mystery to investigate: What makes a circle a circle? Some children may identify that, for any given circle, the distance from the centre of a circle to its circumference is always the same, no matter where this is measured. A high attaining child may even tell you it is a regular polygon with an infinite number of very small sides! Both of these show great insights into the properties of a circle. Tell the children we are going to investigate to see if there is anything else we can find out. Meena and Charlie work their way through the collection of objects. For each one, they measure and record both the diameter of the circular face, and the perimeter of that circle's circumference. (The latter will be more easily and accurately undertaken using a tape measure.) At the end of their measures the may have a record such as:	Do the children recognise that there is a fixed relationship with the circumference being equal to *'3 and a bit'* times the diameter? Do the children understand why they get a slightly different result for π from each different object? All measurement is approximate, so even their most accurate measuring is imprecise. Do children realise that this number π is unchanging or ***constant***, and is specific to defining a circle?

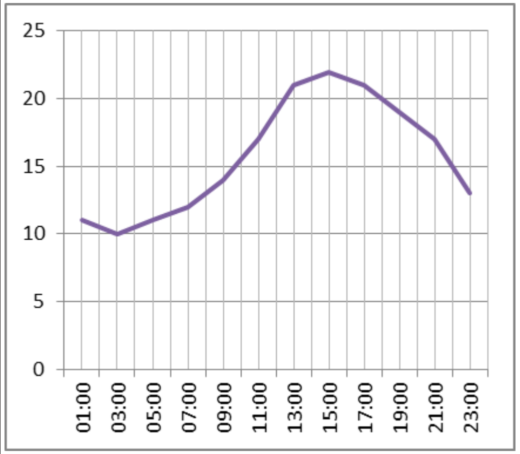
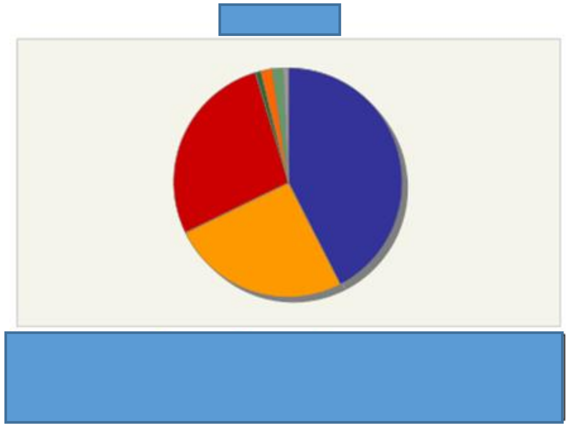
Later, some children could explore the nature of π as an ***irrational*** (never-ending, non-recurring decimal) ***number***? Ask them to research (online or in books) how precisely π has been calculated so far.

object	diameter (cm)	circum-ference (cm)	Possible relationship?
bowl	17.5	54.5	***54.5 ÷ 17.5 = 3.114***
plate	27.3	86.5	
cup	8.5	27.6	
coaster	11	34.6	

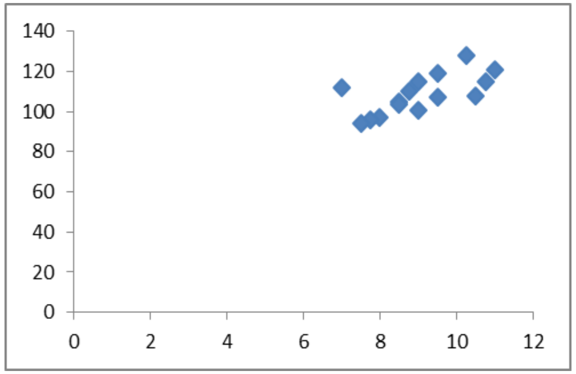
Now ask them to see if they can find any relationship between the measures for each circle. It is appropriate to tell the children they should use calculators for this exploration!

