

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

5-6

NATURE OF THE ACTIVITIES SUGGESTED HERE

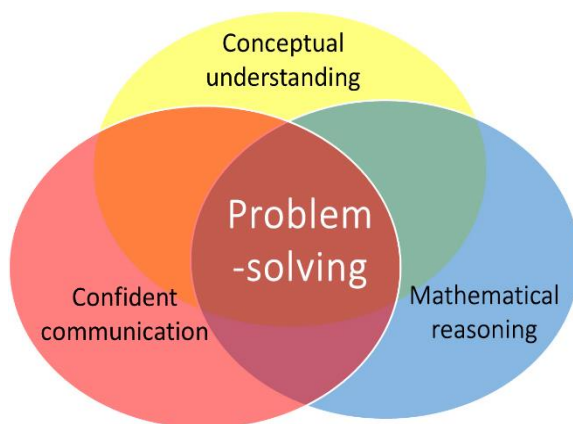
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner*, the **realistic mathematics education** of Freudenthal*, More recently, Lo's* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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11. Mental Strategies for Multiplication and Division

Divide numbers of up to 4 digits by 1 digit, using mental strategies.

The ability to partition numbers is key to the distributive law, which is the underlying strategy for the written methods of division (long and short). This activity is for children to practise solving divisions by 1-digit numbers using mental strategies based upon ad hoc partitioning of the dividend.

This reinforces the inverse relationship with multiplication

Division by partitioning

The teacher models some examples, asking how a number, the dividend could be partitioned (broken up) to make it easier to divide? For example:

$$\begin{array}{r}
 70 \quad + \quad 5 \\
 \hline
 5 \quad \left[\begin{array}{|c|c|} \hline 350 & 25 \\ \hline \end{array} \right.
 \end{array}$$

$375 \div 5 = (350 + 25) \div 5$
 (chosen because we know $35 \div 5 = 7$)
 $= (350 \div 5) + (25 \div 5) = 70 + 5 = 75$

$$\begin{array}{r}
 60 \quad + \quad 3 \\
 \hline
 441 \\
 7 \quad \left[\begin{array}{|c|c|} \hline 420 & 21 \\ \hline \end{array} \right.
 \end{array}$$

$\div 7 = (420 + 21) \div 7$
 (chosen because we know $42 \div 7 = 6$)
 $= (420 \div 7) + (21 \div 7) = 60 + 3 = 63$

$$\begin{array}{r}
 120 \quad + \quad 20 \quad + \quad 4 \\
 \hline
 6 \quad \left[\begin{array}{|c|c|c|} \hline 720 & 120 & 24 \\ \hline \end{array} \right.
 \end{array}$$

$864 \div 6 = (720 + 120 + 24) \div 6$ (chosen because we know $72 \div 6 = 12$)
 $= (720 \div 6) + (120 \div 6) + (24 \div 6) = 120 + 20 + 4 = 144$

Can children adapt the strategy of using the distributive law to division, and see how each part expresses a subset of repeated subtractions of the divisor?

Can children use sketches to illustrate the partitions?

Children need to be confident in scaling multiplication facts by 10 and 100.

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	<p>Charlie and Meena each try the following then compare how they each chose to partition the numbers and check each other's calculation: $552 \div 4$; $276 \div 6$; $660 \div 8$; $747 \div 9$; $1750 \div 7$; $2568 \div 4$; Encourage children to see if there are other ways to partition the same numbers: e.g. $441 \div 7 = (350 + 91) \div 7$. Ask them to explain which partitions they found to be the most helpful or the most efficient and why. Simplify or challenge with simpler/larger numbers for the dividends. Challenge higher attainers with some straightforward 2-digit divisors, for example: $1001 \div 11$; $3756 \div 12$; $2775 \div 25$; or some divisions that yield remainders.</p>	
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Hundreds

Tens

Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

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WORKSHEETS FOR UPPER PRIMARY

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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

WORKSHEETS FOR UPPER PRIMARY

5-6

GO OFY WAYS TO MAKE 1!

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

$\frac{1}{2}$				
$\frac{1}{3}$				
$\frac{1}{4}$				
$\frac{1}{5}$				
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				
$\frac{1}{10}$				

CATALOGUE CHANGES

<u>Barry's Bikes Catalogue</u>		<u>Accessories page</u>	
A. LED light set	\$ 20.59	F. Bike helmet	\$ 14.57
B. Twin mudguards	\$ 16.21	G. 'D' lock	\$ 16.9
C. Tyre pump	\$ 8.34	H. Cycle computer	\$ 9.35
D. Rack	\$ 23.98	I. Basket	\$ 11.93
E. Gel cycle seat	\$ 25.89	J. Puncture repair kit	\$ 2.49

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SPORTS MATHS

<i>Team:</i>										
Event Name:										
Child's Name										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m ²)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

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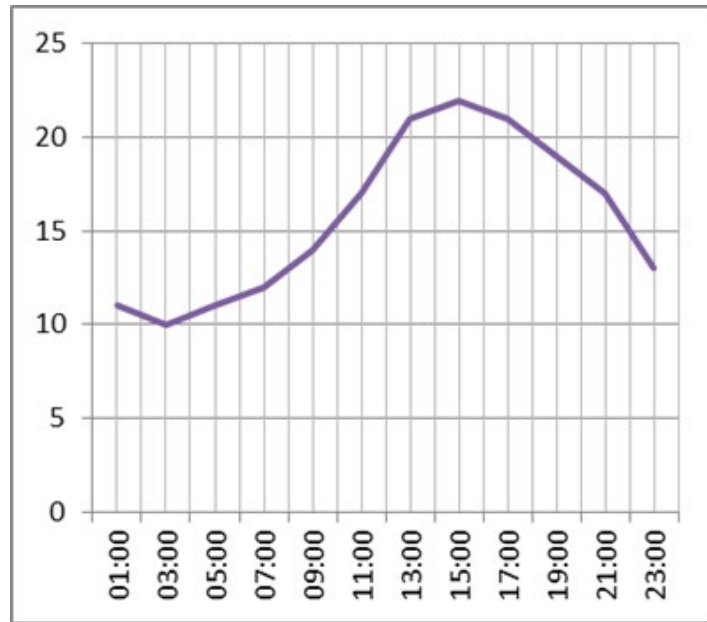
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<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

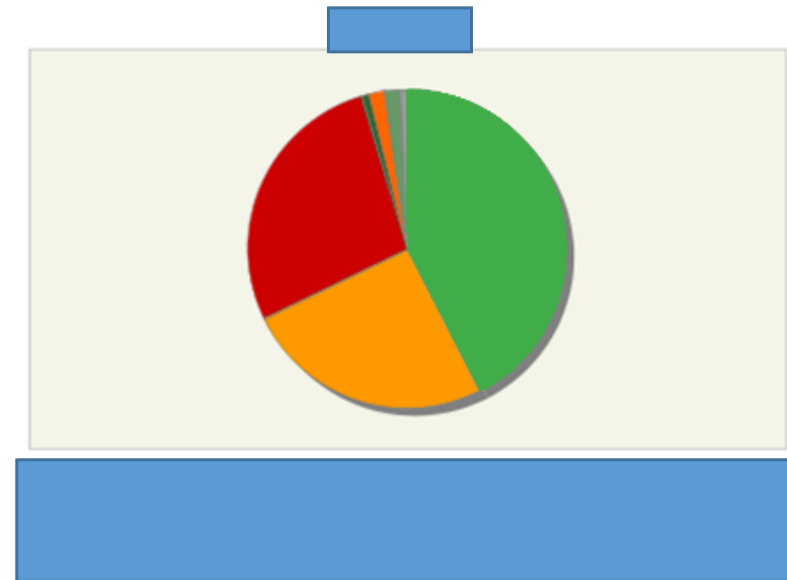
<i>object</i>	<i>diameter (cm)</i>	<i>circum- ference (cm)</i>	<i>Possible relationship?</i>

DATA DETECTIVES

A. Line Graph

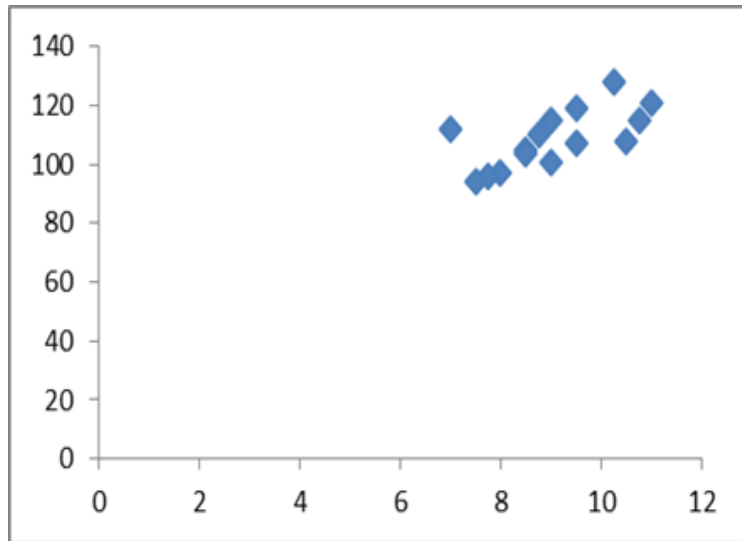


B. Pie Chart

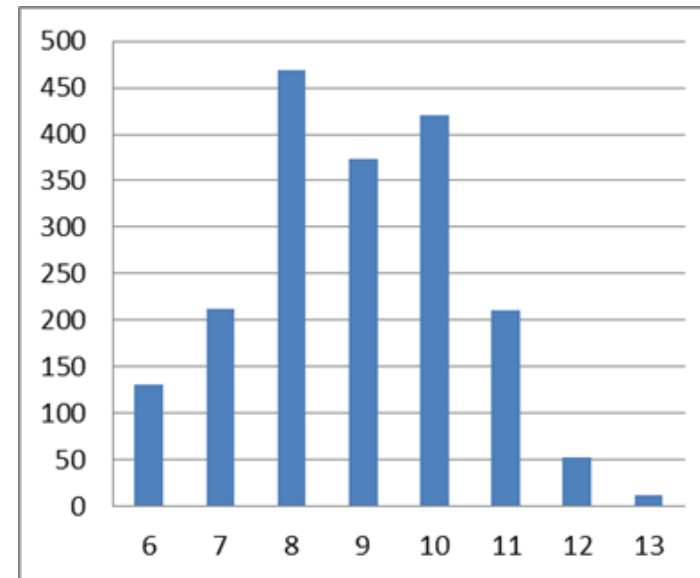


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



TV PROGRAMMES

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	

<i>Programme length</i>	<i>Frequency</i>
<i>Total:</i>	