PROBLEM-SOLVING EXAMPLES FOR DEVELOPING MASTERY IN PRIMARY

NATURE OF THE ACTIVITIES SUGGESTED HERE

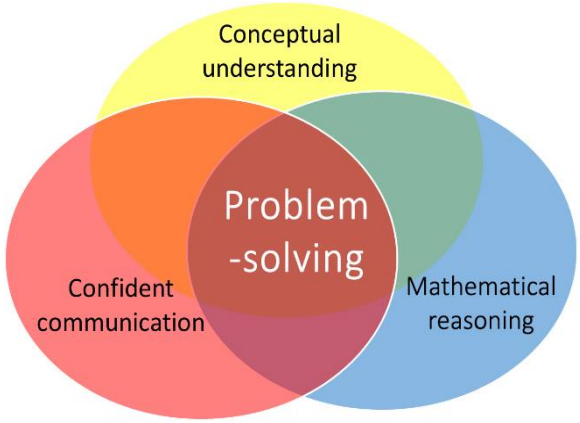
With the surge of interest and sometimes confused interpretations of what is meant by ***mastery*** in mathematics, different claims have been made about ***mastery*** and what is required. The efficacy of different aspects of mastery approaches to learning mathematics in the primary school, as demonstrated by higher performing jurisdictions in East Asia, as measured by PISA* and TIMSS* have been questioned and challenged.

However, there are some essential points which appear to be in common when examining different approaches.

Research in mathematics education, already known for many years, has been used by curriculum developers and educationalists in East Asia, including Bloom's* theories of ***mastery,*** the development of ***deeper conceptual understanding*** through a progression in ***Concrete-Pictorial-Abstract (CPA)*** experiences, first discovered by Bruner* and the ***realistic mathematics education*** of Freudenthal*, More recently, Lo's* research in the subject of ***Variation Theory*** has been prominent in exploring how to plan learning for understanding through small steps in conceptual and procedural variation when teaching.

All of these principles have informed the sample of activities presented here. Proponents of ***mastery*** in mathematics (e.g. Drury*) also argue that teaching and learning must focus on enabling children to develop ***rich connections*** between different facets of their mathematical experience and learning. The diagram below shows how these facets are all inter-related, and how teaching to connect these is crucial to ***deeper mathematical learning.***

Hence, the activities suggested here are designed to promote the following:



- practical activity manipulating concrete resources where possible;
- working in pairs or groups to encourage the confident use of the language of mathematics through explanation and reasoning with other children;
- ensuring that formal written arithmetic develops from secure experiences with concrete, visual and mental understanding of the manipulation of number and the arithmetic operations;
- solving problems (or by playing games) with the potential for a useful or pleasing result;
- opportunities for finding more than one acceptable result, which children can compare and discuss through collaboration or (guided) peer-assessment.

There is an expectation that discussion and exploration of misconceptions or errors is a healthy and productive feature of the classroom and that children are encouraged to explain their thinking and listen to others.

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In some of the activities, one may argue that a written sheet of exercises could be given to produce similar results. However, the use of ***concrete apparatus*** and ***visual images*** provides a medium for discussion and helps to establish a rich conceptual understanding which is often insufficiently developed through an abstract engagement with written exercises alone. In other cases, children are using equipment to generate the problem to be solved, so can be more engaged in its solution.

Where it is suggested pairs or groups of children work together, the groups may of course be varied to suit the teacher's own judgement. For example, in a game intended for pairs, an odd number of children can be accommodated by a changing combination of 2 vs 1.

To make it more accessible when reading the description of the activities, children's names have been used to identify the sequence of interactions between learners working in pairs or groups.

For every activity, it is paramount that the teacher teaches by modelling the activity with the class, so that children see and imitate what they need to do. Simply providing a written instruction sheet or verbal series of instructions is insufficient for the children to understand and engage with most activities.

Each activity has suggestions for extending or simplification. The expectation is that each can be explored comprehensively within one classroom lesson of 45 minutes or more.

For more information about improving the capacity for teaching and learning mathematics in the primary school, visit www.MathematicsMastered.org

*References

Bloom, B. S. (1971) 'Mastery learning', in J. H. Block (ed.), *Mastery Learning: Theory and Practice*, New York: Holt, Rinehart & Winston.

Bruner, J. S. (1960) *The Process of Education*, Cambridge, Mass.: Harvard University Press.

Drury, H. (2014) *Mastering Mathematics*, Oxford: Oxford University Press.

