PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN LOWER PRIMARY

1-2

NATURE OF THE ACTIVITIES SUGGESTED HERE

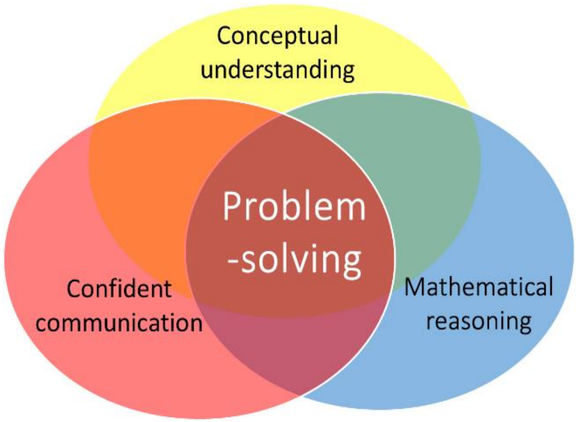
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points that appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of *Mastery,* the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner*, the ***realistic mathematics education*** of Freudenthal*. More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding, which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

18. Proportionality and Percentages

Recognise proportions within a set of objects or quantity.

To enable children to develop a solid understanding of ratio and proportion, it is important to gain practical experience in which they see how this is maintained with concrete examples. This activity can be done with children in lower primary as it demonstrates proportionality and equivalence without using symbolic notation. It is a prior activity to *Creative Cuboids* recommended for Year 3 and Year 4 in activities for Chapter 15.

<u>**Proportional patterns**</u> Working in pairs, children will need the following:

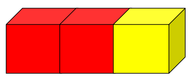
- Multilink cubes in several colours;
- Prepared sheets with tables to record results (see worksheets):

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

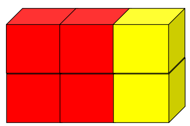
Colour:		
	in every	
	in every	
	in every	

First, model an example of a row of 3 multilink cubes, choosing from 2 colours:



Say that for this pattern there are ‘2 red cubes in 3 cubes’ and ‘1 yellow cube in 3 cubes’.

Now explain that you want to keep using the same pattern, in another row. How many cubes would there be altogether? How many red? How many yellow? Demonstrate this:



Now use the language of proportion to describe the pattern: ‘2 red cubes *in every* 3, will be 4 red cubes *in every* 6. 1 yellow cube *in every* 3, will be 2 yellow cubes *in every* 6’.

Do the children see the idea of ***ratio*** in the continuity between ‘2 ***in every*** 3’, and ‘4 ***in every*** 6’, etc.?

Do the children to see that the step increase in each column is by the same number as in the first row every time?

Do they make connections with counting up groups of 2s, or 3s?

Can the children predict how many they will have of each colour in total, before they add the next row?

Now ask the children to predict how many cubes of each colour would be needed altogether if there is another row.

Demonstrate this, and express it: '6 red cubes *in every* 9. 3 yellow cubes *in every* 9.' Write this into a copy of the tables:

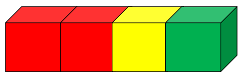
Colour: red		
2	in every	3
4	in every	6
6	in every	9

Colour: yellow		
1	in every	3
2	in every	6
3	in every	9

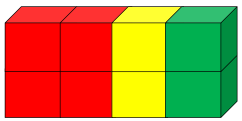
Colour:		
	in every	
	in every	
	in every	

Ask the children if they can see any patterns in the columns of numbers.

Now Luke and Emily each create their own patterns based on a row of 4 multilink cubes, choosing from 3 colours. For example, Luke creates:



then:



… and so on.

