PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN UPPER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

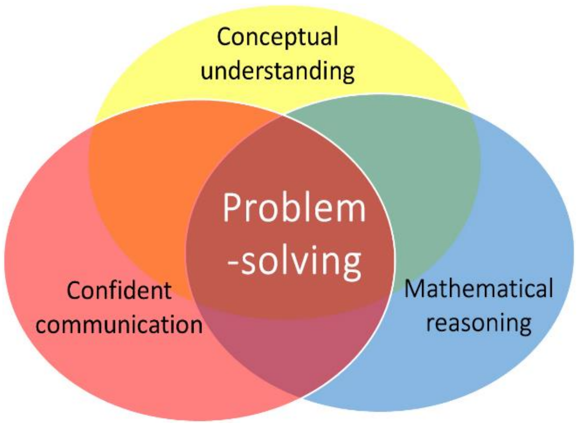
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia,including Bloom's* theories of ***mastery***, the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of concrete apparatus and visual images provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

15. Fractions and Ratios

Identify, name and write equivalent fractions of a given fraction.

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

This activity is to help children understand how equivalent fractions can be substituted during addition and subtraction of fractions.

Goofy ways to make 1! Children work in pairs. Each pair will need:

- A blank table for completing equivalent fractions (see worksheets):
- Fraction wall (to explore combinations visually and manipulatively, if helpful).

$^{1}/_{2}$				
$^{1}/_{3}$				
$^{1}/_{4}$				
$^{1}/_{5}$				

First Meena and Charlie complete a blank table of the 4 equivalent fractions for each of the first 10 unit fractions: $^{1}/_{2}$, $^{1}/_{3}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{1}/_{6}$, $^{1}/_{7}$, $^{1}/_{8}$, $^{1}/_{9}$ and $^{1}/_{10}$:

$^{1}/_{2}$	$^{2}/_{4}$	$^{3}/_{6}$	$^{4}/_{8}$	$^{5}/_{10}$
$^{1}/_{3}$	$^{2}/_{6}$	$^{3}/_{9}$	$^{4}/_{12}$	$^{5}/_{15}$
$^{1}/_{4}$	$^{2}/_{8}$	$^{3}/_{12}$	$^{4}/_{16}$	$^{5}/_{20}$
$^{1}/_{5}$	$^{2}/_{10}$	$^{3}/_{15}$	$^{4}/_{20}$	$^{5}/_{25}$
$^{1}/_{6}$	$^{2}/_{12}$	$^{3}/_{18}$	$^{4}/_{24}$	$^{5}/_{30}$
$^{1}/_{7}$	$^{2}/_{14}$	$^{3}/_{21}$	$^{4}/_{28}$	$^{5}/_{35}$
$^{1}/_{8}$	$^{2}/_{16}$	$^{3}/_{24}$	$^{4}/_{32}$	$^{5}/_{40}$
$^{1}/_{9}$	$^{2}/_{18}$	$^{3}/_{27}$	$^{4}/_{36}$	$^{5}/_{45}$
$^{1}/_{10}$	$^{2}/_{20}$	$^{3}/_{30}$	$^{4}/_{40}$	$^{5}/_{50}$

Next model, with the children, how the number 1 can be the sum of two or more fractions and then how any of these fractions could be replaced by an equivalent fraction of a different denomination. For example:

$$1 = {}^{1}/_{2} + {}^{1}/_{2} = {}^{1}/_{2} + {}^{2}/_{4} = {}^{1}/_{2} + {}^{3}/_{6} = {}^{1}/_{2} + {}^{2}/_{6} + {}^{1}/_{6} = {}^{1}/_{2} + {}^{2}/_{6} + {}^{2}/_{12}$$

Now challenge Meena and Charlie to find as many different ways to make 1 as they can – the more creative, the better! Tell them to check their solutions by writing down the simplest equivalent fraction under each fraction in the sum. For example:

$1 = {}^{3}/_{15} + {}^{5}/_{25} + {}^{1}/_{10} + {}^{3}/_{18} + {}^{4}/_{12}$ and $1 = {}^{1}/_{5} + {}^{1}/_{5} + {}^{1}/_{10} + {}^{1}/_{6} + {}^{1}/_{3}$ … and so on.