Charlie and Meena find that for every circle they have measured, dividing the circumference by its diameter always gives an answer of '*3 and a bit'* or '3.1-*something*'. Discuss with the children how this is true for every circle, and this special number is the ***ratio*** of any circle's diameter to its circumference which is always '*3 and a bit': 1.* The number is so special that mathematicians have even given it its own symbol (π) and name 'pi' (a letter from the Greek alphabet).

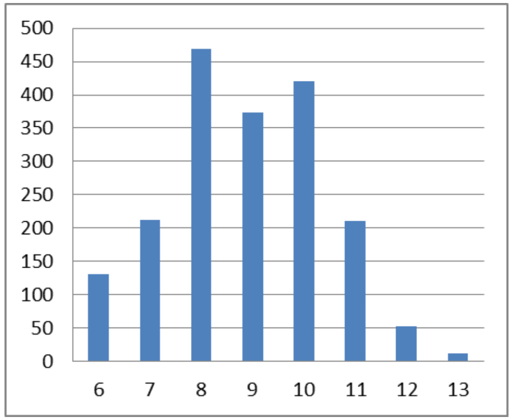
Do they see that as the calculation for the circumference of a circle has been established as ***π × diameter***, and ***diameter*** = ***2 × radius***, we can also calculate the circumference as: ***2 × π × radius***

26. Handling Data **To interpret the *story* told by a graph, and draw conclusions from the data.** This activity is to challenge children in developing their skills in applying and using the data presented in graphical form.	<u>**Data detectives**</u> Children work in groups of 3 or 4. They will need: • Selection of different, but incomplete, graphs, so that the information presented may be subject to more than one interpretation or explanation (see photocopiable resources). Show some simple untitled graphs with some missing labels and/or scales. For example: A. Line Graph  B. Pie Chart 	Can the children in the group *convince* one another of the logical suitability of their explanation and their reasons for this? The children may have more than one valid explanation that fits the data. Do the children know the important attributes and uses of each type of graph? When is it appropriate to use one type rather than another? Does this have an impact on the potential explanations for each? For example, only a graph with a continuous relationship between two variables should be represented by a line graph. Of these, showing a variable related directly to time is one of the most common uses. Can children draw conclusions from the graphs according to their interpretation: what questions could each graph answer? For example:

C. Scatter graph



D. Bar Chart



Meena, Charlie, Alexi and Woljca examine each graph in turn. They discuss and agree between them a convincing story for the data presented in the graph, and complete the missing title, label and scales.

The graphs invite the children come up with potentially more than one convincing story. For the record the originals are:

A. Line graph: temperature in °C recorded at different times over a 24-hour period.
B. Pie chart: Votes recorded for different parties at a general election: Labour – red, Liberal Democrat – yellow, Conservative – blue, then smaller parties grouped into other colours.
C. Scatter graph: relationship of age (year) to height (cm) for different children in a group.
D. Bar chart: Number of pairs (the ***frequency***) of men's shoes in different sizes sold by a shoe shop during the same period.

- Why is it warmest mid-afternoon, rather than when the sun is highest in the sky?
- Who won the election? If the pie chart represented the proportion of MPs each party has in Parliament could the governing party be outvoted?
- Do children grow steadily according to age? If so what would be the height/length of a baby when born? Does that make sense?

