

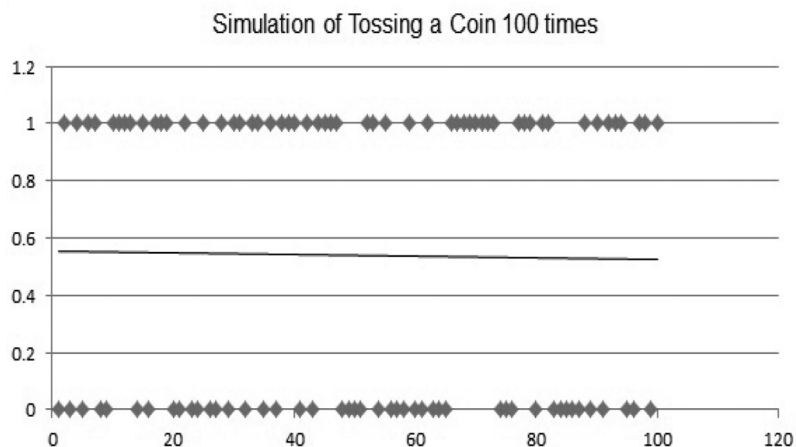
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RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS

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QUESTION 1.

The diagram below shows the result of using Excel to model the tossing of a fair coin 100 times (Have a go at recreating it by letting 0 represent a tail and 1 a head, using `=randbetween(0,1)` and finding the average of the result).



The trend line shown in the diagram represents how the probability of getting a head (or tail) tends to: 0.5

QUESTION 2.

You have been asked the feasibility of conducting a survey of families regarding the number of children they have. The company who wish to conduct the survey are developing a product which should appeal to both boys and girls and would like to know if a sample of 2000 families will give them the information they require to develop a marketing campaign.

Complete the following calculations

$$P(\text{at least 1 boy}) = 1 - P(\text{number of boys}) = 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$$

$$\text{Expected number of families with at least 1 boy} = 2000 \times \left(\frac{15}{16}\right) = 1875$$

$$\text{Expected number of families with 2 boys} = 750$$

$$P(1 \text{ or } 2 \text{ girls}) = P(1 \text{ girl}) + P(2 \text{ girls})$$

$$\text{Expected number of families with 1 or 2 girls} = 1250$$

$$\text{Expected number of families with no girls} = 125$$

QUESTION 3.

You are in charge of the quality control department for a company that manufactures machine tools. The company has received complaints that too many of the sold tools are defective. You have been asked to investigate the quality control sampling procedures.

Complete the following

The binomial distribution is appropriate for these calculations.

The probability of finding exactly 2 defective machine tools from a sample of 10 machine tools, if 10% of all machine tools, produced are defective, is: 0.19

If 5% of all produced machine tools are found to be defective, the probability of finding less than 2 from a sample of 10 is: 0.32

QUESTION 4.

$$\text{The distribution function } F(x) = P(X \leq x) = \begin{cases} 0 & x < a \\ (x - a) / (b - a) & a \leq x < b \\ 1 & x \geq b \end{cases}$$

The equation means 'The function x equals the probability of the random variable $X = 0$ if x has a value less than a , $X = \frac{x-a}{b-a}$ if x has a value greater than or equal to a or if x has a value less than b , $X = 1$ if x has a value greater than or equal to b .'

The equation describes the uniform distribution.

QUESTION 5.

The time taken, in minutes, for candidates to answer some statistics questions is given by:

$$f(x) = \begin{cases} \frac{1}{17-4} & 4 \leq x \leq 17 \\ 0 & \text{otherwise} \end{cases}$$

The area between 4 and 17 is equal to 1.

This area represents the probability of an event occurring.

The probability of a candidate responding between 4 and 10 minutes is: $\frac{6}{13} = 0.46$

QUESTION 6.

The area under the normal distribution curve is: 1

The area between $\pm 3z = 99.7\%$

With a mean of 0 and variance of 1, the standardised variable $z = \frac{X - \mu}{\sigma}$ where X is the random variable, μ is the mean and σ is the standard deviation.

QUESTION 7.

You work for a company that sells sugar. The mean weight of 500 bags is 151 g with a standard deviation of 15 g. Assuming the weights are normally distributed, complete the following:

The number of bags weighing between 120 g and 155 g is: $500 \times 0.6 = 300$

The number that weigh more than 185 g is: $500 \times 0.0107 = 5$

QUESTION 8.

Using the information from question 7, use SPSS to calculate the probabilities and then answer the questions (remember SPSS takes a value equal to or less than the value you specify as the variable).

In order to use SPSS, I require the following information:

the variable: weight

the value of the mean: 151 g

the value of the standard deviation: 15 g

The number of bags weighing between 120 g and 155 g is: 300

The number that weigh more than 185 g is: 5

QUESTION 9.

For this question use Excel to calculate the answers to question 7.

In order to use Excel, I require the following information:
The variable weight
The value of the mean 151 g
The value of the standard deviation 15 g
The value I enter for cumulative is TRUE
The number of bags weighing between 120 and 155 g is: 300
The number that weigh more than 185 g is: 5

MINI PROJECT

You are the CEO of a company that manufactures products for the mining industry. In a competitive tendering process to supply diamond tipped rock drills, you have been asked to sign and agree a supply contract with an international company with an annual turnover of £ 323.5 million. This contract would be a major achievement for your company and would allow you to expand with the creation of more jobs.

Currently the design specification dictates that the drills must be between 48 and 52 mm to be acceptable.

The drills cost £ 25.00 to manufacture.

The quality control manager has given you the following information:

The mean diameter of the drills is 50 mm.

The standard deviation is 2 mm.

The international company has asked you to agree to the following terms:

They will buy the drills for £ 60.00 each.

You are expected to buy back any drills outside of the specification at a cost of £ 65.00.

Initially the contract is for a thousand drills. You need to work out if this contract is going to be profitable and, if it isn't, what you need to do to make it profitable.

And finally...

Is passing a statistics test a random event or can you stack the odds in your favour?