PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

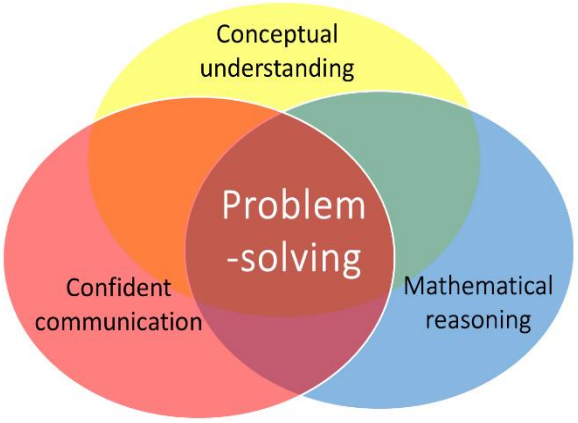
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery,* the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*. More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

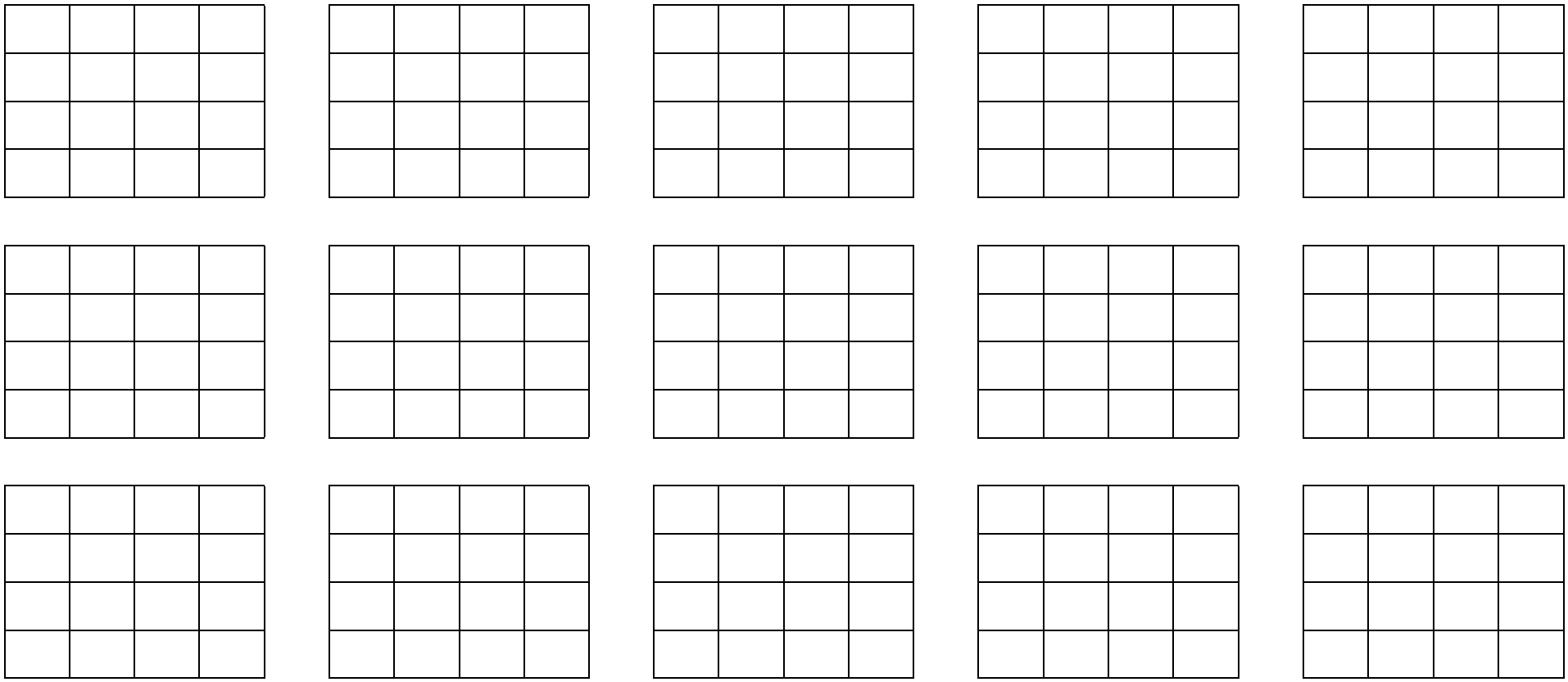
Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

