12

COMPARING TWO POPULATIONS

QUESTION 1.

In the box below write down what you think 'associations between variables' means and what 'differences between groups' means.

Associations between variables means: the relationship between the variables within the population.
Differences between groups means: comparison of groups using a variable of interest.

QUESTION 2.

In the box below write down the statistical analysis techniques offered by Excel for two-sample testing. Also give a brief explanation of when each test would be most appropriate.

t-test:paired two-sample for means: *t*-test is used for small samples, typically less than 30.
t-test: two-sample assuming equal variances.
t-test: two-sample assuming unequal variances.
z-test: two-sample for means.
Variances of each sample is known.

QUESTION 3.

Write down the statistical analysis techniques offered by SPSS for comparing means and give a brief explanation of when each test would be most appropriate.

Means. Calculates and compares the means of different samples.
One sample *t*-test. Tests the sample mean against a known value.
Independent samples *t*-test. Used for samples which are independent.
Paired samples *t*-test. Assumes both samples have a normal distribution.
One-way ANOVA. Used to test the hypothesis that several means are equal.

QUESTION 4.

The table below gives the heights of 10 boys and 10 girls. Using a suitable statistical technique, test the assumption that the boys, on average, are taller than the girls.

Boys	Girls
72	67
72	67
79	67
74	62
71	64
66	62
71	64
65	69
67	69
72	66

Complete the following:

Due to the size of the sample, the most appropriate test would be the *t*-test. If the sample had been larger, a more appropriate test would be the *z*-test.

In the box below, write down 2 assumptions you would make concerning this test.

Samples are independent.
Samples are selected from normally distributed populations.

Complete the following:

The null hypothesis H_0: $\mu_b = \mu_g$

The alternative hypothesis H_1: $\mu_b > \mu_g$

A one-tailed test can be used since the alternative hypothesis states that the mean height of the boys is greater than the mean height of the girls.

The test will be conducted with an α value of 0.05

The mean height of the boys is 70.94

The mean height of the girls is 65.67

The standard deviation for the boys is 4.09

The standard deviation for the girls is 2.52

The number of degrees of freedom is 18

The value of the test statistic is 3.46

The *p*-value is 0.001

The results from this analysis show that H_0 can be rejected which means it can be inferred that, on average, boys are taller than girls.

Complete the following:

The analysis can be interpreted to mean that there is a probability of 0.001 that the sample means could be this far apart or further if the null hypothesis was accepted.

QUESTION 5.

200 people who have been diagnosed with a chronic illness have been divided into two groups to test the effectiveness of a new medicine. Group A have been given the medicine and Group B have been given a 'sugar' pill (a placebo). The results show that 75 people from Group A reported beneficial effects from taking the medicine and 65 people from Group B reported similar beneficial effects.

Before the medicine can be sold in large quantities, the manufacturer needs to know if these results are significant. Your task is to select and carry out a suitable test to find out if the medicine is indeed effective.

You decide to test at three levels of significance: (1) 0.01, (2) 0.05 and (3) 0.10.

The null hypothesis is that the observed frequencies are due to chance which implies the medicine is ineffective.

The alternative hypothesis is the medicine is effective.

If p_1 represents the population proportion who reported benefits from using the medicine and p_2 the population proportion who reported no benefits from using the medicine, the hypotheses can be written as:

$$H_0: p_1 = p_2$$
$$H_1: p_1 > p_2$$

p is the estimated average of people who reported beneficial effects in the two sample groups and is given by p_1 and p_2.

In the box below evaluate the formula:

$$\sigma_{p_1 - p_2} = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\sigma_{p_1-p_2} = 0.0648$$

The test statistic $Z = \dfrac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sigma_{p_1-p_2}} = \dfrac{0.75 - 0.65}{0.0648} = 1.54$

Complete the following:

On the basis of a one-tailed test at a 0.01 level of significance, H_0 would be rejected. This means the medicine is ineffective and the results are due to chance.