27. Comparing Sets of Data **To find the *mean*, *median* and *mode* for a set of data.** This chapter is mostly focused on the professional needs of teachers, rather than the learning needs of children. However, it is important that by the end of Key Stage 2 children understand and use the terms ***range*, *minimum*, *maximum*** and the ***mean*** as a measures of average. So an activity is suggested here on these ideas, but only for children in upper Key Stage 2. Children will also practically encounter the ***median*** and ***mode*** as alternative, yet valid interpretations of average from ***mean.*** This activity combines data collection and analysis, for which the children will use a TV guide to explore the average	**TV programmes** Children work in groups of 3 or 4. They will need: • TV guide or PC/tablet to access this online. • Blank frequency table (see photocopiable resources). Set out the problem that the Controller of a new TV channel wants to know what the average length of a *daytime* programme (9am–6pm) should be by comparing the averages of programmes on the other channels: BBC1, BBC2, ITV1 and Channel 4. (Concentrating on these times helps avoid the children reading about potentially inappropriate programmes broadcast at later times in the evening! It may be wise just to photocopy the earlier parts of the evening schedules and distribute these.) Show how to find the difference in minutes between the starting time of one programme and the starting time of the next, using a number of examples. Each child in the group: Meena, Charlie, Alexi and Woljca is to take one of the four channels and calculate the length of time in minutes allocated to each programme to be broadcast on that channel over the same period (say two days). Each child records the raw data in minutes. For example, Charlie collects this data is for BBC1 for two days between 9am–6pm: 60, 10, 25, 30, 35, 40, 45, 45, 30, 50, 10, 15, 195, 25, 60, 60, 30, 40, 45, 45, 75, 45, 55 Charlie then arranges his data in order of increasing programme length: 10, 10, 15, 25, 25, 30, 30, 30, 35, 40, 40, **45**, 45, 45, 45, 45, 50, 55, 60, 60, 60, 75, 195 Then each child identifies the ***median*** (middle data item) programme length for their sample. For Charlie's data, the median is the 12th item in the order list, a value of 45 minutes. Each child then calculates the ***mean*** programme length, by adding all the items in the set and dividing by the total number of items. For Charlie's data the mean is 1080 ÷ 23 = 47 minutes (rounded to the nearest minute). Each child then draws up a ***frequency table***, grouping the data in increasing intervals of, say, 15 minutes, as shown below. Depending on the data collected a larger or smaller interval might be used. They use this the frequency table to identify the group with the highest frequency – which in this case is 31–45 minutes.	Do the children understand why the data has to be sorted in order to identify the ***median*** programme length? Do they see how to find the one in the middle of an ordered list? What if there is an even number of items? (Calculate the value halfway between the two middle items.) Do the children realise that it is not necessary to order the data to calculate the ***mean?*** Do the children see why a ***frequency table*** is needed to calculate the ***mode***? Do they see why grouping the data into intervals is helpful to managing this?

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length of TV programmes using the three measures of average.

In reality, for the use of the ***mode*** to be valid a much larger set of data should be collected than in the example here.

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This group can be identified as the **mode**, in the sense that more programmes have a length in the interval 31–45 minutes than any other interval of 15 minutes.

Programme length	Frequency
1-15 minutes	3
16-30 minutes	5
31-45 minutes	8
46-60 minutes	5
61-75 minutes	1
76-90 minutes	0
91-105 minutes	0
106-120 minutes	0
121-135 minutes	0
136-150 minutes	0
151-165 minutes	0
166-180 minutes	0
180-195 minutes	1
Total	23

As a class discuss the children's findings. What could this data tell the Controller of the new TV channel about people's viewing habits for the different channels?

Extend the challenge by looking at some further channels with different patterns, for example, some of the film channels. What would be appropriate intervals for a frequency chart of those programme lengths?