16. Decimal Numbers and Rounding **The use of rounding in the context of money.** **Combine amounts to make a particular value.** **Find different combinations of coins that equal the same amounts of money.** To enable children to develop a proper understanding of rounding, they will need to understand that the nearest number to round towards depends on the context of the problem. They can begin to see this in real contexts at an early age, for example when paying for things they wish to buy. In this problem, they have only a few coins from which to choose and they need to offer the smallest amount necessary to pay for the intended item.	**Pay the least amount** Children to work in groups of 4, possibly with some adult support. They will need: • A number of items labelled with different prices: 3c, 6c, 8c, 11c, 13c, 17c, 19c; • Three purses or bags each containing one each of the following coins only: 5c, 10c and 20c; • One small tray of mixed coins and/or a number line to assist in calculating the change; • Counters to award to the 'winners' of each round. First model how we may not have the exact money when we pay for something, and how to find the smallest value coin to offer for an item. In each case, first decide which value it should round to: for example, an item costing 3c is rounded (up) to 5c, an item costing 6c is also rounded (down) to 5c. The item costing 8c would be rounded up to 10c. In each case select the appropriate coin to pay for the item – 5c or 10c. For higher value items, e.g. 17c, first agree that it must be rounded to 15c, then identify the appropriate smallest combination of coins 10c + 5c. Emily is the shopkeeper with the items for sale. Starting with the cheapest of these, she places any ***two*** items item in front of her. Luke, Kasia and Nathan select the smallest amount that they would need to offer to pay for the combined price of these items. • For example, Emily chooses two items 6c and 8c. All should agree that the total to pay is 14c, which rounds up to 15c. All should all choose 10c + 5c. Those who offer the correct amount from their coins are winners and may take one counter each. • Note that the children do not hand over the coins, only show what they would offer. They keep the coins to use again for subsequent items. No change is actually given so that the coins in each purse or bag remain the same. • Luke passes his bag to Emily, and he now becomes the shopkeeper. Luke chooses the 13c item and the 11c item, so the children should calculate the combined price of 24c, then round this (up) to 25c to pay. They should offer 20c + 5c to pay. To extend the exercise, vary the coins in the purses to enable different combinations of coins to be used for the same amount, e.g. to provide 25c as 5 x 5c or (2 x 10c) + 5c. For higher attainers extend the amounts up to 50c or allow children to overpay and the shopkeeper to give change, e.g. pay 20c for a 15c price, and receive 5c in change.	Do the children correctly recognise the *values* of different coins and add these *values* correctly? Do the children realise that it is always possible to offer an amount more than the price required, but we are trying to find the ***nearest*** possible amount to the cost of the item?

Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

Colour:		
	in every	
	in every	
	in every	

Colour:		
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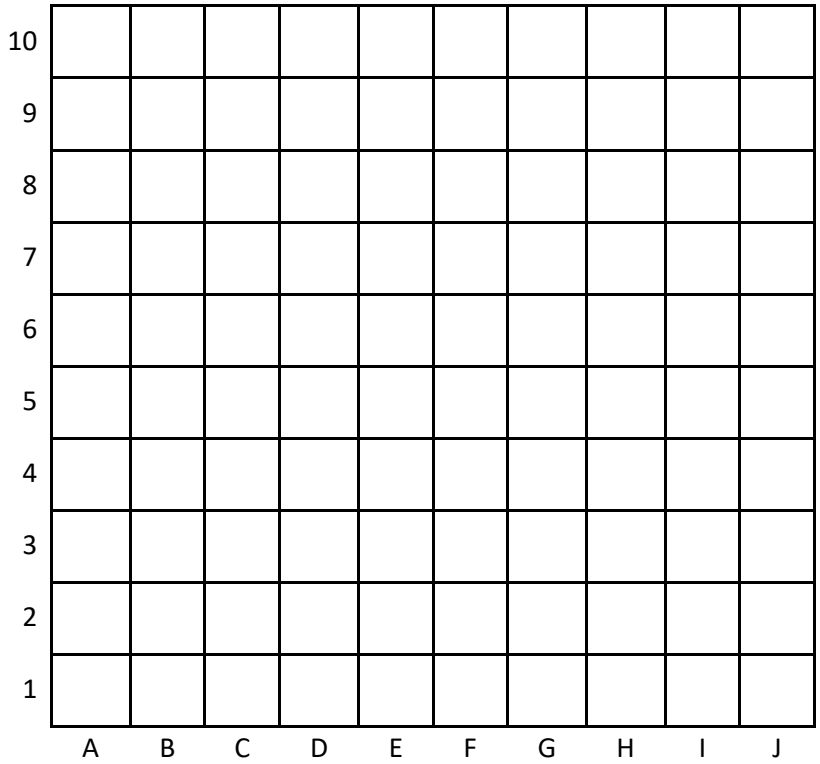
ODDS AND EVENS

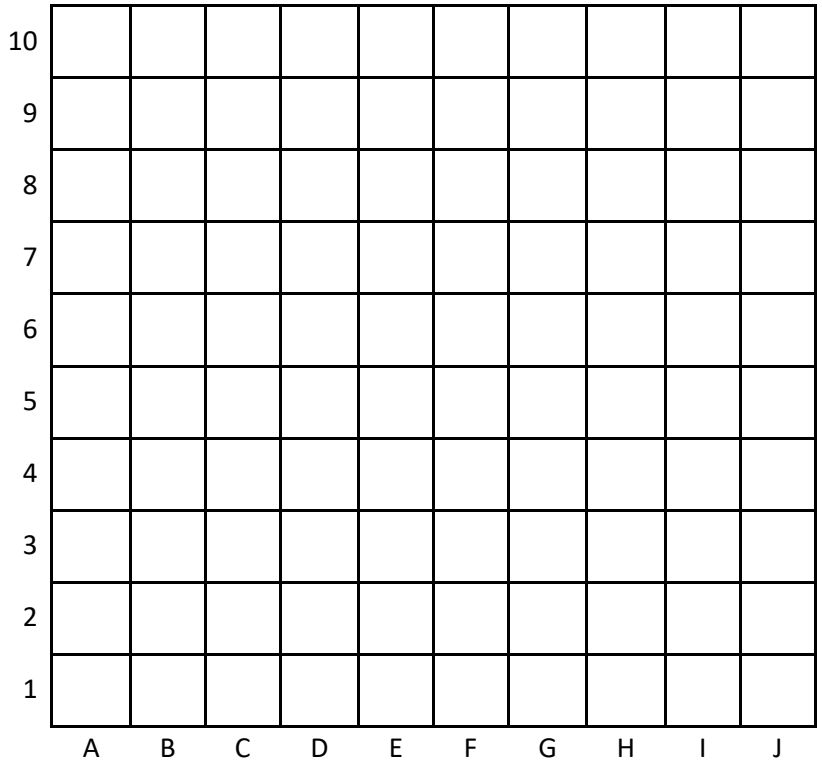
Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