Simplify or challenge some by reducing or increasing the denominators involved.

Do the children have a sufficient understanding of ratio yet, that it is possible to exchange equivalent fractions as these maintain the same ratio between numerator and denominator?

Do the children have enough experience of substituting equivalent fractions?

A concrete fraction wall will help here if they have not. It will enable them to swap and substitute difference pieces physically.

Hundreds	Tens	Ones

DIVISION BY MULTIPLYING

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
Kilometres/litre	6	8	11	12	14

Division by Multiplying

The Nissota Car Manufacturer

Model of car	SuperExec	GazGuy	Missive	Yazz	Wego
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100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

GO OFY WAYS TO MAKE 1!

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

$^1/_2$				
$^1/_3$				
$^1/_4$				
$^1/_5$				
$^1/_6$				
$^1/_7$				
$^1/_8$				
$^1/_9$				
$^1/_{10}$				

CATALOGUE CHANGES

Barry's Bikes Catalogue		Accessories page	
A. LED light set	$ 20.59	F. Bike helmet	$ 14.57
B. Twin mudguards	$ 16.21	G. 'D' lock	$ 16.9
C. Tyre pump	$ 8.34	H. Cycle computer	$ 9.35
D. Rack	$ 23.98	I. Basket	$ 11.93
E. Gel cycle seat	$ 25.89	J. Puncture repair kit	$ 2.49

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SPORTS MATHS

Team:

Event Name: / Child's Name										

PAINTING WALL AREAS

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

Section of wall	Length (m)	Height (m)	Area of section (m^2)
Total area to be painted:			

HOW MANY DEGREES?

Shape	Angles	Sum of angles

Shape	Angles	Sum of angles

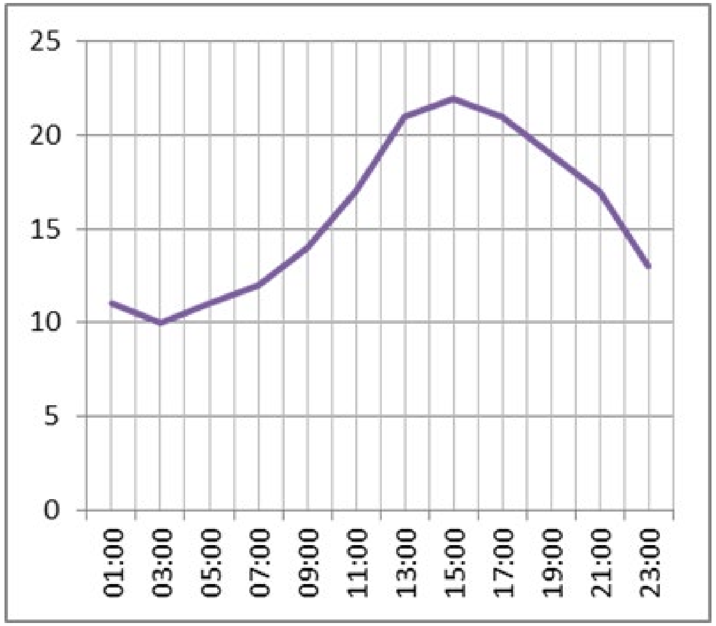
THE LIFE OF PI

object	diameter (cm)	circum-ference (cm)	Possible relationship?