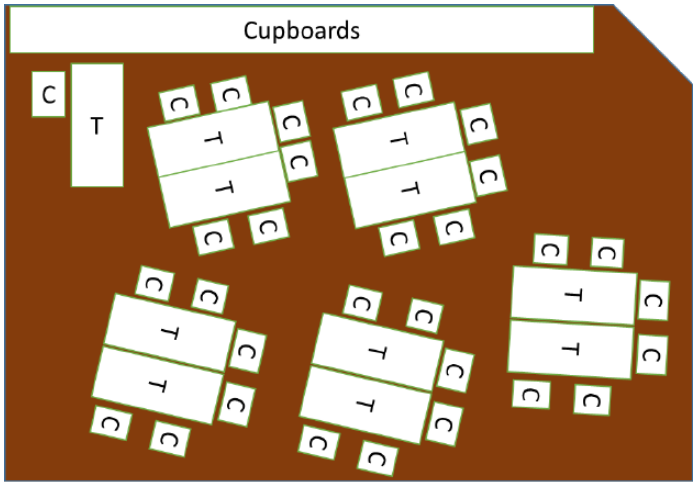
Freudenthal, H. (1991) *Revisiting Mathematics Education – China Lectures*, Dordrecht: Kluwer.

Lo, M. L. (2012) *Variation Theory and the Improvement of Teaching and Learning*, Gothenburg studies in educational sciences 323, Gothenburg University.

Programme for International Student Assessment (PISA), [Organisation for Economic Cooperation and Development (OECD)]

Trends in International Mathematics and Science Study (TIMSS), [International Association for the Evaluation of Educational Achievement (IEA)]

18. Proportionality and Percentages **Solve simple measure problems involving fractions and decimals.** **Find the effect of dividing a two- or three-digit number by 10.** **Convert between different units of measure [m to cm].** **Round a length to the nearest cm.** This activity reinforces the idea of division or fractions as a 'scaling/ratio' structure (see Chapter 10), and uses this to maintain the proportions of room furniture within the dimensions of a room plan. A plan view of a room is used by architects, designers,	**Classroom Designers** Groups of 4 co-operate to make one plan. Each group will need: • Metre sticks or tape measures; • 1 large sheet of sugar paper e.g. 1 m^2, upon which a 1:10 scale outline of the classroom walls has been drawn. Use a smaller section of the classroom if it is too large; • Sheets of 1 cm^2 paper; • Blu Tack. The children work for *Classroom Designers*. The company has been asked to create a new classroom design, but first they must make a plan view of the existing room layout. The children need to make smaller 2D shapes for the different pieces of furniture in the room, so that things are in proportion with one another. First show the children the 1:10 scale outline of the classroom plan view on the piece of sugar paper. Next model how to make a 1:10 scale plan view outline of an item of furniture, for example a chair. N.B. • Simplify objects to rectangles, circles or other simple 2D shapes wherever possible, using the item's maximum width and length. • With the children, scale these measurements by a tenth and round each resulting figure to the nearest cm. • Then using 1 cm^2 paper, to make it easier to count by squares, mark and cut out a scaled rectangle for the chair. • Write on the shape which piece of furniture it is, and use some Blu Tack to place it in an appropriate place within the outline of the room on the sugar paper.	Do the children see the plan as representing a view of the classroom from above? Although the resulting plan will not be exact, do the children see the need for proportionality in scale drawings? Do the children see that every item has been 'shrunk' proportionally to the others? Do the children see how this can be applied to their own needs: a plan of their bedroom furniture, for example?

kitchen planners and others to work out how and where everything fits into the space. Show some examples – for example, plans of new homes designs from internet estate agents.	Rohan, Shelley, Alice and Marek identify and measure different pieces of furniture, to create and place different shapes to create their plan of the classroom. For example:  When the children have created their plan of the actual classroom, they can be challenged to plan a new classroom design by moving those pieces of furniture which are not fixed items. The children will need to be aware of the scaled measurements needed to allow access and movement between items of furniture. The children's designs can also make for an interesting display.	

Hundreds	Tens	Ones

THE CHEAP COACH COMPANY

The Cheap Coach Company

Bathurst													
Bowral–Mittagong		259											
Forster–Tuncurry													
Goulburn	148												
Mudgee			325										
Muswellbrook			354	252	248								
Newcastle		324			345								
Orange		56	282	530			309	392					
St Georges Basin		323	87	505			453	355	341				
Sydney					195		247		254				
Taree			420										
Ulladulla		363							380			558	
Wollongong			80				326	240					
	Batemans Bay	Bathurst	Bowral–Mittagong	Forster–Tuncurry	Goulburn	Mudgee	Muswellbrook	Newcastle	Orange	St Georges Basin	Sydney	Taree	Ulladulla

100-SQUARES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

The Cosy Café			
Savoury			
Sandwich	$1.95	Baguette	$2.75
Tortilla wrap	$1.45	Pizza slice	$2.10
Sweet			
Scone and jam	$2.10	Cake	$2.90
Shortbread	$1.25	Tiffin	$1.75
Drinks			
Squash	95c	Juice	$1.35
Tea	$1.65	Coffee	$2.15

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ESTIMATING IN PRACTICE

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

Item	Estimated mass	Predicted order of increasing mass	Actual mass when measured	Actual order of increasing mass

PROBABLE PROPORTIONS

Colour	*Tally*	*Total*

Colour	*Tally*	*Total*