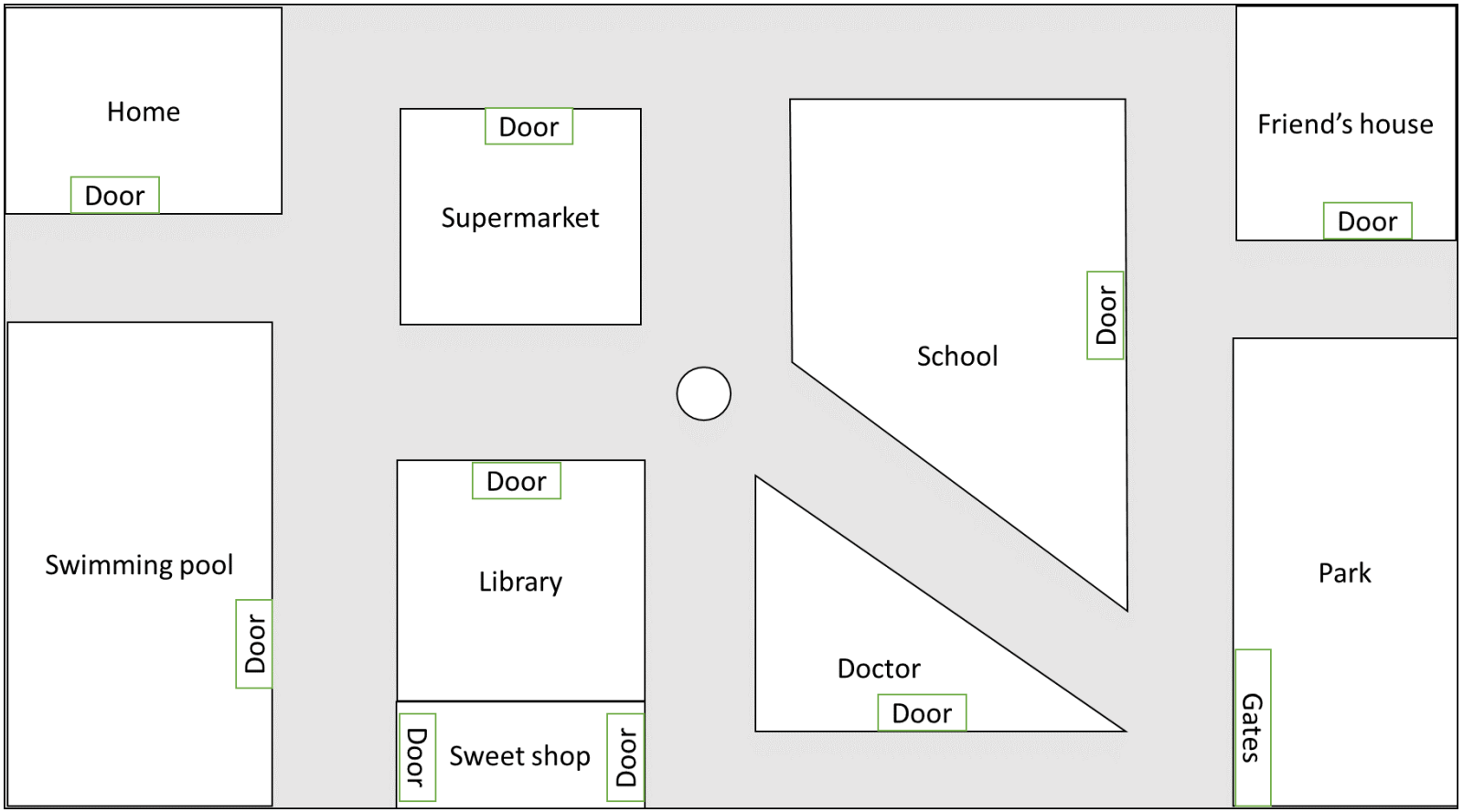


List of squares I have fired at:



List of squares I have fired at:

ROBOTS


Home
Door
Door
Supermarket
School
Door
Friend's house
Door
Swimming pool
Door
Door
Library
Park
Doctor
Door
Door
Sweet shop
Door
Gates

SHAPE SORTER

TRAFFIC SURVEY

Vehicle	*Tally*	*Total*

Vehicle	*Tally*	*Total*