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28. Probability **To apply probability to develop and use strategies for an outcome.** Although *Probability* does not presently appear in the English Primary curriculum, it is a worthwhile and interesting area of mathematics for children, and it is included here as it is still featured in other international curricula.	**<u>Simple Trumps</u>** Children play in groups of 4 or 5. Each group will need: • A pack of playing cards, with the picture cards (*King*, *Queen*, *Jack* and *jokers*) removed. This simplified version of the card game *Trumps* is an interesting and informal application of probability to develop playing strategies to help win a game. Meena shuffles and deals the cards out to Charlie, Alexi, Woljca, Danielle and herself. Each child will have eight cards, which they may look at but not show to anyone else. Advise the children to organise the cards into suits in ascending order of number. N.B. Explain that the Ace is very special, and is treated as the highest card of each suit. The first player to Meena's left (Charlie) lays a card *face up* in the centre of the table. Every other player in turn must now lay a card from their hand which is in the *same* suit as Charlie led. After every player has had a turn, the player who laid the highest card has won the round or *trick.* The winner of the trick puts the stack of cards just won to one side nearby, to count the number tricks they have won at the end of the game. They then lead the next round, by laying a card from their hand in the centre of the table for the player to their left to follow suit. If a player cannot follow suit, because they have none of that suit in their hand, they must play a card from another suit, but this is counted as a zero. When all the cards have been played, the winner is the child who has won the most tricks. To extend the challenge: • include the picture cards (*King*, *Queen*, *Jack* but excluding *jokers*); • allow each child in turn to choose one suit for a round which beats all the others (*trumps*), so that a two in that suit is higher than an ace of any other. However, a player must not *revoke*, that is, they may not lay a trump if they hold also card in the suit which was laid.	Do the children understand the simple *etiquette* of card games: turn-taking, not revealing cards to others, not revoking? Do they remember how many cards there are in each suit (10 in the simple game)? Given the actual number of cards of a particular suit which they have been dealt, do they recognise that someone has fewer or more of that same suit? Do they see how this affects the game? Can the children predict any tricks they may be able to win? Why? For example, if a child has, say, both the Ace and ten of a suit, they should be able to win both of the tricks in this suit when they play these. Encourage children to use the language of probability in discussing strategy: ***more likely, less likely, certain, impossible,*** and so on.

Common equipment recommended

Notes about some of the common equipment recommended for use with the activities.

100 square

It is helpful to have available class sets of paper 100 squares of size about the width of a piece of A5. Two sets are useful in different arrangements: 1–100 and 0–99.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Counters

A large box of 500–1000 counters in many colours, of about 1.2cm diameter.

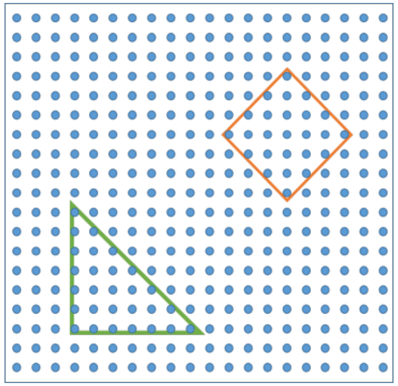
Cuisenaire Rods

These coloured rods of different lengths are great for helping children to develop a practical sense of ratio and of algebra from an early age. The rods were invented by a Belgian primary school teacher, Georges Cuisenaire. The right to use the name 'Cuisenaire' is now owned by a specific company, but other manufacturers supply this resource by different descriptions.

Fraction Wall

A rectangular tray containing plastic or wooden strips, each strip sub-divided into equal fractions of a different denominator, enabling the pieces to be moved and recombined in different ways to make 1. It is a physical model of the fraction chart pictured here.

Children can be encouraged to create their own Fraction Wall with identical paper strips which they fold into $^{1}/_{2}$, $^{1}/_{3}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{1}/_{6}$, and $^{1}/_{8}$ and paste to a backing sheets.



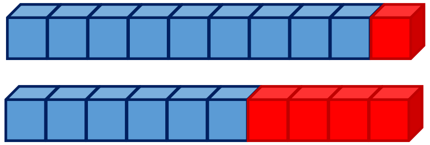
Geoboard

Geoboards
Typically plastic or wooden boards with raised pins or points in a square matrix formation around which elastic bands can be looped to create 2-dimensional shapes of various shapes and sizes.

Maths mat
Simply ***a sheet of A4 or A3 paper***, on which children arrange counters or other concrete apparatus they actually use in a calculation. This helps children to identify the counters which form the calculation, separately from the 'spare'/unused counters on the table.

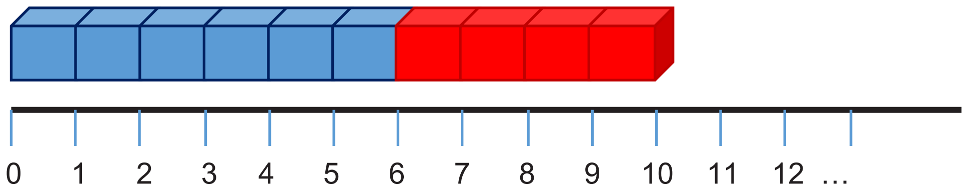
Money
Plenty of coins of all denominations. It is more realistic and better for learning about money, if these can be real coins. However, plastic money will suffice if it is a reasonable representation of actual coins in size and appearance. It is also surprising how easy it is to come across out-of-date coins in sets when the Royal Mint has changed them.