On the basis of a one-tailed test at a 0.05 level of significance, H_0 would be rejected. This means the medicine is ineffective and the results are due to chance.

On the basis of a one-tailed test at a 0.1 level of significance, H_0 would be accepted. This means the medicine is effective and the results are reliable.

From the above results decide whether a Type I error has been committed. In the box below, write down what a Type I error is, its relevance to this question and explain how it is related to the level of risk the company is prepared to take.

A Type I error is when the results are due to chance and the company concludes they are due to the medicine and proceeds to mass produce an ineffective medicine.
It is relevant to this question because the cost of mass producing the medicine would be enormous.
The level of risk is inferred from the level of significance.

From the above results decide whether a Type II error has been committed. In the box below, write down what a Type II error is, its relevance to this question and explain how it is related to the level of risk the company is prepared to take.

A Type II error is when the medicine does actually help and the company concludes it doesn't.
It is relevant to this question because the company would lose out on making a profit and people with chronic illnesses could have benefited from it.

QUESTION 6.

The company you work for plan to install new equipment which the supplier claims will improve productivity by reducing the design, implement and manufacture times. The table below shows the completion times of 24 engineers, 12 of whom used the existing equipment and the other 12 used the new equipment.

Current equipment	Proposed equipment
300	274
280	220
344	308
385	336

Current equipment	Proposed equipment
372	198
360	300
288	315
321	258
376	318
290	310
301	332
283	263

You have been asked to verify the supplier's claim of improved productivity.

μ_1 is the population mean for the current equipment.

μ_2 is the population mean for the proposed equipment.

Evidence is required to show that μ_2 is less than μ_1.

The null hypothesis can be written in statistical terms as $\mu_1 - \mu_2 \leq 0$ and the alternative hypothesis as $\mu_1 - \mu_2 > 0$.

$H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 - \mu_2 > 0$

You decide to use a level of significance of $\alpha = 0.05$

The sample sizes are $n_1 = 12$ and $n_2 = 12$.

The sample means are $\bar{x}_1 = 325$ and $\bar{x}_2 = 286$.

The sample standard deviations are $s_1 = 40$ and $s_2 = 44$.

The test statistic, $t = 2.27$.

The degrees of freedom, $df = 21$

Using an upper tail test, the *p*-value is between 0.025 and 0.01.

H_0 is therefore rejected since the *p*-value is less than $\alpha = 0.05$.

The statistical evidence suggests that the new equipment does improve productivity.

QUESTION 7.

Currently you are employed as the Marketing Manager for a medium-sized company. You are now looking for an opportunity to earn more money. You are considering moving to either London or Birmingham. Being a competent statistician you have investigated average salaries in both cities.

The table below shows your summary statistics.

Birmingham	London
$n_1 = 40$	$n_2 = 50$
$\overline{x}_1 = £56,100$	$\overline{x}_2 = £59,400$
$s_1 = £6000$	$s_2 = £7000$

Complete the following:

$$H_0: \mu_1 - \mu_2 \geq 0$$

$$H_1: \mu_1 - \mu_2 < 0$$

The value of the test statistic is -2.41.

The p-value is 0.009 which is between 0.005 and 0.01.

The conclusion is that salaries in Birmingham are lower than salaries in London, therefore reject the null hypothesis.

MINI PROJECT

You have been employed by a company who are investigating the best medium for advertising their products. You decided to compare the amount of time people spent watching satellite television and/or terrestrial television. You used a sample of 15 people.

The table below shows the data you collected:

Person	Sat TV	Ter TV
1	22	25
2	8	10
3	25	29
4	22	19
5	12	13
6	26	28
7	22	23
8	19	21
9	21	21
10	23	23
11	14	15
12	14	18
13	14	17
14	16	15
15	24	23

You are expected to produce a report which uses statistical analysis to make a recommendation on the best medium to use for advertising your company's products.

And finally...

At Christmas time, who's the meanest, Scrooge or the Grinch?