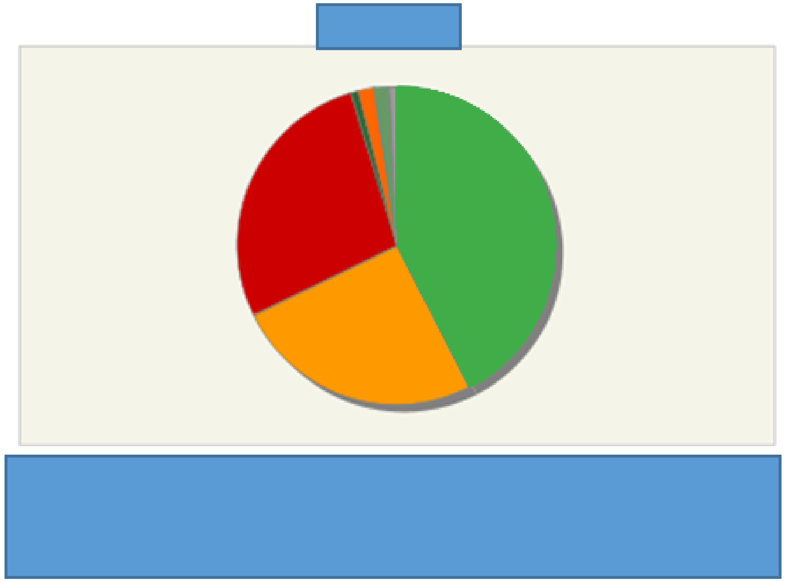
object	diameter (cm)	circum-ference (cm)	Possible relationship?

DATA DETECTIVES

A. Line Graph

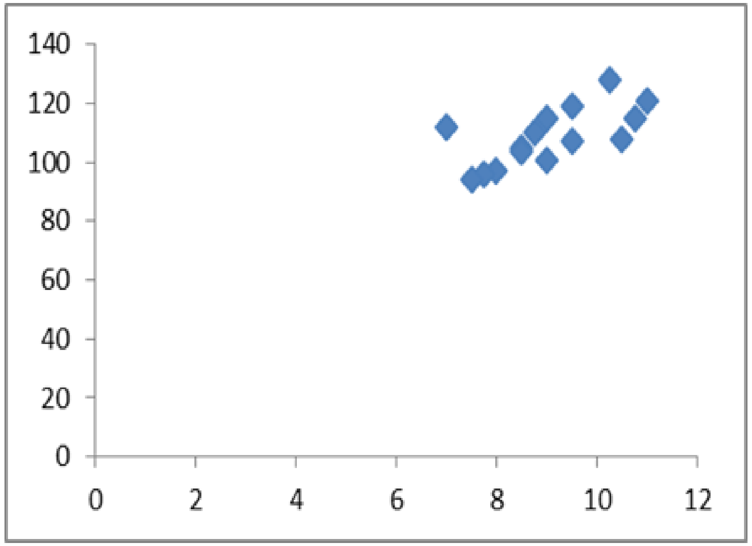


B. Pie Chart

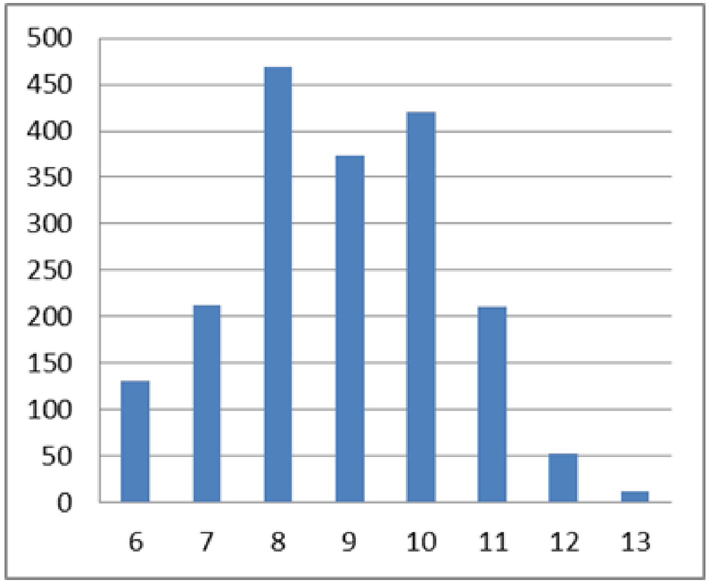


DATA DETECTIVES

C. Scatter graph



D. Bar Chart



TV PROGRAMMES

Programme length	Frequency
Total:	

Programme length	Frequency
Total:	