Multilink ©
Plastic, interlocking cubes (edges about 1.8 cm in length) of different colours, which can be connected to one another in different ways to construct an endless variety of solid 3D creations. The name 'multilink' is the trademark of a particular manufacturer, and other variants of interconnecting cubes are available, though we have found *multilink* to be the most reliable.



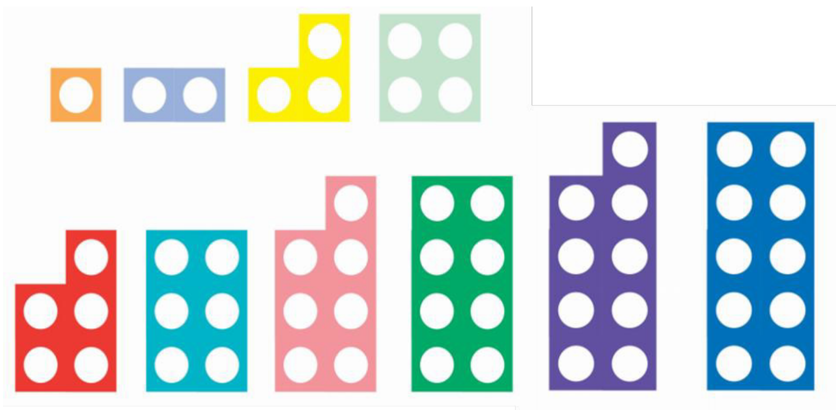
Number lines

In the early years and Y1 it is helpful to have a number line which is scaled to fit the multilink pieces, so children learn to match the edge of a piece of multilink with a complete whole number:



After that in KS1 it is helpful to have wipe-clean laminated number lines up to 20 or 30 which children can use individually at their desks, and a longer number line from 0 to 100 along a wall at a comfortable height for a child to point and touch individual numbers. It is important al so to display vertically arranged number lines, and certainly from KS2 if not earlier, to show negative numbers to -10 on number lines.

Numicon templates ©



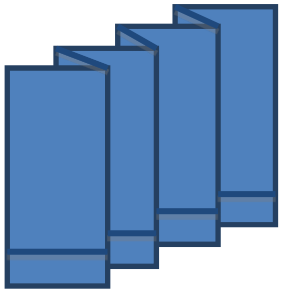
These are another commercially produced resource for exploring number and calculation. We have not seen an alternative provider of a similar resource of the same quality. It is especially helpful in reinforcing odd/even numbers, and for overlaying numbers upon others, for example when exploring multiplication and division as repeated groups.

Paper/plastic cups

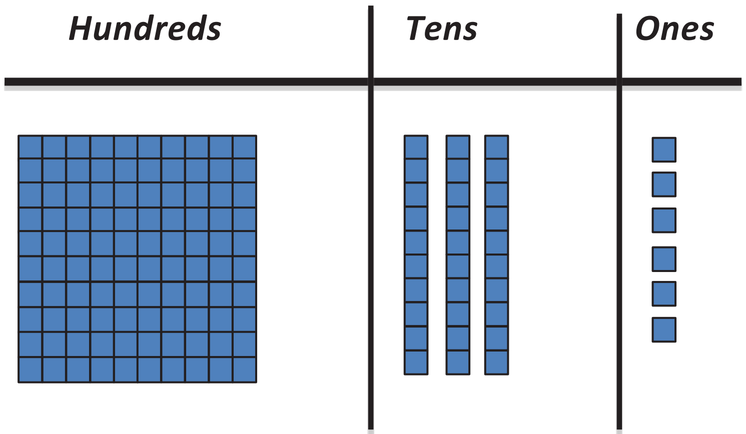
When demonstrating/modelling the use of counters in calculations to the whole class, it is helpful for the class teacher to use a larger and more visible resource to represent the counters, such as paper cups.

