

# PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

3-4

## NATURE OF THE ACTIVITIES SUGGESTED HERE

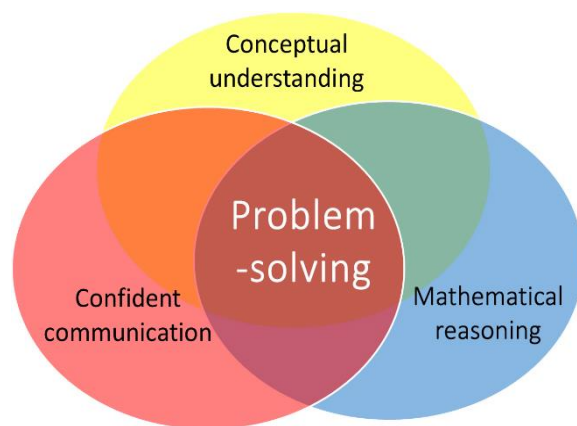
With the surge of interest and sometimes confused interpretations of what is meant by **mastery** in mathematics, different claims have been made about **mastery** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA\* and TIMSS\* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's\* theories of **mastery**, the development of **deeper conceptual understanding** through a progression in **Concrete-Pictorial-Abstract (CPA)** experiences, first discovered by Bruner\* and the **realistic mathematics education** of Freudenthal\*, More recently, Lo's\* research in the subject of **Variation Theory** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of **mastery** in mathematics (e.g. Drury\*) also argue that teaching and learning must focus on enabling children to develop **rich connections** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to **deeper mathematical learning**.

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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## NATURE OF THE ACTIVITIES SUGGESTED HERE

In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of **concrete apparatus** and **visual images** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit [www.MathematicsMastered.org](http://www.MathematicsMastered.org)

### \*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

# PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

<p><b>16. Decimal Numbers and Rounding</b></p> <p><b>Estimate to check answers to a calculation.</b></p> <p><b>Round decimals to the nearest whole number.</b></p> <p><b>Estimate and calculate money in pounds and pence.</b></p> <p>When dealing with sums of money combining both \$s and cent, it is very useful to be able to estimate the final amount in order to be sure that a mistakes is not made. It's often helpful to round amounts to the nearest dollar. This enables a reasonably good estimate to be made simply from adding up a number of whole dollars.</p>	<p><b>Rounding to estimate</b> After demonstration, children work in pairs. They will need:</p> <ul style="list-style-type: none"> <li>Menu of items to select in a café, for example, the Cosy Café menu:</li> </ul> <div data-bbox="609 534 1505 1050" style="background-color: #c0392b; color: white; padding: 10px; text-align: center;"> <p><i>The Cosy Café</i></p> <table border="0"> <tr> <td colspan="4"><u>Savoury</u></td> </tr> <tr> <td><i>Sandwich</i></td> <td><i>\$1.95</i></td> <td><i>Baguette</i></td> <td><i>\$2.75</i></td> </tr> <tr> <td><i>Tortilla wrap</i></td> <td><i>\$1.45</i></td> <td><i>Pizza slice</i></td> <td><i>\$2.10</i></td> </tr> <tr> <td colspan="4"><u>Sweet</u></td> </tr> <tr> <td><i>Scone and jam</i></td> <td><i>\$2.10</i></td> <td><i>Cake</i></td> <td><i>\$2.90</i></td> </tr> <tr> <td><i>Shortbread</i></td> <td><i>\$1.25</i></td> <td><i>Tiffin</i></td> <td><i>\$1.75</i></td> </tr> <tr> <td colspan="4"><u>Drinks</u></td> </tr> <tr> <td><i>Squash</i></td> <td><i>95c</i></td> <td><i>Juice</i></td> <td><i>\$1.35</i></td> </tr> <tr> <td><i>Tea</i></td> <td><i>\$1.65</i></td> <td><i>Coffee</i></td> <td><i>\$2.15</i></td> </tr> </table> </div> <p>In this activity Shelley and Rohan practise how to make suitable estimates before they add amounts of money, and see how close the estimate was to their calculation.</p> <p>First model, with the class, how to choose a helpful or 'friendly' number to which they can round an amount. For example, when adding say, \$1.20 and \$3.65: \$1.20 is nearer \$1 than \$2, while \$3.65 is nearer \$4 than \$3. Thus, a reasonable estimate would be \$1 + \$4 = \$5. The addition of the actual amounts would of course be \$4.85.</p> <p>Now Shelley and Rohan each choose two items from the menu.</p>	<u>Savoury</u>				<i>Sandwich</i>	<i>\$1.95</i>	<i>Baguette</i>	<i>\$2.75</i>	<i>Tortilla wrap</i>	<i>\$1.45</i>	<i>Pizza slice</i>	<i>\$2.10</i>	<u>Sweet</u>				<i>Scone and jam</i>	<i>\$2.10</i>	<i>Cake</i>	<i>\$2.90</i>	<i>Shortbread</i>	<i>\$1.25</i>	<i>Tiffin</i>	<i>\$1.75</i>	<u>Drinks</u>				<i>Squash</i>	<i>95c</i>	<i>Juice</i>	<i>\$1.35</i>	<i>Tea</i>	<i>\$1.65</i>	<i>Coffee</i>	<i>\$2.15</i>	<p>Do they round up or down individual items to the nearest whole dollar correctly?</p> <p>Do children understand that an estimate is not intended to be exact, but should be close enough to the amount to enable a reliable estimate?</p> <p>Do they realise that the maximum difference between the actual and rounded amount in each case can be no more than 50c? In the case of adding two items do they see that the final answer should be less than \$1 different (50c + 50c) to the estimated answer?</p> <p>How should they round an amount which is, say \$1.50? Does it makes sense to round this or not?</p>
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- They first round the price up or down as appropriate to write their estimate of the total in each case, then carry out the addition, using whatever informal or formal written method they are currently practising;
- If there is a difference of more than \$1 between their estimate and the answer they need to check the calculation, possibly by an inverse calculation (subtraction), and then check their estimate again;
- They repeat this for different combinations from the menu.

The estimates and calculations can be extended by selecting three or more items from the menu. It is also possible to round items to the nearest 50c, so that \$1.65 is rounded to \$1.50. This will help make estimates more precise.

Hundreds	Tens	Ones

## THE CHEAP COACH COMPANY

*The Cheap Coach Company*

Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					
	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla

## 100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

## ROUNDING TO ESTIMATE

<i>The Cosy Café</i>		
<u>Savoury</u>		
Sandwich	\$1.95	Baguette \$2.75
Tortilla wrap	\$1.45	Pizza slice \$2.10
<u>Sweet</u>		
Scone and jam	\$2.10	Cake \$2.90
Shortbread	\$1.25	Tiffin \$1.75
<u>Drinks</u>		
Squash	95c	Juice \$1.35
Tea	\$1.65	Coffee \$2.15

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Squash	95c	Juice \$1.35
Tea	\$1.65	Coffee \$2.15



## ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

## PROBABLE PROPORTIONS

<i>Colour</i>	<i>Tally</i>	<i>Total</i>

<i>Colour</i>	<i>Tally</i>	<i>Total</i>