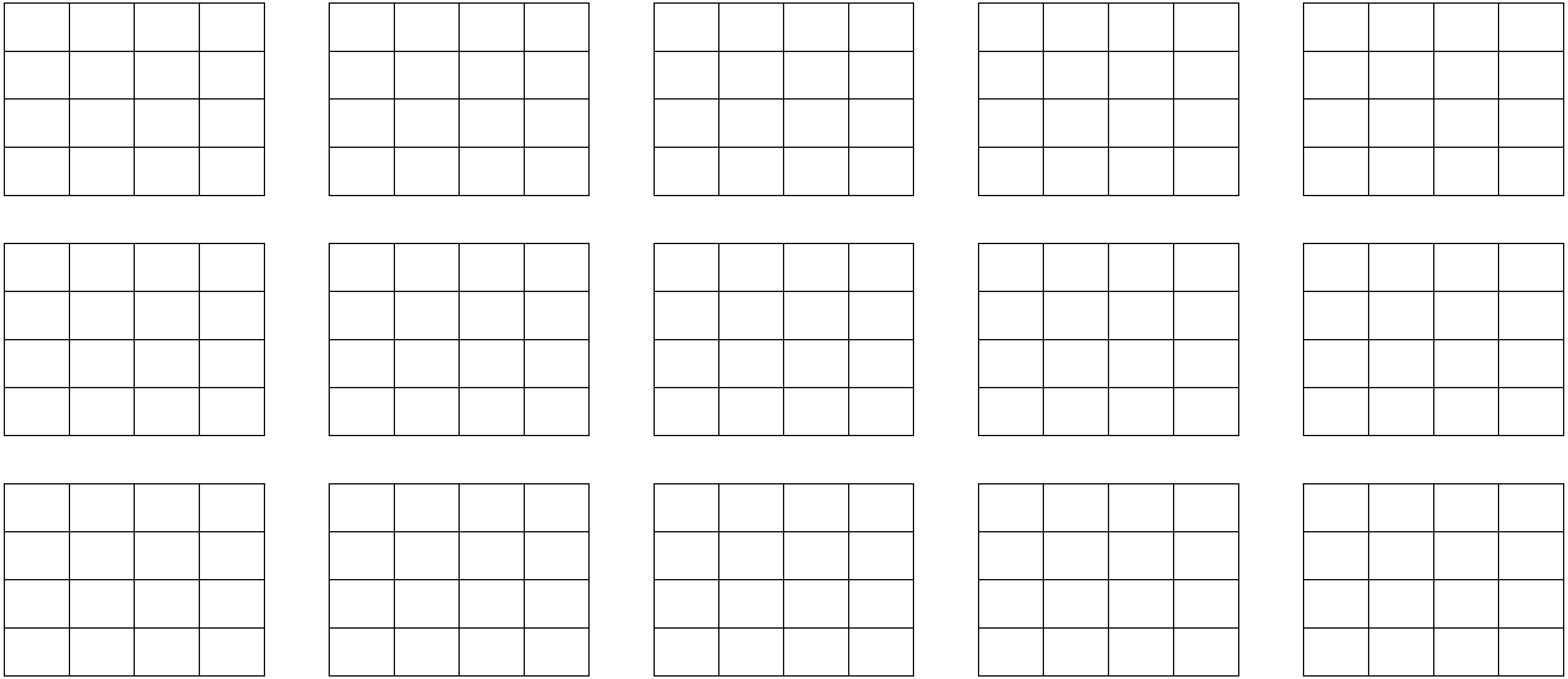
The children each complete their own tables, then compare their patterns, to see what they notice about how the numbers increase in each column and any relationships between them.

After this, they can explore rows of 5 choosing from 3 colours, and perhaps rows of 6 choosing from 3 colours.

Hundreds	Tens	Ones

SEEING SQUARES

Cut into separate grids – 1 for each child



100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

COOK'S CHERRY SHORTCAKES

Cook's cherry shortcakes (for ten children)
250 g plain flour
65 g butter
25 g castor sugar
150 ml milk
2 eggs
140 ml whipped cream
500 g cherry pie filling

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65 g butter
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Colour:		
	in every	
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Colour:		
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	in every	