Paper screens

This is a simple and useful device for enabling a pair of children to hide something from one another for the purposes of a game. It can be made simply from a sheet of A3 (or larger) sugar paper with a series of vertical 'concertina' folds and one small 'turn-up' folded horizontally along the bottom. For example:



Place value base-10 blocks or Dienes' apparatus



These are described in Chapter 6, 'Numbers and Place Value'. Invented by Zoltan Dienes, base-10 apparatus is a set of scaled manipulative blocks, in 1s, 10s, 100s and 1000s which are used to make scaled concrete representations of place value in number, e.g. the number 136 is represented concretely by the arrangement pictured left.

Place value (p.v.) mat

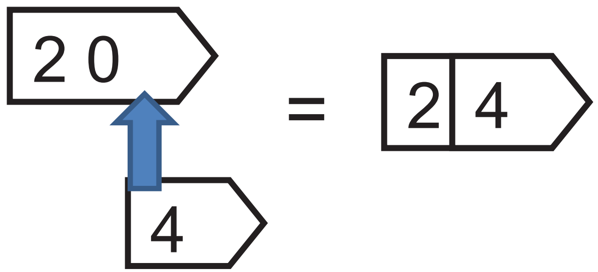
When using base-10 apparatus, it is helpful for the child to have a simple A4 or A3 mat upon which Hundred, Tens and Ones columns have been created in which to place the different denominations of the base-10 apparatus.

A free photocopiable resource for this is supplied.

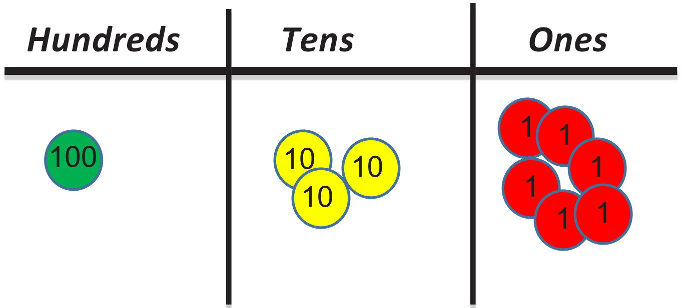
Place value (p.v. or 'arrow') cards

These are described in Chapter 6. Place value cards are often called 'arrow' cards because of their shape, and are number cards which can be assembled to represent a number so that the number can also be partitioned into hundreds, tens and ones, to show the actual place value of each digit, e.g. for 20 + 4 = 24

There are many variations of this resource freely available on the internet, and sets which can be purchased ready-made for schools to provide for pupils.



Place value (p.v.) counters



These are an abstraction from base-10 apparatus, so that equal-sized labelled counters are used to represent place value on a place value mat, rather than having items which are scaled in size. The counters usually represent each place value with a different colour, as for the number 136 in the example here. The significant learning developments are that the value of an item is not proportional to its size compared with other items and that one counter ***unitises*** or has a ***1:many*** relationship with other counters which may be of equivalent size or even larger. A similar experience which children will encounter before using this resource is when exchanging coins of different values.

Playing cards

Playing cards are a very cheap, reusable and shared resource, so it is not expensive to buy sufficient packs to equip a class. Even quite young children enjoy learning to shuffle cards properly, but as long as they can make sure the cards are 'mixed up' to some extent, that is sufficient!

Polydron frameworks ©

We are not paid to advertise *Polydron,* and you may find a cheaper alternative, but this really is the most effective resource we have come across for helping children to easily construct their own 3-dimensional shapes from pre-formed 2-dimensional shapes that click together. The skeletal faces in the *Frameworks* also have the benefit of being able to open into one flat open net, allowing children to draw around the inside of each face to create an approximate representation of the whole net. For example, a net for a triangular prism could look like any of the 'opened' shapes below: