

PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

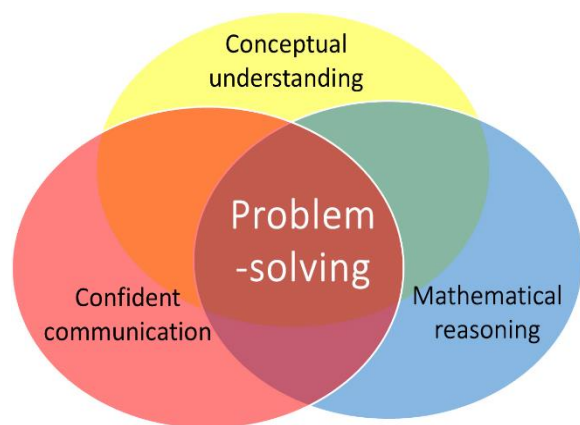
With the surge of interest and sometimes confused interpretations of what is meant by *mastery* in mathematics, different claims have been made about *mastery* and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery*, the development of *deeper conceptual understanding* through a progression in *Concrete-Pictorial-Abstract (CPA)* experiences, first discovered by Bruner*, the *realistic mathematics education* of Freudenthal*. More recently, Lo's* research in the subject of *Variation Theory* has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of *mastery* in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop *rich connections* between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to *deeper mathematical learning*.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

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Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

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Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

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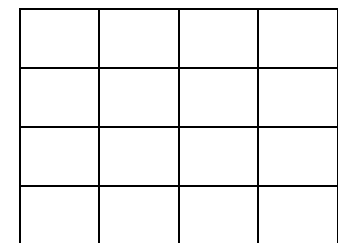
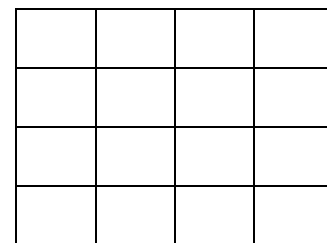
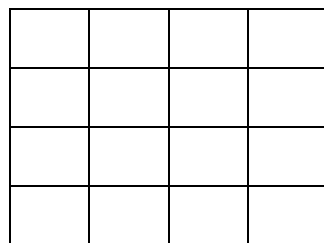
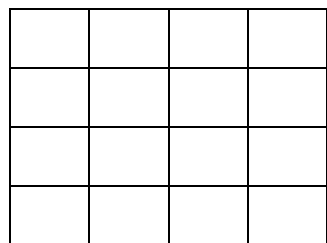
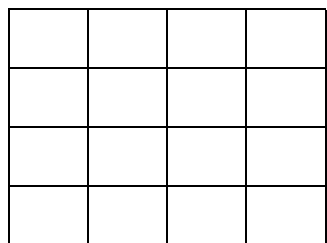
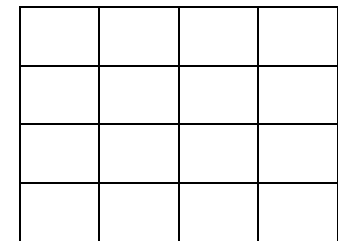
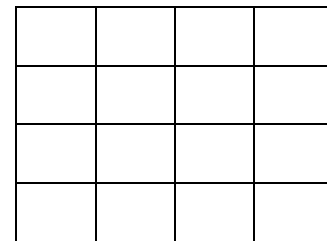
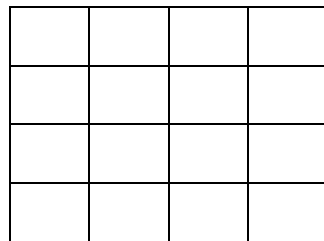
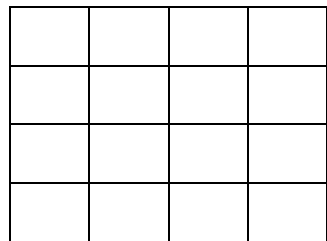
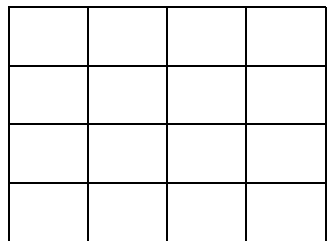
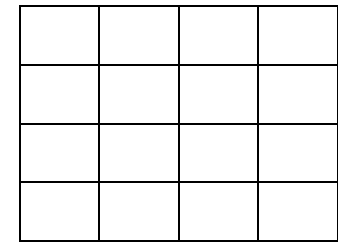
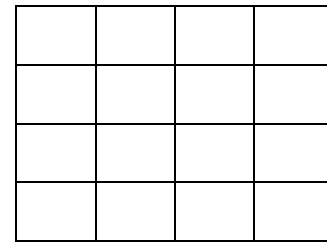
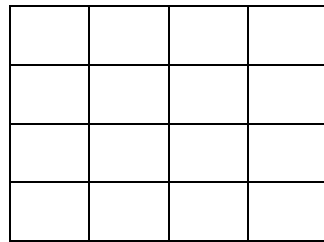
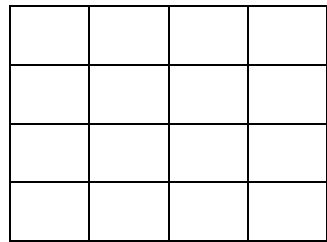
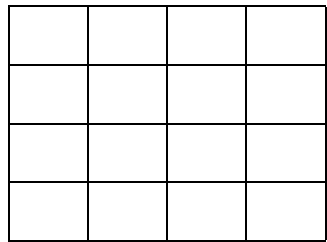
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<p>14. Integers: Positive and Negative</p> <p>Understand ordinal use of number can be extended to negative numbers.</p> <p>Read and interpret temperature scales.</p> <p>Temperature is a real-life context in which we can introduce children to positive and negative whole numbers in a meaningful way.</p>	<p>Comparing temperatures An on-going activity, a regular observation to carry out with the whole class. You will need:</p> <ul style="list-style-type: none">• Large classroom number line: if possible, a vertical line with a range of at least -20 to $(+)50$;• a large, easy-to-see, air-temperature thermometer, and a freezer thermometer;• 'Internet access to find temperatures in major cities in other parts of the world.' <p>Rather than one in-depth specific lesson, it is helpful to introduce children as early as possible to the everyday use of a thermometer to measure the outside temperature as a real-life exercise in mathematical/scientific data-gathering. In Australia temperatures rarely fall below 0°C, so in order to encounter negative numbers in the context of temperature, it is worth comparing the outside air temperature with the temperature of frozen food in the school kitchen.</p> <p>Use the number line to mark the outside air temperature and the temperature in the freezer (usually between -18°C and -20°C). Explain that as the temperature rises, bacteria multiply, spoiling food which becomes unpleasant and also dangerous to eat.</p> <p>Compare the two temperatures on the number line, then together find the difference (<i>comparison</i> structure of subtraction) between them. Use the number line to illustrate differences in an informal, visual way. It is also a very natural way to help them see that you can find the difference between a positive and a negative number, by seeing the visual space between them on the number line. This is very helpful to see the difference of 2° between, say, $+1^{\circ}\text{C}$ and -1°C !</p> <p>As an extension, look up the recorded/forecast temperatures of cities in other parts of the world, particularly those where temperatures below 0°C are common in Winter, for example in Canada or Russia. Together, work out the difference between the local temperature and that of the distant city.</p>	<p>It does not matter at this stage that children may not know what is actually meant by degrees, just that they see that we measure temperature in degrees, just like we measure money in cents, and length in metres.</p> <p>Do the children realise that in this context 0°C does not represent the <i>absence</i> of temperature (or heat)?</p> <p>Do they see that 0 is simply a point in ordering the values of temperature, from which we count numbers positively in one direction (getting warmer) and negatively in the other (getting colder)?</p> <p>Can the children see the difference between two temperatures as the numerical 'space' between them on the number line, regardless of whether the temperatures are positive or negative values?</p>
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Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



WORKSHEETS FOR LOWER PRIMARY

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
 250 g plain flour
 65 g butter
 25 g castor sugar
 150 ml milk
 2 eggs
 140 ml whipped cream
 500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
 250 g plain flour
 65 g butter
 25 g castor sugar
 150 ml milk
 2 eggs
 140 ml whipped cream
 500 g cherry pie filling

