PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

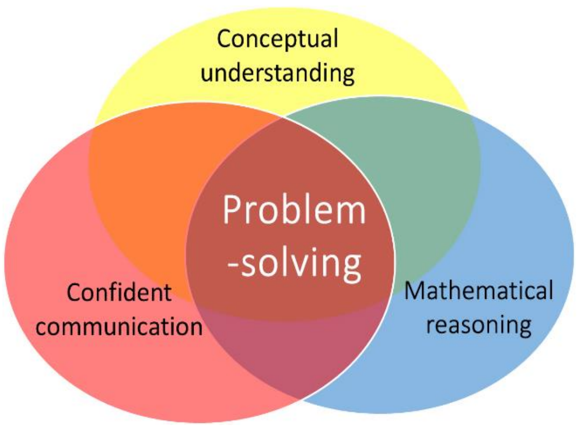
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of ***mastery***, the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner* and the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

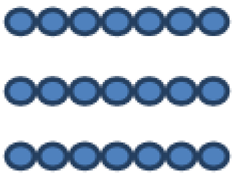
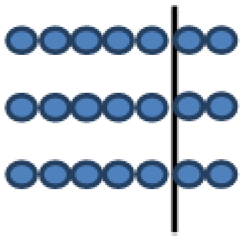
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

11. Mental Strategies for Multiplication and Division **Solve problems involving multiplying and adding, using the distributive law.** This activity provides practical concrete experience of the distributive law. It is important for children to understand how the distributive law works when they encounter written methods of multiplication – and subsequently in algebra.	**<u>The any-times table</u>** The teacher demonstrates using a multiplication table that some may find challenging but others think they are getting to know, say, 7-times. Set out ‘counters’ in several rows of 7 on the teacher’s ‘maths mat’ (for example, by using paper cups on a table). Write down the number sentence for the total each time another row is added:  $1 \times 7 = 7$ $2 \times 7 = 14$ $3 \times 7 = 21$, … Explain that if you did not know the 7-times table you could work it out another way using tables you do know. Partition the columns into 5s and 2s. Then write the number sentences for the total each time another row is added like this:  $1 \times (5 + 2) = 7$, $2 \times (5 + 2) = 14$, $3 \times (5 + 2) = 21$ (and so on).	The children can arrange a multiplication *m x n* as a rectangular array. Children understand that partitioning the array does not change the total, but restructures the array into two smaller component arrays. The total can be expressed as the sum of these arrays. Children can adapt the strategy of using the distributive law to other multiplications, and hence it is possible to construct any times table from multiplication facts they already know.

	Then discuss how the pattern here could also be written as: (1 × 5) + (1 × 2) = 5 + 2 = 7, (2 × 5) + (2 × 2) = 10 + 4 = 14, (3 × 5) + (3 × 2) = 15 + 6 = 21, … So it is possible to work out any fact for the 7-times table from knowing the 5-times and the 2-times tables! Shelley and Rohan now calculate, say, 9 × 7 from their knowledge of 9 × 5 and 9 × 2; and then 12 × 7 from 12 × 5 and 12 × 2. They go on to explore how to use this approach to work out other multiplication tables, for example: 8-times, 12-times, 13-times, 14-times, 15-times, 17-times. This activity could be simplified by starting with the illustration that the 5-times table is made up of the 3-times and 2-times tables.	Children need to write the distributed calculation for each multiplication table fact, to establish the connection between the number sentence and its concrete representation. Ensure the language is also reinforced, for example: '3 × 7 is the same as 3 × 5 added to 3 × 2'.

Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla
Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

ROUNDING TO ESTIMATE

The Cosy Café

<u>Savoury</u>			
Sandwich	$1.95	Baguette	$2.75
Tortilla wrap	$1.45	Pizza slice	$2.10
<u>Sweet</u>			
Scone and jam	$2.10	Cake	$2.90
Shortbread	$1.25	Tiffin	$1.75
<u>Drinks</u>			
Squash	95c	Juice	$1.35
Tea	$1.65	Coffee	$2.15

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Tea	$1.65	Coffee	$2.15

ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

PROBABLE PROPORTIONS

Colour	*Tally*	*Total*

Colour	*Tally*	*Total*