Colour:		
	in every	
	in every	
	in every	

Colour:		
	in every	
	in every	
	in every	

ODDS AND EVENS

Odd	Even

Odd	Even

SIMPLE BATTLESHIPS

	A	B	C	D	E	F	G	H	I	J
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										

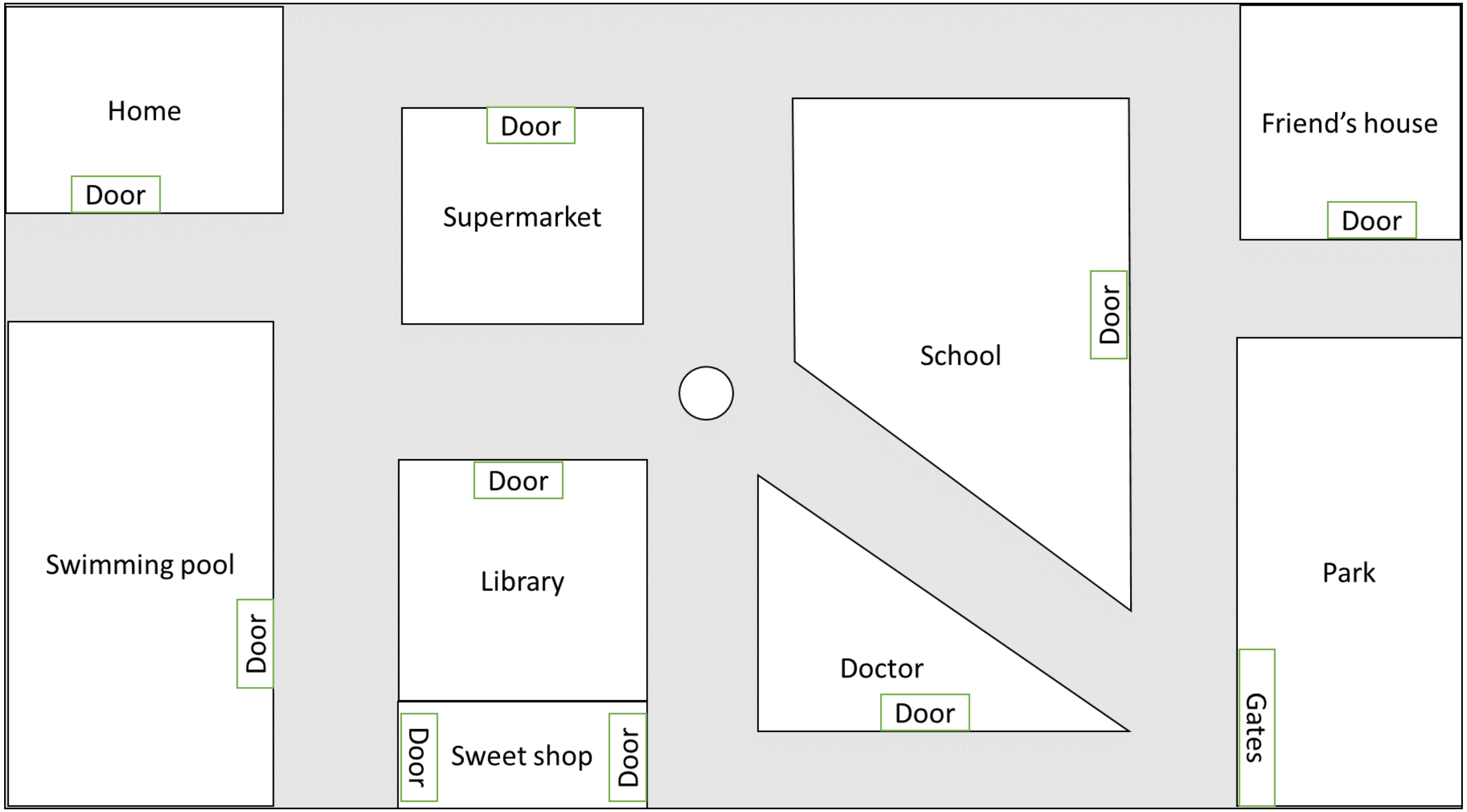
List of squares I have fired at:

	A	B	C	D	E	F	G	H	I	J
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										

List of squares I have fired at:

WORKSHEETS FOR LOWER PRIMARY

ROBOTS



SHAPE SORTER

TRAFFIC SURVEY

Vehicle	Tally	Total

Vehicle	Tally	Total