Colour:		
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ODDS AND EVENS

Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

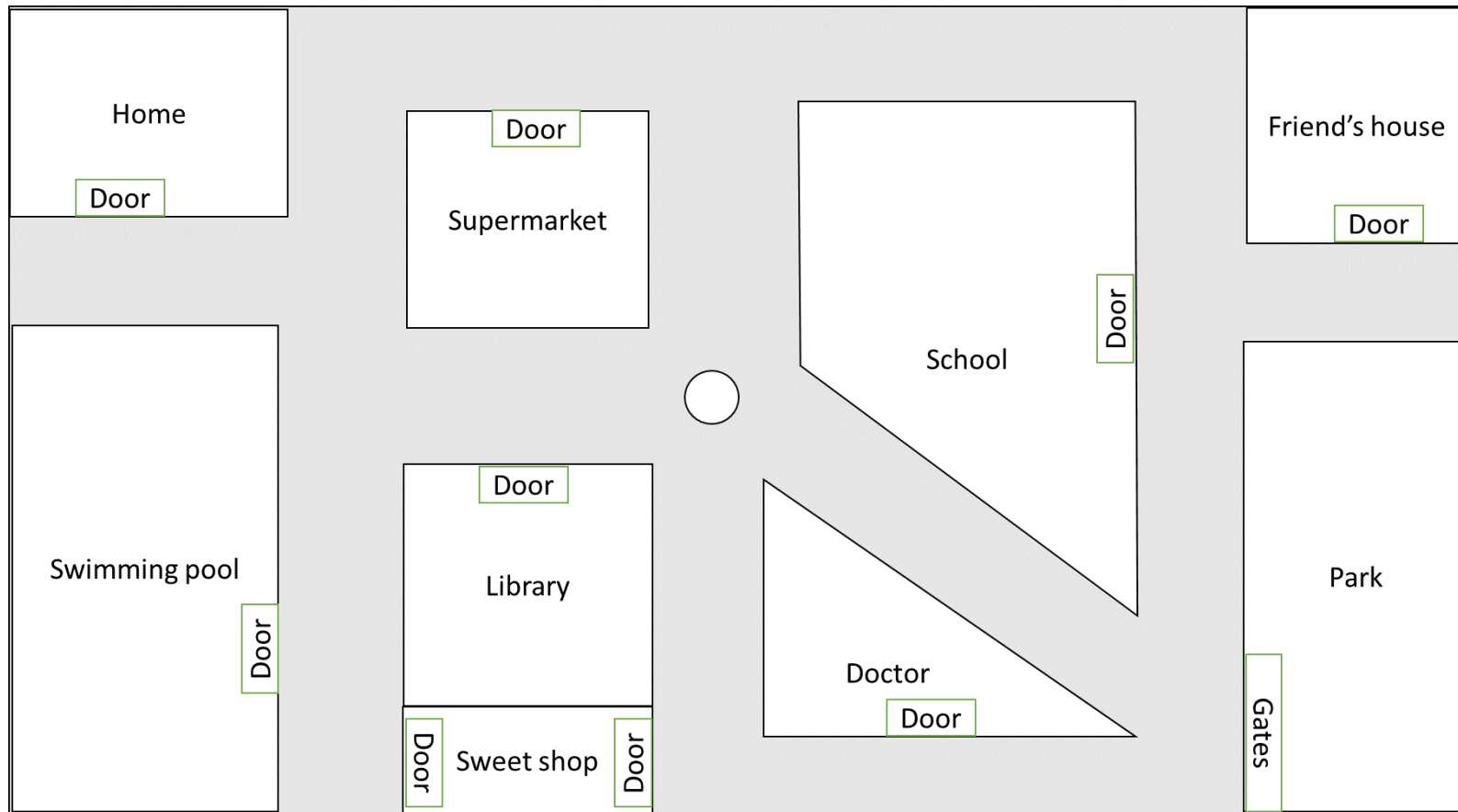
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

List of squares I have fired at:

ROBOTS



SHAPE SORTER

TRAFFIC SURVEY

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>

<i>Vehicle</i>	<i>Tally</i>	<i>Total</i>